Customization of low-cost hexapod robots based on optimal design through inverse dynamics computation

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Abstract—The Stewart parallel mechanism is used in various applications due to its high load-carrying capacity, accuracy and stiffness, such as flight simulation, spaceship aligning, radar and satellite antenna orientation, rehabilitation applications, parallel machine tools. The dissemination of such parallel robots is however limited by three factors: the limited workspace, the singularity configurations existing inside the workspace, and the high cost. In this work, a simulation environment to support the design of a cost-effective Stewart Platform-based mechanisms for specific applications and to facilitate the choice of suitable components, e.g., linear actuators, plate sizes, is presented. The optimal design here presented has multiple objectives. It intends to maximize the payload and minimize the forces at each leg needed to counteract external forces applied to the mobile platform during positioning or manufacturing applications. These objectives can be achieved through a dynamic optimization. It also aims at avoiding reduction of the robot workspace through a kinematic optimization.

I. INTRODUCTION

In recent years, great interest has been devoted to parallel manipulators based on the Stewart platform, also named hexapod. In comparison with a serial manipulator, the Stewart parallel manipulator, capable of providing six degrees of freedom (DOF) movements, has advantages [1] of high structural rigidity, high positioning accuracy, fast dynamic response, and large load-to-weight ratio, which makes it find a wide range of applications in flight simulation [2], spaceship aligning, radar and satellite antenna orientation [3], [4], rehabilitation applications [5], robots [6], parallel machine tools [7], [8]. Unfortunately, there are factors limiting the application of parallel mechanisms. First, the limited workspace that reduces the number of tasks the robot can execute and the singularities configurations existing inside the workspace in which the manipulator gains one or more degrees of freedom and therefore loses its stiffness. Moreover, although the versatility of the hexapod has been recognized, its acceptance by industry as production equipment has not yet occurred. Some obstacles to this include the high cost and unproven performance in a production environment for a specific task. Hence, development of efficient tools that allow to maximize the robot workspace, to reduce the singularities inside the workspace and to optimize the design of the parallel platform reducing the hexapod costs, e.g., by choosing a suitable set of linear actuators, becomes a very important issue. During the past decade, the structural design and optimization of parallel robots have been carried out by many researchers [9], [10]. Given the number of performance parameters to consider (i.e., workspace volume, manipulability, dexterity, singularity, accuracy, actuator interferences, actuation forces) it is still difficult to find an optimal general design for a 6-DOF parallel manipulator. Stoughton and Arai [11] proposed a new design that offers an improved dexterity over the traditional Stewart platform mechanism by optimizing the manipulator considering a weighted sum of dexterity and workspace volume. Merlet [12] proposed a complete analysis on the 6D-workspace of a Stewart-type parallel manipulator by comparing different robot geometries. Su et al. [4] presented a methodology for the design of optimal kinematical characteristics of Stewart platform using Genetic Algorithm (GA) to minimize the condition number of the Jacobian matrix to get accurate trajectory tracking. Chen [13] designed a safety mechanism capable of moving within the range of its full link lengths based on the link space by minimizing the average condition number of extreme poses.

In the present work a simulation environment to support the design of a low cost Stewart platform-based mechanism for specific applications is presented. In order to maximize the payload and improve the rejection of external forces exerted on the mobile platform during positioning or manufacturing applications, a dynamic optimization has been carried out. Moreover, in order to increase the robot workspace also a kinematic optimization has been performed. To this aim, the design is optimized by determining the leg attachment points on both top and base plates. In particular, the leg attachment points on either mobile (or top) and fixed (or base) plates are optimized satisfying the mechanical constraints introduced in the design (such as leg attachment point geometry, distances between the legs/actuators, minimum and maximum top and base plate dimensions, minimum and maximum leg strokes). A GA is used to combine two or more different optimum objectives by properly defining a cost function to minimize. In order to minimize the maximum leg force value and to equally distribute among the legs the forces exerted by each linear actuator during a positioning and/or machining task, the maximum root-mean-square (RMS) value of the forces is selected as optimum objective. A second optimum objective can be taken into account to maximize (or do not penalize) the robot workspace volume.

II. BASIC MODELING OF STEWART PLATFORMS

The Stewart Platform is a closed kinematic chain manipulators comprising six linear actuators, each connected by a universal joint to the manipulator base and by a spherical joint to the top platform. This arrangement of actuators allows the
platform to be placed in any position and orientation within a certain volume of space. Let denote with $l_i, i = 1, ..., 6$, the six actuated prismatic leg lengths, with $\mathbf{a}_i = (a_{xi}, a_{yi}, a_{zi})^T$ and $\mathbf{b}_i = (b_{xi}, b_{yi}, b_{zi})^T$ the position vectors of the center of the universal $(A_i)$ and spherical $(B_i)$ passive joints given in base and mobile platform reference frames, respectively (see Fig. 1). Given that the pose of the platform can be defined by $R_{ps}$ to the base frame, the platform attachments $\mathbf{b}_i$ can be written in the base frame as

$$\mathbf{b}_i = \mathbf{p} + R\mathbf{b}_i, \ i = 1, ..., 6. \quad (1)$$

The following subsections illustrate the inverse kinematics (IK) problem, the Jacobian computation, the statics problem and the inverse dynamics problem of a general Stewart platform.

A. Inverse Kinematics

The inverse kinematics problem of a Stewart platform [14] consists in computing the leg lengths $l_i$ given the position $\mathbf{p}$ and the orientation $(\varphi_p, \theta_p, \psi_p)$ of the mobile platform. So, the IK can be computed as described in eq. (2), where $\mathbf{b}_i$ is calculated as in (1).

$$l_i = \|\mathbf{b}_i - \mathbf{a}_i\| \quad (2)$$

B. Statics

Let $\mathbf{x} = [x_p, y_p, z_p, \varphi_p, \theta_p, \psi_p]^T$ be the vector with the six Cartesian coordinates of the mobile platform and $\mathbf{q} = [l_1, l_2, l_3, l_4, l_5, l_6]^T$ be the vector with the six leg lengths. If $\mathbf{x}(t)$ represents the pose of the mobile frame with respect to the base frame at any time $t$, the leg lengths can be computed as:

$$l_i = \sqrt{\frac{y_p - a_{yi} + b_{xi}(c_{\varphi_p}s_{\psi_p} + c_{\psi_p}s_{\varphi_p}s_{\theta_p}) + b_{yi}(c_{\varphi_p}c_{\psi_p} - s_{\varphi_p}s_{\theta_p}) - b_{zi}c_{\psi_p}s_{\theta_p}}{x_p - a_{xi} + b_{zi}s_{\psi_p} + b_{xi}c_{\psi_p}s_{\theta_p} - b_{yi}c_{\psi_p}s_{\theta_p}} \quad (3)$$

where the symbols $s_\alpha$ and $c_\alpha$ denote $\sin(\alpha(t))$ and $\cos(\alpha(t))$, respectively. Note that in the (3) the time dependence is omitted.

The analytical Jacobian of the generic Stewart platform [8] is given by

$$\mathbf{J}_A(\mathbf{q}) = \frac{\partial \mathbf{q}(\mathbf{x})}{\partial \mathbf{x}} \quad (4)$$

From the inverse kinematics equation, it is possible to compute the inverse differential kinematics mapping between the vector of the generalized velocities $\mathbf{\dot{x}} = [\dot{x}_p, \dot{y}_p, \dot{z}_p, \dot{\varphi}_p, \dot{\theta}_p, \dot{\psi}_p]^T$ and the vector of leg velocities $\mathbf{\dot{q}} = [\dot{l}_1, \dot{l}_2, \dot{l}_3, \dot{l}_4, \dot{l}_5, \dot{l}_6]^T$ as

$$\mathbf{\dot{q}} = \mathbf{J}_A(\mathbf{x})\mathbf{\dot{x}}. \quad (5)$$

The matrix $\mathbf{J}_A$ is the analytical Jacobian to be distinguished from the geometric Jacobian $\mathbf{J}$ relating the joint velocity vector to the end-effector velocity $\mathbf{v} = [\mathbf{p}_e, \omega_e]^T$, being $\mathbf{p}_e$ the linear velocity and $\omega_\mathbf{e}$ the angular velocity. The inverse differential kinematics in terms of the geometric Jacobian is

$$\mathbf{q} = \mathbf{J}(\mathbf{x})\mathbf{v}. \quad (6)$$

By comparing (5) with (6), the relationship between the two Jacobians becomes

$$\mathbf{J}(\mathbf{x}) = \mathbf{J}_A(\mathbf{x})\mathbf{T}_A^{-1}(\mathbf{x}), \quad (7)$$

where $\mathbf{T}_A(\mathbf{x})$

$$\mathbf{T}_A(\mathbf{x}) = \begin{bmatrix} 1 & 0 & s_{\varphi_p} & c_{\varphi_p}c_{\theta_p} & c_{\varphi_p}s_{\theta_p} & -s_{\varphi_p}\theta_p \\ 0 & c_{\varphi_p} & -s_{\varphi_p}c_{\theta_p} & -c_{\varphi_p}s_{\theta_p} & s_{\varphi_p}\theta_p & 0 \\ 0 & s_{\varphi_p} & c_{\varphi_p}c_{\theta_p} & s_{\varphi_p}s_{\theta_p} & -c_{\varphi_p}\theta_p & 0 \end{bmatrix}. \quad (8)$$

By virtue of the duality established by the principle of virtual works, the inverse statics mapping between the vector of leg forces $\mathbf{\tau}$ and the vector of generalized forces $\mathbf{h}_e$ on the mobile base is

$$\mathbf{\tau} = \mathbf{J}_A^T(\mathbf{x})\mathbf{h}_e = \mathbf{J}_A^T(\mathbf{x})\mathbf{T}_A^T(\mathbf{x})\mathbf{h}_e. \quad (9)$$

C. Inverse Dynamics

The dynamic analysis of a Stewart platform is more difficult in comparison with the serial manipulator because of the existence of several kinematic chains all connected by the moving platform. Several methods were used in the literature to describe the problem and obtain the dynamic model of the manipulator, but there is still no consensus on which formulation is the best to describe the dynamics of the manipulator. A Lagrange formulation was presented in [15] to provide an analytical and orderly model, while a closed form solution for the inverse and direct dynamics models based on the Newton-Euler formulation was presented in [16]. In the present paper, the inverse dynamics (ID) of the Stewart platform has been computed using a physics-based approach by modeling the parallel mechanism in the Matlab/Simulink environment using the SimMechanics Toolbox. This solution has been selected to obtain an easy-to-use software tool also owing to the availability of an optimization toolbox in the same software environment.

III. DYNAMIC SIMULATOR FOR ID COMPUTATION

The proposed simulator provides a simulation environment to support the design of a Stewart platform-based mechanism for specific applications. A dynamic optimization is carried out to minimize a cost function that will be defined in Section IV-A. In particular, the simulator has been developed in Matlab/Simulink environment using the SimMechanics Toolbox and the Global Optimization Toolbox. Given an initial configuration of the Stewart platform in terms of leg attachment points on base and top plates and given two sets of bounds properly defined on the base of the mechanical constraints, a GA is used to solve a non-linear constrained
The dynamic model of the parallel mechanism is computed as illustrated below. The base and top plates and all components of the legs have been implemented as rigid bodies with a proper mass and inertia tensor. A leg consists of a linear actuator attached to the base and top plates using an universal joint and a spherical joint, respectively. In detail, an actuator has been implemented in SimMechanics using three elements: a chassis and a shaft, that are rigid bodies, and a prismatic joint that gives a single dof and that connects the chassis to the shaft. Moreover, all the objects have been considered to satisfy specified maximum dimensions of the plates or other mechanical constraints.

A. Construction of the SimMechanics model

The dynamic model of the parallel mechanism is computed as illustrated below. The base and top plates and all components of the legs have been implemented as rigid bodies with a proper mass and inertia tensor. A leg consists of a linear actuator attached to the base and top plates using an universal joint and a spherical joint, respectively. In detail, an actuator has been implemented in SimMechanics using three elements: a chassis and a shaft, that are rigid bodies, and a prismatic joint that gives a single dof and that connects the chassis to the shaft. Moreover, all the objects have been considered to be of aluminium material, with a mass density of 2700 kg/m$^3$; the inertia tensors of both top and base plates have been computed considering the cuboids that include their shapes; the inertia tensors of the chassis and the shaft of the actuators have been implemented in SimMechanics using three elements: a chassis and a shaft, that are rigid bodies, and a prismatic joint that gives a single dof and that connects the chassis to the shaft. Moreover, all the objects have been considered to satisfy specified maximum dimensions of the plates or other mechanical constraints.

B. Parameterizations of the Hexapod geometry

In order to find the best hexapod geometry that should provide the maximum workspace volume and the minimum RMS value of the leg forces exerted by the actuators along a desired trajectory, the optimization process has been carried out by considering symmetric and un-symmetric hexapod geometries. The choice of a geometry affects the time required for the optimization process; the use of a large number of variables allows to optimize the platform minimizing the number of constrains but it increases the computational time.

Stewart-Gough Geometry (4 Variables) – Symmetric geometry (SG4).

The geometry is defined by using two variables for the base plate and two variables for the top plate (see Fig. 2). In particular, $\rho_1$ and $\rho$ are the circle radius of the base and top plates, respectively, and $\theta_1$ and $\theta_2$ are the half angle between two pairs of joints on the base and on the top plates, respectively. The leg attachment point positions $\mathbf{a}_i$ and $\mathbf{b}_i$, with respect to base frame and mobile frame, can be easily computed in the Cartesian space as a function of the geometry parameters defined above. The explicit expressions are not reported for brevity.

Note that only $x$ and $y$ coordinates are considered since the $z$ coordinate is fixed by the mechanical design.

Stewart-Gough Geometry (8 Variables) – Un-symmetric geometry (SG8).

The previous geometry can be modified as in Fig. 3 by relaxing some constrains and by defining each leg attachment point position individually. Four variables are used to define the leg attachment points on the base plate and other four variables are used to define the leg attachment points on the top plate. In particular, $\rho_1$ and $\rho$ are the circle radius of the base and top plates, respectively; $\theta_{a_i}$ and $\theta_{b_i}$, with $i = 1, 2, 3$ are the angles that define the three joint positions in the half-plane of positive $x$ axis on the base plate and on the top plate, respectively. The joint positions of the three legs in the left half-plane are calculated by symmetry with respect to the $y$ axis. The leg attachment point positions, with respect to the base plate frame and top plate frame, can be computed in the Cartesian space as a function of the parameters just defined.

Generic One-Axis Symmetry (Un-Symmetric) Geometry (OA12).
IV. OPTIMIZATION ALGORITHM

A GA has been used to combine two different optimum objectives by properly defining a cost function to minimize. In order to minimize the maximum leg force value and to equally distribute among the legs the forces exerted by the linear actuators during a positioning and/or machining task, the maximum RMS value of the forces has been selected as a metric. A second optimum objective has been taken into account to maximize (or do not penalize) the robot workspace volume.

A. Cost Function

The cost function $F$ can be defined as in (10), where the parameters $k_1$ and $k_2$ determine the weight of each optimum objective in $F$.

$$F = k_1W_1 + k_2W_2$$

The first contribution $W_1$ takes into account the leg forces necessary to follow a given position and orientation trajectory of the mobile plate, that depends on the specific application, e.g. a part positioning during an assembly process, and to balance external forces applied to the top plate, e.g. during the handling of a part subject to a machining process. The contribution $W_2$ takes into account the workspace volume. A possible choice of the two terms $W_1$ and $W_2$ can be as in (11), where $\tau(k)$ is the vector of the leg forces at the $k$th time instance of the task execution and $V_W$ is the volume of the robot workspace for a given design.

$$W_1 = \frac{1}{N} \sum_{k=1}^{N} \|\tau(k)\|, \quad W_2 = -V_W$$

The leg forces are computed by solving the inverse dynamics of the Stewart platform using the dynamic model described in Section III-A, using as input the leg positions, velocities and accelerations computed by solving the IK problem given the desired top plate trajectory. The platform workspace volume is computed by considering the geometrical approach proposed by [17]. In particular, the considered workspace is the positional workspace (or fixed-orientation workspace), computed maintaining the top plate orientation aligned with the base frame. The choice of such $W_1$ allows to reduce the maximum value of the forces required by the actuators and it allows to equalize the mean value of the six forces along the entire considered trajectory.

B. Genetic Algorithm

The optimization parameters have been set to be:

- Population Size = Number of variables * 15
- Number of Generations = 50

The optimization process stops if the maximum number of generations is reached or if in two successive generations the cumulative change in the cost function value is less than the termination tolerance value set to $10^{-6}$.

For each geometry illustrated in Section III-B, in order to avoid collisions between the legs, actuators and fixtures, sets of variable boundaries have been properly defined. Moreover, such boundaries have been defined to satisfy other mechanical design constraints, e.g., maximum dimensions of both base and top plates (0.3500 m and 0.1500 m of radius, respectively), introduced assuming a limited space available for robot installation. All the proposed setting parameters have been adjusted in successive simulations and they have been chosen so that the algorithm has been provided the best result.

V. RESULTS AND DISCUSSION

This section reports the results of the optimization process. The optimization of the Stewart platform has been carried out by considering the geometries illustrated in Section III-B for the case study described in the Section V-A. Moreover, in order to compute the workspace volume of the Stewart platform, the motor strokes, and then, the minimum and maximum leg lengths are required and they have been chosen as $Motor_{stroke} = 0.2500 \text{m}$, $l_{min} = 0.4205 \text{m}$, $l_{max} = 0.6705 \text{m}$, respectively.

A. Case study definition

The considered case study consists of two phases: in the first phase the hexapod moves a work-object in a desired position; in the second phase the hexapod holds the work-object during a manufacturing process, e.g. a drilling process. In detail, in the positioning task the hexapod moves a work-object of 50 kg weight along a desired trajectory. The work-object is attached to the mobile plate through a weld joint simulating a tight grasp. The trajectory is planned in the Cartesian space and it is defined by imposing the initial and final pose of the mobile frame (positioned in the center of gravity (COG) point of the top plate and oriented to be parallel to the base frame when the top plate is parallel to the base plate). The robot moves from the initial configuration $x_i$ to the final configuration $x_f$ reported in (12) (position expressed in meter and orientation in degrees) in a given time $t = 5 \text{s}$. The motion timing law is of third order polynomial type and it is implemented using the “spline” command in Matlab. Using the IK algorithm, the leg lengths have been computed and the inverse dynamics of the hexapod has been solved by controlling the position of each linear actuator.

$$x_i = [0, 0, 0.5125, 0, 0, 0]^T$$

When the mobile plate reaches the final pose, the manufacturing task starts. The hexapod holds the work-object fixed in the final position and a force of 500 N is applied along the $x$ direction at $t = 6 \text{s}$ for 2 seconds on a given point $p_0$. This point is reported in eq. (13), where $WorkObj_{COG_f}$ denotes the final position of the work-object COG.

$$x_f = [0.0064, -0.0043, 0.6930, 8, -3, -1]^T$$

$$p_0 = WorkObj_{COG_f} + [0.1000 \text{m} 0 \text{m}]^T$$

The results of the optimization process are reported below. The analysis has been carried out by changing the values of the weight parameters $k_1$ and $k_2$ in the cost function (10) and by considering a set of initial joint positions obtained from a purely mechanical design accomplished by reducing the overall dimensions of the Stewart platform and its incumbance. The selected values of $k_1$ and $k_2$ have been calibrated in successive simulations until the desired result has been obtained. For brevity, two optimization processes for the SG4 geometry
and one optimization process for SG8 and OA12 geometries are reported. The computational time required to optimize the hexapod design increases by increasing the number of unknown variables and it ranges from half an hour for the SG4 geometry to one hour and half for the OA12 geometry. The presented results are summarized in Table I. Figure 5 shows the initial workspace of the Stewart platform and the leg forces required to execute the assigned task.

**SG4**
1. $k_1 = 0.1, k_2 = 100$ (Figure 6).

   The maximum actuation force is about 1251 N in the manufacturing task.

2. $k_1 = 0.1, k_2 = 1$ (Figure 7).

   A higher emphasis is given to the leg force contribution in the cost function during the optimization process by decreasing the weight parameter $k_2$. The leg forces and the volume workspace decrease accordingly.

   The maximum actuation force is about 656 N in the manufacturing task.

**SG4**

1. $k_1 = 0.1, k_2 = 1$ (Figure 8).

   The maximum actuation force is about 468 N in the manufacturing task.

**OA12**

1. $k_1 = 0.1, k_2 = 1$ (Figure 9).

   The maximum actuation force is about 452 N in the manufacturing task.

<table>
<thead>
<tr>
<th>TABLE I. SIMULATION RESULTS</th>
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<tr>
<td>Max pos. force [N]</td>
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<tr>
<td>Initial Design</td>
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<tr>
<td>SG4 - $k_2 = 100$</td>
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<td>SG4 - $k_2 = 1$</td>
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<td>SG8 - $k_2 = 1$</td>
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<td>OA12 - $k_2 = 1$</td>
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### B. Discussion

Adjusting the weight parameters $k_1$ and $k_2$ the presented tool permits to obtain an optimized solution tailored to the spe-
cific application decreasing significantly the forces exerted by the linear actuators and, therefore, increasing the robot workspace volume. In fact, given the initial design in Fig. 5 that requires a maximum leg force value of about 2650 N and allows to obtain a $V_W = 0.20086 m^3$, the presented optimized designs surely provide a smaller maximum leg force (up to 85% of reduction of the maximum force in the OA12 geometry). This results into a more accurate and cost-effective choice of the mechanical components of the platform. Moreover, a larger workspace can be obtained by choosing a larger $k_2$, e.g. in SG4-a, although an higher maximum leg force is achieved. Note that the un-symmetric geometry, such as SG8 or OA12, provides a smaller force value than the symmetric one, but a symmetric geometry could better balance external forces applied along directions not considered in the optimization process. So, a symmetric design may prove more reliable than the un-symmetric one if it is used in a generic application and in a task not considered in the optimization process.

VI. CONCLUSION

The proposed solution provides a useful simulation environment to facilitate the design of an ad-hoc cost-effective Stewart platform-based robot and to allow a proper choice of its components. By using a GA, a cost function, that takes into account the leg force values and the workspace volume, is minimized. Three different geometry representations are investigated and analyzed obtaining a reduction up to 85% of the maximum force required at the actuators compared to the proposed initial design. New hexapod geometries will be investigated and new simulations will be carried out in order to find a more suitable solution in terms of required actuator forces, i.e., MSP and Griffis-Duffy geometries. Furthermore, a sensitivity analysis of the obtained solution with respect to the given trajectory will be performed.

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