Evaluating solvency versus efficiency performance and different forms of organization and marketing in US property—liability insurance companies

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Abstract

Solvency is a primary concern for regulators of insurance companies, claims paying ability is a primary concern for policyholders, and return on investment is a primary concern for investors. These interests potentially conflict, and the decision-makers for the firm must trade off one concern versus another. Here we examine the efficiency of insurance companies via data envelopment analysis using solvency, claims paying ability, and return on investment as outputs and using a financial intermediary model for the insurance company. The effect of solvency on efficiency is then examined. These efficiency evaluations are further examined to study stock versus mutual form of organizational structure and agency versus direct marketing arrangements, which are examined separately and in combination.

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1. Introduction

This paper uses the non-parametric properties of DEA (data envelopment analysis) coupled with rank order statistics (which are distribution free) to study the relative efficiency of the different organization structures (cross classified by their marketing distribution systems) used by US property and liability insurance companies. Here we focus on the relations (if any) between solvency and efficiency, because, as noted in the literature,
this can present a potential for conflict in that solvency is a prime concern for regulatory agencies and consumers whereas efficiency tends to be of prime concern to management and investors. See, e.g., BarNiv and Raveh (1989), Barrese (1990) and Brockett et al. (1994).

A question of interest is whether, and the extent to which, these two objectives interact. In particular, an issue we address is whether solvency conditions, such as might be imposed by regulatory agencies, interfere substantially with efficiency and, if so, what are the magnitudes of these reductions in efficiency, which input or output variables are most affected, and how might they be ameliorated. In an attempt to better understand and clarify these results, we go further to dichotomize our results by organizational form into mutual versus stock companies to allow for the possibility that those two organization types of structures might be differentially affected by imposed solvency constraints. Rationales for potential differences in the efficiency and solvency relationship between stock and mutual insurers can be derived from the different incentive structures inherent in the two types of organizational forms; in stock companies return on shareholder investment dominates incentives, while solvency and claims paying ability considerations can dominate considerations of mutual insurance company decision makers. Possible effects on mutual versus stock types of organization are also intrinsically intertwined with agency versus direct sales type of marketing systems. One can also conjecture efficiency differences between these marketing distribution systems that have substantially different cost structures, and are correlated with product line differences (commercial versus personal lines) wherein solvency constraints could have different priorities and regulatory and social imperatives. This further breakdown allows for the possibility of differential interactive effects between these types of organizations and marketing systems, which are generally used in the provision of insurance services.

2. Background

The Savings and loan crises of the 1980s aroused public concern over the stability of all types of financial institutions in the United States and this spilled over into insurance as one institution that functions as a financial intermediary. This caused lawmakers to open inquiries and to hold hearings on insolvency cases that occurred in the insurance industry. The ostensible objective was to forestall the occurrence of solvency crises in that industry and, thereby, avoid the risk of situations like those that arose in the savings and loan and banking industries.

Table 1 can provide perspective. This table shows that a total of 257 insurance companies became insolvent during the period 1980–1990. We use this as a justification for the interest aroused in this topic during the period (1989) covered by our data. Changes in the economy were, of course, responsible for many of the problems experienced by the insurance industry during this period. For instance, the high interest rates offered in the late 1970s and 1980s brought a substantial influx of capital to the insurance marketplace. Competition within the industry resulted in price-cutting and, when interest rates fell, insurance companies did not have sufficient cash flow to support all of their obligations. In any case, insolvency within the insurance industry became (and continues to be) of major interest and the identification of potentially troubled firms has become a major regulatory research objective. See Brockett et al. (1997a,c) and Harrington and Niehaus (1998).

Table 1
Property and liability insurers insolvency*

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Insolvencies</td>
<td>5</td>
<td>9</td>
<td>8</td>
<td>11</td>
<td>26</td>
<td>49</td>
<td>25</td>
<td>19</td>
<td>35</td>
<td>38</td>
<td>32</td>
</tr>
</tbody>
</table>

This brings up the issue of how “solvency” might be identified and measured before a crisis occurs. Unlike other measures such as ROI (return on investment) the identification and measurement of solvency involves a complex of considerations. For instance, the National Association of Insurance Commissioners (NAIC) developed a system called the Insurance Regulatory Information System (IRIS) which was designed to provide an early warning system for insurer insolvency.

We might similarly use these IRIS ratios for our measure of insolvency, with “insolvency” being indicated when a certain number of these ratios are deemed to be out of line. Although complex and awkward to use, we might nevertheless have used this system except that the adequacy of IRIS for predicting troubled insurers was investigated empirically and found to be somewhat arbitrary and not strongly predictive. In part, it was found to yield results that were only a little better than would be secured from the toss of an unbiased coin. The NAIC has subsequently constructed a new system called the FAST. However, until very recently (after the writing of this paper) these variables and techniques were kept confidential by the NAIC and hence could not be examined or used for measuring solvency in this paper. To characterize solvency propensity, we therefore turned to research done by Brockett et al. (1994) which reported results from uses of neural network artificial intelligence methods for providing early warning signals for insurer insolvency. This was done since these results strongly dominate the IRIS system as well as all of the statistical methods examined for solvency monitoring and early warning. More details on this approach are supplied later so, for the moment, we simply note that this neural network approach iteratively utilizes a series of logistic regressions to provide a “solvency propensity score” which is in the form of “probability of remaining solvent” for each of the 1524 companies included in our sample for the year of our study (1989).

The frequency distribution for these solvency score are skewed to the left (as shown in Fig. 1). However, this skew is not sufficiently serious to warrant replacing a use of averages in favor of other statistical measures such as modes or medians. In fact, as this figure shows, the bulk of the frequencies (80% = 26% + 40% + 14%) occur in a range where the solvency scores are between 0.6 to 0.9 which is well represented by an average of 69% and a standard deviation of 12% for this distribution. See the column headed “Total” in Table 2.

Table 2 breaks this solvency score index down further by organizational form (stock versus mutual) and by the marketing distribution systems employed. Regarding the marketing system variable, we designate two types in the usual way, e.g., as given by Rejda (1998, pp. 503–505). The first type (agency) consists of independent agents, which can represent more than one insurer. We distinguish this from the other types which we refer to as “direct”. The “direct” form of distribution directly represents a single insurer and includes exclusive agencies (which can represent

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5 The IRIS system identifies a property casualty insurer for further regulatory evaluation if four of eleven computed financial ratios for the company lie outside a given “acceptable” range of values. IRIS thus uses univariate (one at a time) tests in its approach and the acceptable range of values is determined such that, for any given univariate ratio measure, only approximately 15% of all firms have results outside of a specified “acceptable” range.

6 A detailed description of IRIS (Insurance Regulation Information System) and its use as an early warning system may be found in Brockett and Cooper (1990), and summarized in Brockett et al. (1994).

7 A description of the FAST variables is available in Cummins et al. (1999).
only one company or one company group), direct writers (wherein the sales person is an employee of the insurer) and direct response (a company that sells through mail order or other mass media such as the internet). As can be seen from Table 2, the solvency proxy variable values generally do not differ much by marketing distribution type. In fact, these ratios are all almost identical for stock/agency versus stock/direct as well as mutual/agency and mutual/direct.

### 3. DEA Model—RAM Version

The DEA (Data Envelopment Analysis) model we use has been selected for its suitability to the problems that are of interest here. This model is a variant of the “additive DEA model” first presented in Charnes et al. (1985). This model is presented in Cooper et al. (1999) and possesses some very desirable properties vis a vis other DEA models. To make the paper self-contained, and because it is new, we briefly develop this model. We start with the following formulation (whose terms are defined below):

$$\max \quad \sum_{i=1}^{m} (s_i^- / R_i^-) + \sum_{r=1}^{s} (s_r^+ / R_r^+)$$

subject to

$$x_{i0} = \sum_{j=1}^{n} x_{ij} \lambda_j + s_i^- , \quad i = 1, \ldots, m,$$

$$y_{r0} = \sum_{j=1}^{n} y_{rj} \lambda_j - s_r^+ , \quad r = 1, \ldots, s,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$0 \leq \lambda_j, s_i^-, s_r^+ \text{ all } i, j, r.$$ (1)

Here $j = 1, \ldots, n$ indexes the entities being compared and contrasted for efficiency determination. In the vast literature of DEA these entities are referred to as decision making units (DMU). These could be an actual decision conducting entity such as a company, but they might also be an entity such as a marketing system if the focus of the diagnosis is to compare and contrast the efficiency of different marketing systems. At any rate, the DMUs are responsible for converting input resources into output characteristics as may be appropriate to the class of entities being examined for efficiency. These inputs and outputs take the form of given (observed) quantities or quantitative characteristics represented as follows in this model:

$$x_{ij} = \text{amount of input } i = 1, \ldots, m \text{ used by DMU}_j,$$

$$y_{rj} = \text{amount of output } r = 1, \ldots, s \text{ yielded by DMU}_j,$$

while $x_{i0}, y_{r0}$ represent the corresponding input and output values for DMU$_0$, the DMU whose efficiency to be evaluated by a suitable choice of the variable $0 \leq \lambda_j, s_i^-, s_r^+$ in (1).

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**Table 2**

<table>
<thead>
<tr>
<th>Type</th>
<th>Stock/agency</th>
<th>Stock/direct</th>
<th>Mutual/agency</th>
<th>Mutual/direct</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.3500</td>
<td>0.3500</td>
<td>0.3500</td>
<td>0.3500</td>
<td>0.3500</td>
</tr>
<tr>
<td>Max</td>
<td>0.9100</td>
<td>0.9100</td>
<td>0.6687</td>
<td>0.9100</td>
<td>0.9100</td>
</tr>
<tr>
<td>Average</td>
<td>0.7037</td>
<td>0.6911</td>
<td>0.6687</td>
<td>0.6827</td>
<td>0.6939</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.1275</td>
<td>0.1127</td>
<td>0.1363</td>
<td>0.1279</td>
<td>0.1280</td>
</tr>
<tr>
<td>Var</td>
<td>0.0162</td>
<td>0.0127</td>
<td>0.0186</td>
<td>0.0164</td>
<td>0.0164</td>
</tr>
</tbody>
</table>
We note that any choice of \(x_i's \) which results in \(x_{0i} > \sum_{j=1}^n x_{ij} \lambda_j \) is interpreted to mean that the empirical evidence shows that with some (convex) combination of inputs other DMUs could have improved this input in amount \( s_i^- = x_{0i} - \sum_{j=1}^n x_{ij} \lambda_j > 0 \) without worsening any other input or output. In this instance DMU_0 is identified as having used an excessive amount of this input. Similarly \( s_i^- = \sum_{j=1}^n y_{ij} \lambda_j - y_{0i} \) identifies a shortfall in output characteristic \( r \) in amount \( s_i^- \), by reference to the outputs recorded for this same convex combination of other DMU_j's.

In each case, these non-zero “slack” variable values provide an estimate of the input excesses and the output shortfalls that could have been improved without worsening any other input or output. Hence the results are wholly data dependent in that they do not require any prior specification of weights (or other methods) for determining the relative importance of the different input and outputs being considered. These evaluations are also non-parametric. No explicit specification of functional relations between inputs and outputs (such as a Cobb–Douglas production function) is required to arrive at these results.

Returning to (1), we see that the objective is oriented toward maximizing these slack values. This ensures that all such inefficiencies are identified. Hence we have a test for efficiency which we can state in terms of the following:

Definition of Efficiency: DMU_0 is fully efficient if and only if all slack variables are zero at an optimum in (1). This indicates that for this DMU_0 no other DMU (or combination of DMUs) can produce the same output with smaller inputs, or use the same set of inputs to produce more output.

We now use the objective of (1) to show that it yields a measure that comprehends all of the thus identified inefficiencies and their magnitudes. To obtain this measure in “dimensionless form” we define \( R_i^- \) and \( R_r^+ \) to be the range defined by the lowest and highest observed values in each row—viz.,

\[
R_i^- = \bar{x}_{ij} - x_{ij}, \quad i = 1, \ldots, m,
\]

\[
R_r^+ = \bar{y}_{jr} - y_{jr}, \quad r = 1, \ldots, s,
\]

where \( \bar{x}_{ij}, \bar{y}_{jr} \) are the highest and \( x_{ij}, y_{jr} \) are the lowest of the \( j = 1, \ldots, n \) values in rows \( i \) and \( r \), respectively. 

Since \( \sum_{j=1}^n \lambda_j = 1 \), as given in (1), we also have

\[
0 \leq s_i^- = x_{0i} - \sum_{j=1}^n x_{ij} \lambda_j \leq \bar{x}_{ij} - x_{ij} = R_i^-, \quad i = 1, \ldots, m,
\]

\[
0 \leq s_r^+ = \sum_{j=1}^n y_{ij} \lambda_j - y_{0i} \leq \bar{y}_{jr} - y_{jr} = R_r^+, \quad r = 1, \ldots, s.
\]

Therefore,

\[
0 \leq \sum_{i=1}^m (s_i^- / R_i^-) + \sum_{r=1}^s (s_r^+ / R_r^+) \leq 1 \tag{2}
\]

can be used as a measure of inefficiency for DMU_0 in which (a) the lower bound is achieved only when no inefficiencies are detected via (1) (i.e., no nonzero slacks in inputs or outputs) and (b) the upper bound is achieved only when \( s_i^- = R_i^- \) for all \( i = 1, \ldots, m \) and \( s_r^+ = R_r^+ \) for all \( r = 1, \ldots, s \) (i.e., complete slack or shortfall on output and overuse of inputs). As seen in (1c) \( s_i^- \) and \( R_i^- \) and the \( s_r^+ \) and \( R_r^+ \) are stated in the same units. Hence the value specified in (2) is dimensionless and a choice of different units of measure for any input or output will not affect the optimum value of (1). In addition, from (1) and (1b) it is clear that the measure of inefficiency is invariant to changes in location or scale of both inputs and outputs. 

We now have a measure of inefficiency defined over all inputs and outputs with a natural zero occurring if and only if all slacks are zero so that (in accordance with the above definition) the maximization in (1) does not detect inefficiencies in
any of DMU₀’s inputs or outputs. The upper bound of unity is achieved, on the other hand, only if the constraints allow a best-to-worst comparison for every one of DMU₀’s inputs and outputs. In general, therefore, the objective in (1) defines a maximal value that represents an average of these relative inefficiencies.

Now DMU₀, the entity being examined, is also one of the DMUs, say DMUⱼ, and hence is represented on the right as well as the left in (1). A solution therefore always exists and, via the conditions in (2), an optimum solution is bounded on both sides. This continues to be the case as successive DMUⱼ are positioned on the left for evaluation. Hence these statements are not vacuous and in all cases solutions to (1) are similarly bounded. As noted in Cooper et al. (1999), this measure is also strongly monotonic. This means it can be used for rankings. That is, strong monotonicity, together with the occurrence of a natural zero, unity and direction, allows this measure to be used for the rankings we will subsequently employ. Ranking of the evaluated entities according to their efficiency is a frequent managerial desire of DEA analysis, but is not generally possible with other DEA formulators.

The above measure of inefficiency may also be complemented by the following measure of efficiency:

\[
0 \leq \Gamma = 1 - \frac{\sum_{i=1}^{m} (s_i^- / R_i^-) + \sum_{r=1}^{s} (s_r^+ / R_r^+)}{m + s} \leq 1
\]

(3)

which Cooper et al. (1999) refer to as RAM (= range adjusted measure) of efficiency. The properties noted for (2) also carry over to (3). Thus, the values of \( \Gamma \), as obtained from (1) are not dependent on the units in which the inputs and outputs are measured and, as in (2), the measure is interpretable in term of the average inefficiency that is present in the performance of the DMU₀ being evaluated. Finally, the value of this measure is not origin dependent. This, too, is a great convenience for the uses we will make of this measure, because it means that an arbitrary constant may be added to the data, in any row without affecting the optimal solutions in (1). That is, we can, if we please, replace the \( x_{ij} \) by new values \( \tilde{x}_{ij} = x_{ij} + d_i \) and the \( y_{ij} \) by \( \tilde{y}_{ij} = y_{ij} + c_r \) where the \( d_i \) and \( c_r \) are arbitrary constants without affecting the solutions because of the condition \( \sum_{r=1}^{s} \lambda_r = 1 \). This allows us to deal with the possibility of negative values—such as losses compared to profits—without losing contact with results from prior research in DEA which generally assume an absence of negative values in the observations.

4. Data and variable selections

The companies covered in our study consist of 1114 stock and 410 mutual companies based on data obtained from the 1989 property and liability tape we secured from the A.M. Best company—who were also compensated for making error checks and other improvements in the data. This total of 1524 companies decomposed into 1201 using “agency” and 323 using “direct” types of marketing and this enable us to study both efficiency and solvency in the various combinations of company and marketing system types represented in Table 3.

Our DEA model can accommodate non-financial as well as financial variables without concern for their commensurability. This provides us with the flexibility required to evaluate the multiple dimensions in which these insurance companies (or financial institutions in general) need to be evaluated. The selection of variables to represent inputs and outputs is nonetheless crucial and is particularly difficult for financial services firms (as opposed to manufacturing firms which use physical natural resources to produce physical completed products as outputs). Generally speaking, the inputs represent resources that an output producing
entity—or, more generally a DMU—employs in order to conduct its operations. The outputs reflect the results which are desired from the inputs utilized. Thus, most importantly, *ceteris paribus*, an increase in an output should improve the efficiency score and so should a decrease in input. An important contribution of the DEA methodology is its ability to incorporate multiple outputs, allowing a DMU to have multiple objectives or goals (outputs) which they may consider and trade-off with not all companies (DMUs) using the same trade-off values.

The above criteria for input and output selection provide general guidance but more is needed for the output and input choices to evaluate insurance company (or financial services firms) performances. Here we adopt the viewpoint that regards an insurance company as a financial intermediary, and the input and output selection are made accordingly. It is worth noting that the intermediary approach to efficiency determination of insurance companies is not uniformly adopted, and it will, as with all scientific disagreements, ultimately be decided as correct or incorrect by the judgement of history and future research (and use). The alternative approach (referred to as the “production approach” by Berger and Humphrey (1997), would treat financial institutions just as they would treat manufacturing companies. General Motors, they would say, uses capital and labor (inputs) to produce automobiles (output) and investors hope to make a profit, but profit in this viewpoint is a goal, not an output. In the context of insurance companies, this production approach might use premiums, investment income supplied capital and labor costs, as outputs and use losses paid as an output (saying that paying future losses is the “product” being sold). We disagree with this viewpoint. For one thing, if losses changed upward dramatically—say due to a hurricane or an earthquake, or a terrorist attack, or an environmental catastrophe, or all four simultaneously—with no change in inputs, this would be bad (not good) and not efficiency enhancing for an insurance company. Similarly, if invested income increased with no changes in loss payments, it would be a good rather than a bad (efficiency lowering) happening. In our view an insurance firm (as with other product and service lines, and as is consistent with the marketing investor and regulator research viewpoint) provides a bundle of attributes to the consumer, only one part of which may be the end physical product (a loss payment for an insurance firm). We view the loss payment as a vehicle or intermediate step by which the investors get rewarded, consumers get a valued promise of quick claim payment, and consumers, regulators, and employees get a promise of future solvency of the firm (making the insurance IOU worth paying for in the first place). Berger and Humphrey (1997, p. 197) differentiates the “production approach” from the “intermediation approach” which they believe is best used to evaluate entire financial institutions (firms) which are concerned with “intermediating funds between savers and investors”. The production approach is most useful, they say, for evaluating efficiency of branches or subsidiaries. In this paper we are evaluating entire companies and use an intermediation approach. Fig. 2 (using and extending Witt, 1994) summarizes the view of the insurance company as a financial intermediary.

Table 3
Types of companies and marketing systems

<table>
<thead>
<tr>
<th>Organization structure</th>
<th>Marketing system</th>
<th>Number of companies</th>
<th>% of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>All stock companies</td>
<td></td>
<td>1114</td>
<td>73%</td>
</tr>
<tr>
<td>All mutual companies</td>
<td></td>
<td>410</td>
<td>27%</td>
</tr>
<tr>
<td>All agency companies</td>
<td></td>
<td>1201</td>
<td>79%</td>
</tr>
<tr>
<td>All direct companies</td>
<td></td>
<td>323</td>
<td>21%</td>
</tr>
<tr>
<td>Stock</td>
<td>Agency</td>
<td>919</td>
<td>60%</td>
</tr>
<tr>
<td>Stock</td>
<td>Direct</td>
<td>195</td>
<td>13%</td>
</tr>
<tr>
<td>Mutual</td>
<td>Agency</td>
<td>282</td>
<td>19%</td>
</tr>
<tr>
<td>Mutual</td>
<td>Direct</td>
<td>128</td>
<td>8%</td>
</tr>
</tbody>
</table>

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As a financial intermediary, an insurer issues contingent claims to policyholders and uses the proceeds to purchase a portfolio of assets. Management is charged with investing these assets not only to maximize a risk adjusted return on capital but also to do this in a way that maximizes the value of ownership claims. This is one objective to be considered in evaluating performance. In offering insurance an insurer effectively leverers ownership capital by “borrowing” from the policyholders. A critical role of equity (= ownership) capital is therefore the creation of “insurer surplus” to serve as a buffer against the possibility that losses may exceed the net premiums collected.
plus the interest and dividends earned between the
time of premium receipt and the time of dis-
bursement. The greater an insurer’s capital, the
more certain policyholders can be that they will
receive compensation for insured losses. Compe-
tition in insurance markets requires premiums to
be set at levels that compensate policyholders for
the use of their funds and also provide a compet-
titive return to the shareholders or owners as
compensation for their role as residual risk bear-
ers.

The choice between a “production” and an
“intermediary” approach applies to banks and
other financial institutions. However, as pointed
out by McCabe and Witt (1980), Lai and Witt
(1992), and Staking and Babbel (1995), differences
between insurers and other financial intermediaries
must be taken into account in analyzing the
structure of the property–liability insurance in-
dustry. Thus, although the claim against the in-
surer by policyholders is similar to the claims of
depositors and debtors at other financial institu-
tions, it also differs because the claim of each
individual policyholder is contingent upon expe-
riencing a loss. Furthermore, insurance contracts
are usually set up to cover losses incurred during a
specified time period, while actual loss payments
are made over a much longer time period. Poli-
cyholders are, in effect, purchasing a long-term
financial commitment by the insurer. They can-
not cancel past coverage and obtain refunds if
they perceive that the riskiness of the insurer is
increasing. Future business can be transferred
to another insurer, but past exposures cannot
be transferred without payment of additional
 premiums.

The special nature of the fiduciary relations in
insurance and the importance of insurance to
economic and social performances, as well as the
regulatory contexts in which insurance companies
operate, make it advisable to recognize that a
single objective such as “maximize profit” does not
provide an adequate basis for evaluating perfor-
manence. We therefore use multiple goals which in-
clude not only profitability, but also short term
claim paying ability and the longer term ability to
discharge fiduciary responsibilities as represented
by solvency. This leads us to the four inputs and
three outputs selected for this study and listed in
Table 4, which we now discuss.

**Input 1: Surplus previous year:** Surplus, the
excess of assets over obligations, represents the
owners’ stake or equity in the firm. This surplus is
the total of capital and unassigned funds, includ-
ing voluntary and general reserved funds, plus
special reserved funds that are not in the nature of
liabilities. It represents the amount beyond liabil-
ities available to meet obligations to policyholders.

**Input 2: Change in capital and surplus:** Change
in capital and surplus includes the following items:
net income (both investment and underwriting),
et net realized capital gain or loss, change in excess of
statutory reserve over statement reserves, and so
on.

Notice that input 1 (surplus previous year) plus
input 2 (change in capital and surplus) together
provide the amount that is pertinent to the current
year. We include them separately, however, be-
cause of the additional information this can pro-
vide.

**Input 3: Underwriting and investment expenses:**
Underwriting expenses are the costs to an insur-
ance company that arise from the function of un-
derwriting. Investment expenses are the costs
associated with the productive employment of
capital under conditions that provide reasonable
assurance of both income and repayment of the
principle. Both types of expense are incurred to

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13 Under statutory accounting principles (SAP) capital is
called surplus, under generally accepted accounting principles
(GAAP) capital is called equity.
accomplish the objectives of an insurance company.

**Input 4: Policyholders supplied debt capital:** The policyholders supplied debt capital (debt capital of insurers) consists primarily of funds borrowed from policyholders. In our study, these funds are measured in real terms as the sum of the following three items: unpaid net losses, unpaid loss adjustment expenses (the first two of these together represent a company’s obligations for unpaid losses), and unearned premium reserves (which represent the company’s obligations for premiums held for coverage not yet provided).

The first two are “input conserving” in the sense that they make it unnecessary to incur expenses when the service is supplied. The last one, unearned premium reserves, an accrued for an output with its associated benefits to be recognized only when the service is supplied at some future date. In any case, all three have attenuating effects on solvency and ROI. This difference in the properties usually ascribed to inputs (as in our preceding discussions) leads us to utilize our model’s flexibility by using the complement of the observed values of this input. See the discussion of Tables 6 and 7 below.

Referring back to Table 2, in order to maintain comparability with our use of probabilities to represent solvency as output 3, the other two outputs are also put on a 0–1 scale by stating them in ratio form as follows:

**Output 1: Rate of return on investments:** Rate of return on investment \(^{14}\) is a general indicator of the quality of a company’s investment performance. Called “investment yield” by NAIC (National Association of Insurance Commissioners), it is defined as net investment income divided by the average amount of cash and invested assets for current and prior year.

**Output 2: Liquid assets to liabilities:** \(^{15}\) Liquid assets are cash and short-term investments in securities. We follow the NAIC definition where liquid assets is defined to be the sum of deferred premiums booked but not yet due, cash, invested assets, and accrued investment income, minus investment in affiliated companies, and excess of real estate over 5% of assets. Liabilities are the probable future sacrifices of economic benefits stemming from present legal, equitable, or constructive obligations incurred to transfer assets or to provide services to other entities in the future as a result of past events affecting the corporation. This ratio reflects a company’s claim paying ability.

**Output 3: Solvency scores:** As noted earlier, insolvency within the insurance industry is a major issue of public debate and concern, and ways of identifying potentially troubled firms has become a major regulatory and research objective. For warnings of pending insurer insolvency, a regulator has several sources of information. For example, there are reporting and rating services such as those provided by the A.M. Best Company, which rates 3000 property–liability and life health insurers with respect to solvency. However, these ratings are of limited use for our study because many of the insurers of interest (e.g., to state regulators) are not rated by Best’s or by other rating services (e.g., Moody’s or Standard and Poor’s). See Brockett and Cooper (1990).

To our knowledge, the best performing model for predicting property or liability insurer insolvency yet published in the open peer reviewed scientific literature is the neural network model detailed in Brockett et al. (1994). This model used eight financial and operational variables available from the annual statements filed by insurance companies with the NAIC. The neural network model produced a single scalar measure estimating the probability that a company will remain solvent for the next three years. This variable plays a special role in our study so we discuss it further as follows.

The eight financial and operations variables selected for computing the solvency scores are computed from data available in the annual statements filed by insurance companies with the National Association of Insurance Commissioners. These variables are listed in Table 5.

---

\(^{14}\) This is IRIS ratio 5. Although the IRIS system is not very good for early warning of insurer insolvency, this does not mean that certain IRIS ratios are not useful for efficiency determination.

\(^{15}\) This is the reciprocal of IRIS ratio 7.
The early warning system derived by Brockett et al. (1994), as developed for regulatory use in predicting insolvency by the Texas Department of Insurance, is based upon a specially developed neural network model. It uses the same data as the NAIC's IRIS system but has been found to give much better predictions of insolvency, see Brockett et al. (1994). Unlike IRIS (and other) systems, it provides a single (scalar) measure which we use in place of the more numerous measures we would otherwise have to use as listed in Table 5. Notice that the “solvency scores” represents the probability that a company will remain solvent. Hence, a score less than 5, means that the company has less than a 50% chance to remain solvent, and this is less than the probability that the company will become insolvent.

Descriptive statistics for the seven variables (four inputs and three outputs) listed in Table 4 are shown in Table 6. The “Policyholders supplied debt capital” has by far the largest scatter with a range that extends from a minimum of about 17 thousand dollars to more than 15,816 million dollars, and an average of some 334 million dollars. The minimum value for “Change in capital and surplus” has a negative value in excess of $95 million, which means that some companies experienced negative incomes (losses) in this (1989) period. All of the input variables are measured in dollars but the outputs are in the form of ratios as can be seen in the last three rows, where they are listed as follows: “Rate of return on investment”, which ranges from a minimum of 0.38 to a maximum of 26.86%, with a mean value of 7.53%, and a standard deviation of 1.64%. The “Liquid assets to liability” ratio ranges from 0.25% to 9.83%, with an average value of 1.58%; and a standard deviation of 0.96%. The “Solvency scores” range from 0.35 to 0.91, with an average of 0.69 and a standard deviation of 0.13.

As noted in the preceding section, the efficiency scores obtained from our RAM model are invariant to choices of origin so that we are able to eliminate the negative values for “Change in surplus” by adding a constant to the data in this row and then to rescale all inputs and outputs to more convenient units. The results are shown in Table 7 which we relate to Table 6 in the following manner. First, all inputs are rescaled in units of $10,000. Next, to eliminate the negative value in row two of Table 6, a constant in the amount of 9,539.4068 is added to all values. We then complete our discussion by turning to row four in Table 6 where we want to use a complementary value for this input (since an increase in Policyholder supplied debt capital, ceteris parabus, is less desirable rather than more desirable, since if one can get the same output with less policyholder capital, this is preferable). This is accomplished by first multiplying all values by −1 and then adding the result from the same constant (here = 2,000,000) then reverses the relation that previously maintained—e.g., between the minimum and maximum value. No further treatments are conducted, so these are the data as set forth in Table 7 which are used in our analysis.

5. Efficiency and solvency

As already noted, interest in solvency–efficiency relations arises in part because of regulatory and consumer concern with the former and managerial and investor concern with the latter. Rather then review the literature dealing with this topic, we...
concentrate on the treatment in McCabe and Witt (1980) where solvency is modeled as a constraint and profit is modeled as the objective to be optimized. In particular, solvency is modeled by McCabe and Witt in the form of “chance constraints” imposed by a regulatory body with allowable risks of violation. See also Thompson et al. (1974). The objective of management is then to maximize profit while satisfying these constraints.

This modeling of the situation assumes that the thus imposed constraints will, in general, interfere with the maximum profit objective and profit is modeled as the sole objective. Our DEA model, however, is oriented toward three outputs in the form of the following ratios: (1) “Return on investment”, (2) “Liquid assets to liabilities”, and (3) “Solvency”. We justify including the latter as an output objective by noting that investors and managers, as well as regulators, are likely to react favorably to improvements in solvency as well as the other two outputs in our trio of objectives. That is, ceteris paribus, an improvement in the solvency of an insurance company is likely to be viewed with favor by all of these interested parties (investors, managers, and regulators)—as is also true for the other two objectives—and this is also true of present and potential policyholders. Companies can, through policy actions within certain guidelines trade-off one output for another. For example, by investing in higher risk investments, one might increase the first output while decreasing the third output. It is the goal of management to make such parsimonious trade-offs, and different companies, organizational forms, etc. may give greater or lesser emphasis to one versus another output.

Table 6
Descriptive statistics for inputs and output variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inputs (1000 Dollars)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surplus previous year</td>
<td>310,642</td>
<td>4,156,485,463</td>
<td>110,119,221</td>
<td>356,403,362</td>
</tr>
<tr>
<td>Change in capital and surplus</td>
<td>−95,384,068</td>
<td>2,411,221,243</td>
<td>13,276,393</td>
<td>76,875,453</td>
</tr>
<tr>
<td>Underwriting and investment expenses</td>
<td>12,048</td>
<td>3,320,016,506</td>
<td>54,373,509</td>
<td>200,634,024</td>
</tr>
<tr>
<td>Policyholders supplied debt capital</td>
<td>17,002</td>
<td>15,816,993,442</td>
<td>334,444,752</td>
<td>1,330,641,413</td>
</tr>
<tr>
<td>Outputs (ratio)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate of return on investment</td>
<td>0.38</td>
<td>26.86</td>
<td>7.53</td>
<td>1.64</td>
</tr>
<tr>
<td>Liquid assets to liability</td>
<td>0.25</td>
<td>9.83</td>
<td>1.58</td>
<td>0.96</td>
</tr>
<tr>
<td>Solvency scores</td>
<td>0.35</td>
<td>0.91</td>
<td>0.69</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Table 7
Descriptive statistics for inputs and output variables used in DEA

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inputs (1000 Dollars)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surplus previous year</td>
<td>31.06</td>
<td>415,648.55</td>
<td>11,011.92</td>
<td>35,640.34</td>
</tr>
<tr>
<td>Change in capital and surplus</td>
<td>1.00</td>
<td>250,661.53</td>
<td>10,867.05</td>
<td>7,687.55</td>
</tr>
<tr>
<td>Underwriting and investment expenses</td>
<td>1.20</td>
<td>332,001.65</td>
<td>5,437.35</td>
<td>20,063.40</td>
</tr>
<tr>
<td>Policyholders supplied debt capital</td>
<td>418,300.66</td>
<td>1,999,998.3</td>
<td>1,966,556.5</td>
<td>133,063.14</td>
</tr>
<tr>
<td>Outputs (ratio)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate of Returns on investment</td>
<td>0.38</td>
<td>26.86</td>
<td>7.53</td>
<td>1.64</td>
</tr>
<tr>
<td>Liquid assets to liability</td>
<td>0.25</td>
<td>9.83</td>
<td>1.58</td>
<td>0.96</td>
</tr>
<tr>
<td>Solvency scores</td>
<td>0.35</td>
<td>0.91</td>
<td>0.69</td>
<td>0.13</td>
</tr>
</tbody>
</table>

17 See also Harrington and Nelson (1986), Thompson et al. (1974) and Kahane (1977).
On this justification we proceed to examine whether solvency reinforces or attenuates the other objectives as reflected in the efficiency scores. This is done as follows: DEA runs are carried out twice (Case I and II) for the 1524 DMUs (insurance companies) in our sample. One run includes solvency scores as an output that enters directly into the efficiency score. The second run omits it. We then examine whether changes from efficient to inefficient status occur, or vice versa. If no changes occur, we infer that regulation imposed solvency constraints have no effect, at least with respect to whether such changes result in efficiency reclassification at the level at which they are imposed. This is done deterministically, but we also subsequently use statistical techniques to examine the effects on types of organizations and marketing systems when solvency is both included and excluded as an explicitly identified output for evaluation by our DEA efficiency scores.

We start by examining whether changes from efficient to inefficient status (or vice versa) occur with these two treatments of this output. We then conclude this (deterministic) part of our analysis by studying the relative magnitude of the inefficiency in each input and output after which we will turn to formal statistical treatments.

Table 8 records all of the companies where changes occurred in efficient versus inefficient classifications. That is, we omit cases where changes did not occur so that we can then identify the individual companies as is done in this single table. Only 23 (out of 1524) companies actually changed their efficiency classifications, and in the next column we further identify our results by type of company and marketing system. The column headed "company type" shows the affiliations as: 1 = affiliated single company, 2 = unaffiliated single company, and 3 = group company with affiliates. Finally, the fifth column gives the solvency score derived from the neural network model discussed previously.

The last two columns in Table 8 show the efficiency scores. Column six shows this score with solvency included as an output and column seven shows the efficiency score when solvency is re-

<table>
<thead>
<tr>
<th>DMU</th>
<th>Company names</th>
<th>Organization and marketing types</th>
<th>Company type</th>
<th>Solvency score</th>
<th>Efficiency score (with solvency)</th>
<th>Efficiency score (without solvency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>96</td>
<td>UNITED STATES</td>
<td>Stock and agency</td>
<td>3</td>
<td>0.9134</td>
<td>1.0000</td>
<td>0.4685</td>
</tr>
<tr>
<td>98</td>
<td>ZURICH INS</td>
<td>Stock and agency</td>
<td>3</td>
<td>0.9134</td>
<td>1.0000</td>
<td>0.4641</td>
</tr>
<tr>
<td>646</td>
<td>GENERAL RE</td>
<td>Stock and agency</td>
<td>1</td>
<td>0.9134</td>
<td>1.0000</td>
<td>0.4606</td>
</tr>
<tr>
<td>1728</td>
<td>AMER HALLMARK</td>
<td>Stock and agency</td>
<td>1</td>
<td>0.9064</td>
<td>1.0000</td>
<td>0.5072</td>
</tr>
<tr>
<td>2095</td>
<td>CIMARRON GROUP</td>
<td>Stock and agency</td>
<td>1</td>
<td>0.9090</td>
<td>1.0000</td>
<td>0.4385</td>
</tr>
<tr>
<td>2539</td>
<td>UNITED STATES</td>
<td>Stock and agency</td>
<td>1</td>
<td>0.9134</td>
<td>1.0000</td>
<td>0.4647</td>
</tr>
<tr>
<td>2596</td>
<td>NORTH SEA</td>
<td>Stock and agency</td>
<td>2</td>
<td>0.9134</td>
<td>1.0000</td>
<td>0.4797</td>
</tr>
<tr>
<td>3015</td>
<td>SUN ALLIANCE</td>
<td>Stock and agency</td>
<td>3</td>
<td>0.9133</td>
<td>1.0000</td>
<td>0.4112</td>
</tr>
<tr>
<td>3269</td>
<td>NOBEL INS</td>
<td>Stock and agency</td>
<td>3</td>
<td>0.9134</td>
<td>1.0000</td>
<td>0.4535</td>
</tr>
<tr>
<td>3662</td>
<td>FORUM INS</td>
<td>Stock and agency</td>
<td>1</td>
<td>0.9134</td>
<td>1.0000</td>
<td>0.4168</td>
</tr>
<tr>
<td>3666</td>
<td>ALLIANCE INS</td>
<td>Stock and agency</td>
<td>1</td>
<td>0.9133</td>
<td>1.0000</td>
<td>0.5067</td>
</tr>
<tr>
<td>3786</td>
<td>AMER INTERN</td>
<td>Stock and agency</td>
<td>1</td>
<td>0.9117</td>
<td>1.0000</td>
<td>0.5884</td>
</tr>
<tr>
<td>4037</td>
<td>NAT FARMRS</td>
<td>Stock and agency</td>
<td>1</td>
<td>0.9134</td>
<td>1.0000</td>
<td>0.4906</td>
</tr>
<tr>
<td>10588</td>
<td>AMER INTERN</td>
<td>Stock and agency</td>
<td>1</td>
<td>0.9132</td>
<td>1.0000</td>
<td>0.5959</td>
</tr>
<tr>
<td>1813</td>
<td>TORCHMARK GROUP</td>
<td>Stock and direct</td>
<td>1</td>
<td>0.9128</td>
<td>1.0000</td>
<td>0.5254</td>
</tr>
<tr>
<td>1954</td>
<td>ROCKINGHAM GROUP</td>
<td>Stock and direct</td>
<td>1</td>
<td>0.9117</td>
<td>1.0000</td>
<td>0.534</td>
</tr>
<tr>
<td>2496</td>
<td>METROPOLITAN GROUP</td>
<td>Stock and direct</td>
<td>1</td>
<td>0.7185</td>
<td>1.0000</td>
<td>0.7387</td>
</tr>
<tr>
<td>3695</td>
<td>BRITISH AMER</td>
<td>Stock and direct</td>
<td>2</td>
<td>0.9127</td>
<td>1.0000</td>
<td>0.6049</td>
</tr>
<tr>
<td>3701</td>
<td>HEALTH CARE</td>
<td>Stock and direct</td>
<td>2</td>
<td>0.9124</td>
<td>1.0000</td>
<td>0.5108</td>
</tr>
<tr>
<td>4050</td>
<td>FORD MOTOR</td>
<td>Stock and direct</td>
<td>1</td>
<td>0.8029</td>
<td>1.0000</td>
<td>0.7311</td>
</tr>
<tr>
<td>10609</td>
<td>CONCORD GENL</td>
<td>Stock and direct</td>
<td>1</td>
<td>0.8217</td>
<td>1.0000</td>
<td>0.5757</td>
</tr>
<tr>
<td>56</td>
<td>KEMPER GROUP</td>
<td>Mutual and agency</td>
<td>3</td>
<td>0.9134</td>
<td>1.0000</td>
<td>0.2775</td>
</tr>
<tr>
<td>3814</td>
<td>HOCHHEIM PRAIRIE</td>
<td>Mutual and agency</td>
<td>1</td>
<td>0.9134</td>
<td>1.0000</td>
<td>0.2293</td>
</tr>
</tbody>
</table>
moved. Although each DEA run is carried out on 1524 DMUs, only 23 (1.5%) change efficiency classifications. In every case the change is from fully efficient to inefficient with some of the changes being drastic.

Some of these companies may be important regionally and some other may be important nationally. Hence a non-significant number of occurrences need not be unimportant from a regulatory (or public policy) standpoint, so further investigation is warranted. Here, however, we elect a different route to show how additional features of our model in (1) may be exploited. As noted in our discussion of (1)–(3), the objective in (1) yields an efficiency score in non-dimensional units. However nothing is lost because the constraints in (1) also provide information on each input and output in whatever units of measure are employed. The former (i.e., the dimensionless measure in the objective) is of interest for purposes such as scientific research. The latter is likely to be more easily and meaningfully interpreted by managers and public policy officials. It can also help to illustrate what occurred in term of the reclassification that we have just finished examining.

The latter kind of detail information obtained from our stage one and stage two solution is portrayed graphically in Figs. 3–10. Here the non-zero slacks which, represent the inefficiencies in each input and output, are shown for all of the combinations recorded in Table 3. This information allows us to examine the possible effects of omitting the solvency condition in fine detail relative to each input and output, and hence provides information that might be lost from sight in summary measures such as the overall efficiency scores obtained from (3). As noted in the left-hand portion of these charts, the inefficiency in the solvency (output) variable is relatively small in all cases. One might therefore expect that omitting this variable as an output would not have major effects on efficiency scores. This helps to explain what occurred in our overall analysis, as was discussed in connection with Table 8 and, indeed, the omission of this output seems to have had little or no effect on any of the output shortfalls or input excesses recorded in these averages.

6. Effects on types of organizational and marketing systems

We can gain still more insights by building on results from other studies that have been
undertaken with our data and models. These studies were addressed to questions such as whether stock companies are more efficient than mutual companies and whether agency marketing is more effi-

Fig. 4. Effects of solvency on inefficiencies (non-zero slacks) agency versus direct marketing systems.

Fig. 5. Effects of solvency on inefficiencies (non-zero slacks) stock/agency versus mutual/agency.
cient than direct marketing system. These ques-
tions were also addressed in a manner that was
directed to eliminating possibilities of contamina-
tion which can occur because particular managers
are not using this organizations (stock versus mutual) or marketing systems (agency versus direct) to their full (100%) potential efficiency. That is to say, there could be “superior” management in an inferior organizational form, or inferior management in a “superior” organizational form, and
this contamination by uneven distribution of managerial competence may mask, or when averaged, even reverse the observable capabilities of the organizational forms themselves. We wish to know if, for example, an efficiently run mutual firm having adjusted for the managerial (or technical) inefficiencies exhibited by each organizational form.

Most studies of efficiency of stock versus mutual organizational forms do not eliminate the effects of managerial inefficiencies with each organizational form type, and hence their conclusions can not be relied upon as relating to pure performance of the organizational form when operating at their most efficient level for that particular form (e.g., Fama and Jensen, 1983). We overcome this “contamination” deficiency by using the following formulas, as obtained from the DEA literature, to project the original observations for the inefficient firms onto the points on the efficiency frontier for their particular organizational form. This results in an organizationally efficient firm (new DMU) which shows the output which would have resulted for the inefficient companies if they had behaved efficiently for their organizational form.

\[
\hat{x}_0 = \sum_{j=1}^{n} x_{ij}^* \lambda_j^* = x_0 - s_i^+, \quad i = 1, \ldots, m,
\]

\[
\hat{y}_0 = \sum_{j=1}^{n} y_{rj}^* \lambda_j^* = y_0 + s_r^+, \quad r = 1, \ldots, s.
\]

Variants of formula (4), as first given in Charnes et al. (1978) are referred to as “CCR projection formulas” because they project the original observation into a new point with coordinates \(\hat{x}_0 \leq x_0\) and \(\hat{y}_0 \geq y_0\). \(^{18}\) These new coordinates correspond to a point on the efficiency frontier. In fact, this is the point on the efficiency frontier used to evaluate DMU0.

To elucidate the use now to be made with this approach we restrict our immediate discussion to evaluating the relative efficiency of stock versus mutual types of organization and describe the procedure as follows: At stage 1, the two types of organization (stock and mutual) are treated

\(^{18}\) See also Charnes et al. (1981) and Brockett and Golany (1994) where this type of projection formula was used to distinguish whether one program was better than another in the education of disadvantaged children (Project Head-Start).
separately. Separate efficiency frontiers for stock
and mutual, respectively, are then established so
that (4) can be applied to bring each DMU up to
the full efficiency that the frontiers allows for its
type of organization (stock or mutual). Having
removed managerial or technical inefficiency, the
thus adjusted (fully efficient) DMUs are then
combined for simultaneous treatment by (1) so
that a new frontier can be used to identify ineffi-
cencies and yield a resulting efficiency score. If the
two organizational forms are equally efficient (the
efficiency frontier for the two are the same) then
running the DEA with all the projected DMU’s
should yield this common efficiency frontier with
inefficiencies equally interchangeable between the
two groups (organizational forms). If at their best
performance (projected efficiency) the two types
are related, then one type will be more likely than
the other to be inefficient when compared to this
new combined efficiency frontier. After the final
“combined projected: DEA analyses, the scores
which are less than unity are then re-associated
with the “types” of organization (compared to
each other) and we obtain a measure of relative
efficiency performance after managerial inefficien-
cies—i.e., inefficiencies due to the way each type
was managed—have been eliminated.

We could simply examine the number of ineffi-
ciency forms of each type, but this does not
provide a sufficiently rigorous answer to where
differences might have arisen due solely to chance.
Traditional parametric statistics (such as t-tests)
are inappropriate because the efficiency measures
are ordinal but not interval level in data quality.
However, as previously noted, our RAM version
of DEA produces a measure which lends itself to
ranking by reference to the efficiency score which it
provides. This enables us to use a rank order sta-
tistical test in order to determine whether one
type of organization is statistically significantly
more efficient than the other and, of course, the
same procedures can be applied to determine
whether one member of each type for the pairs
listed in Table 3 is more efficient than the other.

Following Brockett and Golany (1994) we use
the Mann–Whitney rank order test statistic for
these purposes because it is (a) non-parametric and
is thus consistent with DEA (which is also non-
parametric), and (b) lends itself to the pairings that
are of interest here.

We briefly describe the procedures as follows.
We have two groups of size, \( n_1 \) and \( n_2 \) for a total of
\( n = n_1 + n_2 \) DMUs which we rank in the ascending
order according to their efficiency ratings. In case
of ties, we use the midrank for all of the tied
DMUs. We then compute the sum of the rankings
for the first subgroup of size \( n_1 \) after which we compute the Mann–Whitney rank order test sta-
tistic for this subgroup via

\[
U = n_1 * n_2 + \frac{n_1 * (n_1 + 1)}{2} - R, 
\]

where \( R \) is the sum of the ranks in the first sub-
group.

We then test whether the two subgroups
have the same distribution of efficiency values
within the pooled collection of \( n \) DMU efficiency
values. The rank statistic used to test the signifi-
cance of the differences between the efficiency
ratings for each subgroup is obtained from the
following statistic (for \( n_1, n_2 \geq 10 \)):

\[
Z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}} - \frac{n_1 (n_1 + 1)}{2} \sqrt{\frac{1}{n_1 n_2 (n_1 + 1)}} \right) 
\]

where \( n_1 \) = number of DMUs in the first subgroup,
\( n_2 \) = number of DMUs in the second subgroup,
\( n \) = number of DMUs in the total, \( n = n_1 + n_2 \),
\( R \) = the sum of the \( n_1 \) rankings of DMU in the first
subgroup, \( U \) = the Mann–Whitney rank test sta-
tistic obtained from (5), \( Z \) = score of an approxi-
mately normal distribution.

\( Z \) has an approximately standard normal dis-
tribution. Specifying a significance level of \( \alpha \) we
reject the null hypothesis that the two subgroups
have the same distribution of efficiency scores at
this significance level if either \( Z \geq Z_{\alpha/2} \) or
\( Z \leq -Z_{\alpha/2} \) where \( Z_{\alpha/2} \geq 0 \) is obtained from a
normal distribution table.

It can be see from (5) and (6) that large values
of \( Z \) will be associated with large values of \( U \) and
small values of \( R \). \( R \) represents the sum of the
ranks from lowest to highest in the first subgroup.
Hence a small value of \( R \) means that there are
many DMUs with lower scores in the first sub-
group. Consequently many of the higher scores

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\text{Following Brockett and Golany (1994) we use
the Mann–Whitney rank order test statistic for
these purposes because it is (a) non-parametric and
is thus consistent with DEA (which is also non-
parametric), and (b) lends itself to the pairings that}
\]
must be in the second group. If the value of \( Z \) is sufficiently positive we will have \( Z > Z_{a/2} \) while if it is sufficiently negative we will have \( Z < -Z_{a/2} \) and the hypothesis that the two subgroups have the same distribution of efficiency scores (after having projected to the efficiency frontier for the organizational form) will be rejected at the \( a \) level of significance. We can then infer that the two subgroups are not on the same efficiency frontiers. We can also conclude that the second subgroup is more efficient than the first one when \( Z > Z_{a/2} \) and that it is less efficient than the first subgroup when \( Z < -Z_{a/2} \). The thus described procedure therefore yields one-sided tests which can identify which subgroup is more efficient as well as symmetrically testing whether the two subgroups differ significantly.

The results of these analyses presented in Table 9 can be summarized as follows: (1) stock companies are more efficient than mutual companies (row 1), (2) agency is more efficient than direct (row 4). The other rows contain information on the combinations spelled out in Table 3. We formalize these results via the following set of relations.

Stock and agency > stock and direct > mutual and agency where “>” means “more efficient than.” Only row six accords a positive values to \( Z \) and we attribute this reversal to the fact that “mutuals” are cooperatives which generally have a culture that emphasizes “belongingness” and this make it easier to manage marketing arms that are identified as a part of their own organization. As can be seen the \( Z \) values in row six are not too different from the values customarily used in significance tests so, although significant, these significant values are much smaller than is the case for the other \( P \)-values.

With these results in hand we are now in a position to extend our previous results to an analyses in which we omitted solvency as an output entering directly into our efficiency score. The question we now address is whether this omission causes any difference to occur when all firms are operated at the level of full (100%) efficiency that their type of organization or marketing systems allow. This is accomplished by repeating the two stage procedure described above—such as was done in Table 8—after which we can ascertain whether any differences emerge when solvency is omitted from the outputs.

The results are displayed in Table 10 and, comparing Tables 9 and 10, no reversals occur. Indeed the results are strikingly similar. We therefore conclude that the solvency score used as an output measure exhibits almost no influence in those rankings. Added to the results reported in Table 8 and the displays in Figs. 3–10 we conclude that inclusion or omission of this output variable has little effect on either the efficiency scores or their component elements.

Putting this all together we conclude that our original assessment is correct. Not only regulators but also the managers, investors and policyholders all devoted much attention to solvency (as well

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19 These results are from Brockett et al. (1997b), and are presented so they can be compared later with the analyses without solvency as an output quantity.
they might) so that very small (if any) inefficiencies were present in this variable. In fact, solvency was accorded the highest priority to ensure an absence of any shortfall as evidenced in the top (output) portions of Figs. 3–10. Table 8 and Figs. 3–10 reflect the inefficiencies of management as well as the organization and marketing system “type” that were utilized. However, these same results continued to apply, as shown in Tables 9 and 10, even after all of these managerial inefficiencies were eliminated.

7. Summary and conclusion

As already noted, the conclusions we just presented are consistent with the justification we provided for treating solvency as an output of managerial interest rather than simply as an externally imposed regulatory condition. More generally our approach, via DEA, allows us to deal very easily with multiple outputs and inputs in a manner that accommodates itself to treatments in which tests can be conducted by omitting (and including) one or more of the variables that might be of interest. Our use of the new RAM model also makes it easy to summarize the results in a scalar efficiency score. It also lends itself to the rankings needed for the significance tests, which we have approached by using the Mann–Whitney statistic. This, in turn, points to additional possibilities in which a natural match is made between the non-parametric properties of DEA and the distribution free statistical methods used to evaluate the results. Finally, we were able to extend our tests to examine actual behavior as in Figs. 3–10 and then go beyond these results to examine what would occur when technical inefficiencies are absent (as is assumed in micro-economic theory) so that each type of organization and marketing system is operated at its full potential—i.e., without the “waste” associated with departures from efficiency frontiers.

The results reported here apply only to the kinds of insurance (Property and Liability) and periods (1989) covered by our data. The methodologies we have introduced, however, can be extended to other periods and to other kinds of insurance. Indeed these methods can be readily extended to study other topics of interest such as return to scale,20 etc., which we have not conducted here. This, in turn, could lead to considering more dynamic phenomena such as changing frontiers over time so that different evaluations could then be made for companies that generally stay close to these changing frontiers over time as distinguished from other companies that attain these frontiers only occasionally.

A start has been made in treating these kinds of dynamic (moving frontier) issues in DEA, (cf., Fare and Grosskopf, 1996)21 but more is needed, including treatment of the stochastic elements that such an extension also invite. All of this can (and should) be done with multiple-input multiple-out-

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20 See Cooper et al. (1996) for a relatively up-to-date treatment of return of scale by DEA in the presence of multiple inputs and outputs.

21 See also Sengupta, 1995.
put models and methods like those we have used in our treatment of insurance companies as financial intermediaries.

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