# Bistatic MIMO radar for near field source localization using PARAFAC

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In this letter, we present a near field targets localization method using PARAFAC decomposition with a bistatic MIMO system. PARAFAC has the interesting properties of uniqueness in the decomposition of tensor constructed from the received signal and automatic pairing of emitter and receiver directional vectors of a bistatic MIMO radar, which allow the estimation of the parameters of each source individually. Furthermore, the proposed method provides very good estimation performance, because it doesn't require the approximation of the directional vectors contrary to the most existing techniques.

*Introduction:* Ground penetration radar and indoor localization are some typical applications of near field source localization. For a near field source, we need to estimate its range and Direction of Arrival (DOA) to perform its localization. The most existing methods use an approximated model to overcome the nonlinearity of the arguments of the elements of the directional vectors [8, 6, 5], which allows to greatly reduce the computational time but sacrifices precision.

A few decades ago, tensor decomposition techniques like Canonical Decomposition and Parallel Factor analysis (PARAFAC) were developed to solve psychometric and chemometric problems [10, 11]. These are similar concepts which were independently proposed for different applications. Unlike matrix decomposition, the tensor decomposition is unique and has attracted a lot of interests [1]. Another advantage of PARAFAC is its automatic pairing of directional vectors [3], which allows to isolate the contribution of each source and estimate its parameters independently from the other sources.

For a near field source, its spatial coordinates provide the minimum and necessary information for its localization. Thus, this letter focuses on the joint estimation of the abscissa and ordinate of targets using PARAFAC with a bistatic MIMO system. The novelty of this method is that it avoids the approximation of near field received and transmit directional vectors, which in return provides better precision.

*Signal Model:* Assume a bistatic MIMO radar with Uniform Linear Array (ULA) of $2M+1$ transmitting and $2N+1$ receiving omnidirectional antennas, separated by inter-element spacing of $d_t$ and $d_r$ respectively. The center of the transmitter and receiver arrays is taken as the reference point ($O$) as shown in Fig. 1.

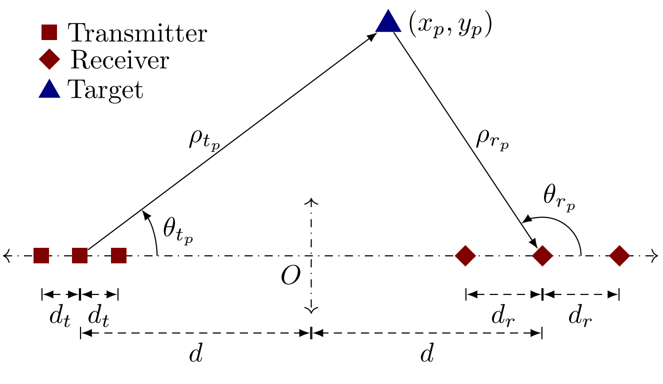


Fig. 1. Bistatic MIMO radar configuration

Let the abscissa and ordinate of the $p^{th}$ target be $x_p$ and $y_p$ respectively, where $p \in [1, 2, 3, \cdots, P]$. Let $d$ represent the distance from the centers of transmitter and receiver arrays to the reference point. Furthermore, to simplify the model, we assume that the two arrays are aligned.

By taking any $L$ samples of the received signal from the output of the matched filters, a three-dimensional tensor ($\boldsymbol{\mathcal{Y}}$) can be created [3].

Let the directional vectors of receiver and transmitter be $\boldsymbol{A} \in \mathbb{C}^{(2N+1)\times P}$ and $\boldsymbol{B} \in \mathbb{C}^{(2M+1)\times P}$ respectively and the signal that contains the reflection coefficients and Doppler shifts of the targets be $\boldsymbol{C} \in \mathbb{C}^{L\times P}$ [4]. $\boldsymbol{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_P]$, $\boldsymbol{B} = [\boldsymbol{b}_1, \boldsymbol{b}_2, \cdots, \boldsymbol{b}_P]$ and $\boldsymbol{C} = [\boldsymbol{c}_1, \boldsymbol{c}_2, \cdots, \boldsymbol{c}_P]$. In terms of $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$, $\boldsymbol{\mathcal{Y}}$ can be written as $\boldsymbol{\mathcal{Y}} = \sum_{p=1}^{P} \boldsymbol{a}_p \circ \boldsymbol{b}_p \circ \boldsymbol{c}_p + \boldsymbol{\mathcal{W}}$, where $\circ$ denotes the outer product and $\boldsymbol{\mathcal{W}} \in \mathbb{C}^{(2N+1)\times(2M+1)\times L}$ is an Additive White Gaussian Noise tensor with zero mean. For a near field target situation, $\boldsymbol{b}_p$ and $\boldsymbol{a}_p$ can be written as $\boldsymbol{b}_p = \left[e^{j\frac{2\pi}{\lambda}\delta_{t_{(-M,p)}}}, \cdots, 1, \cdots, e^{j\frac{2\pi}{\lambda}\delta_{t_{(M,p)}}}\right]^T$ and $\boldsymbol{a}_p = \left[e^{j\frac{2\pi}{\lambda}\delta_{r_{(-N,p)}}}, \cdots, e^{j\frac{2\pi}{\lambda}\delta_{r_{(N,p)}}}\right]^T$, where $\lambda$ is the wavelength of the carrier and, $\delta_{t_{(m,p)}}$ and $\delta_{r_{(n,p)}}$ are, respectively, the path differences with respect to the centers of the receiver and the transmitter arrays, given by

$$\delta_{t_{(m,p)}} = \sqrt{(x_p + d - m\,d_t)^2 + y_p^2} - \sqrt{(x_p + d)^2 + y_p^2} \qquad (1)$$

$$\delta_{r_{(n,p)}} = \sqrt{(x_p - d - n\,d_r)^2 + y_p^2} - \sqrt{(x_p - d)^2 + y_p^2} \qquad (2)$$

where $m \in [-M, \cdots, 0, 1, \cdots, M]$ and $n \in [-N, \cdots, 0, 1, \cdots, N]$.

*Proposed position estimation method:* For the estimation of the location of the targets, an estimate of $\hat{\boldsymbol{A}}$, $\hat{\boldsymbol{B}}$ and $\hat{\boldsymbol{C}}$ is required, which can be obtained by decomposing $\boldsymbol{\mathcal{Y}}$ using a fast algorithm given in [7].

In the following, we propose exact and approximated model based methods to recover target coordinates from $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$.

*Objective function minimization*

Let $\hat{a}_{(n,p)}$ and $\hat{b}_{(m,p)}$ be the elements of $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$ respectively. It is known that the canonical decomposition of a 3-way tensor is unique up to a scaling and a permutation of the components [9]. Therefore, we may find that the estimates $|\hat{a}_{(n,p)}|$ and $|\hat{b}_{(m,p)}|$ are not equal to unity and there may exist an angular rotation per column. However, the middle rows of $\boldsymbol{A}$ and $\boldsymbol{B}$ have zero argument and every element has unity modulus. Therefore, to get rid of that scaling on the estimated matrices, the following operations are required: $\hat{\boldsymbol{A}} = \hat{\boldsymbol{A}}\,\mathrm{diag}\left\{1/\hat{a}_{(0,1)}, 1/\hat{a}_{(0,2)}, \cdots, 1/\hat{a}_{(0,P)}\right\}$ and $\hat{\boldsymbol{B}} = \hat{\boldsymbol{B}}\,\mathrm{diag}\left\{1/\hat{b}_{(0,1)}, 1/\hat{b}_{(0,2)}, \cdots, 1/\hat{b}_{(0,P)}\right\}$, where $\hat{a}_{(0,p)}$ and $\hat{b}_{(0,p)}$ are the middle elements of the $p^{th}$ columns of $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$ respectively.

The following cost function can be used to recover $(\hat{x}_p, \hat{y}_p)$ from $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$.

$$(\hat{x}_p, \hat{y}_p) = \underset{(x_p, y_p)}{\arg\min} \left( \|\hat{\boldsymbol{a}}_p - \boldsymbol{a}(x_p, y_p)\|_F^2 + \left\|\hat{\boldsymbol{b}}_p - \boldsymbol{b}(x_p, y_p)\right\|_F^2 \right) \qquad (3)$$

where $\hat{\boldsymbol{a}}_p$ and $\hat{\boldsymbol{b}}_p$ are the $p^{th}$ columns of $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$ respectively.

Newton's optimization method can be used to solve (3).

*Approximated model*

In terms of range and angle, (1) and (2) can be rewritten as $\delta_{t_{(m,p)}} = \left(\sqrt{\rho_{t_p}^2 + m^2 d_t^2 - 2m\,d_t\,\rho_{t_p}\cos\left(\theta_{t_p}\right)} - \rho_{t_p}\right)$ and $\delta_{r_{(n,p)}} = \left(\sqrt{\rho_{r_p}^2 + n^2 d_r^2 - 2n\,d_r\,\rho_{r_p}\cos\left(\theta_{r_p}\right)} - \rho_{r_p}\right)$ respectively.

For a near field situation, the second-order Taylor expansion is usually used to approximate $\delta_{t_{(m,p)}}$ and $\delta_{r_{(n,p)}}$ [8, 6, 5]. By ignoring the higher order terms, we obtain

$$\frac{2\pi}{\lambda}\delta_{t_{(m,p)}} \approx m\,\omega_{t_p} + m^2\,\phi_{t_p} \qquad (4)$$

$$\frac{2\pi}{\lambda}\delta_{r_{(n,p)}} \approx n\,\omega_{r_p} + n^2\,\phi_{r_p} \qquad (5)$$

where $\omega_{t_p} = -\frac{2\pi d_t}{\lambda}\cos\left(\theta_{t_p}\right)$, $\omega_{r_p} = -\frac{2\pi d_r}{\lambda}\cos\left(\theta_{r_p}\right)$, $\phi_{t_p} = \frac{\pi d_t^2}{\lambda\rho_{t_p}}\sin^2\left(\theta_{t_p}\right)$ and $\phi_{r_p} = \frac{\pi d_r^2}{\lambda\rho_{r_p}}\sin^2\left(\theta_{r_p}\right)$.

The directional vectors for approximated model can be written as $\widetilde{\boldsymbol{a}}_p = \left[e^{j(-N\omega_{r_p} + (-N)^2\phi_{r_p})}, \cdots, 1, \cdots, e^{j(N\omega_{r_p} + (N)^2\phi_{r_p})}\right]^T$ and $\widetilde{\boldsymbol{b}}_p = \left[e^{j(-M\omega_{t_p} + (-M)^2\phi_{t_p})}, \cdots, 1, \cdots, e^{j(M\omega_{t_p} + (M)^2\phi_{t_p})}\right]^T$.

In this signal model, $\theta_{t_p}$, $\theta_{r_p}$, $\rho_{t_p}$ and $\rho_{r_p}$ are related. In fact, any two out of these four parameters can be used to obtain the coordinates. Here, we have used the angles of arrival and departure, which can be recovered from the above model as mentioned below.

Inspired by [2], let $\check{\boldsymbol{a}}_p = \boldsymbol{J_N}\,\widetilde{\boldsymbol{a}}_p \odot \widetilde{\boldsymbol{a}}_p^*$ and $\check{\boldsymbol{b}}_p = \boldsymbol{J_M}\,\widetilde{\boldsymbol{b}}_p \odot \widetilde{\boldsymbol{b}}_p^*$ where $\odot$ is the Hadamard product operator and $\boldsymbol{J_N} \in \mathbb{C}^{(2N+1)\times(2N+1)}$

and $\boldsymbol{J_M} \in \mathbb{C}^{(2M+1)\times(2M+1)}$ are the exchange matrices with all anti-diagonal elements equal to 1, otherwise 0.

$\breve{\boldsymbol{a}}_p = \left[e^{2j\,N\,\omega_{r_p}}, \cdots, e^{2j\,\omega_{r_p}}, 1, e^{-2j\,\omega_{r_p}}, \cdots, e^{-2j\,N\,\omega_{r_p}}\right]^T$ and $\breve{\boldsymbol{b}}_p = \left[e^{2j\,M\,\omega_{t_p}}, \cdots, e^{2j\,\omega_{t_p}}, 1, e^{-2j\,\omega_{t_p}}, \cdots, e^{-2j\,M\,\omega_{t_p}}\right]^T$ show that $\breve{\boldsymbol{a}}_p$ and $\breve{\boldsymbol{b}}_p$ do not depend on $\phi_{r_p}$ and $\phi_{t_p}$ respectively. Therefore, $\omega_{r_p}$ and $\omega_{t_p}$ can be estimated as $\hat{\omega}_{r_p} = \frac{1}{4N}\sum_{n=1}^{2N} \angle\left(\frac{\breve{a}_p(n)}{\breve{a}_p(n+1)}\right)$ and $\hat{\omega}_{t_p} = \frac{1}{4M}\sum_{m=1}^{2M} \angle\left(\frac{\breve{b}_p(m)}{\breve{b}_p(m+1)}\right)$, where $\angle(\bullet)$ stands for the angle (or argument) of the complex number. Furthermore, the angles of arrival and departure can be computed by $\hat{\theta}_{r_p} = \cos^{-1}\left[-\frac{\lambda}{2\pi d_r}\hat{\omega}_{r_p}\right]$ and $\hat{\theta}_{t_p} = \cos^{-1}\left[-\frac{\lambda}{2\pi d_t}\hat{\omega}_{t_p}\right]$ respectively.

Finally, to obtain the position of the target in terms of Cartesian coordinates, the following relations can be used: $\widetilde{x}_p = d\,\sin\left(\hat{\theta}_{r_p} + \hat{\theta}_{t_p}\right) / \sin\left(\hat{\theta}_{r_p} - \hat{\theta}_{t_p}\right)$ and $\widetilde{y}_p = \left|(\widetilde{x}_p + d)\tan\left(\hat{\theta}_{t_p}\right)\right|$, which can be derived by applying basic trigonometry on Fig. 1.

*Simulation results:* Consider a bistatic MIMO radar with following simulation parameters, $M = 2$, $N = 3$ and $L = 100$.

Fig. 2 shows the estimated positions with exact and approximated models at 5 dB SNR for $K = 100$ Monte-Carlo iterations by taking $d = \lambda$ and $d_t = d_r = \lambda/4$.

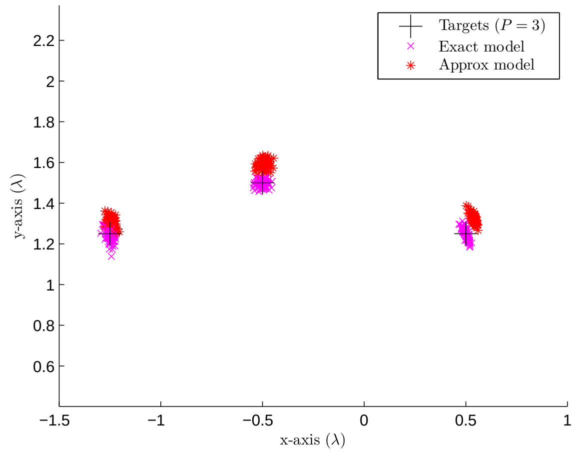


**Fig. 2** *Estimated positions when $d = \lambda$, $d_t = d_r = \lambda/4$, $M = 2$, $N = 3$, $L = 100$, $K = 100$ , $P = 3$ and SNR = 5 dB*

The estimates using approximation show some deviation from the nominal position of the targets, which is no longer true when using the exact model. This deviation can also be observed in Table 1, which is constructed by calculating the coordinates from the approximated model in a noiseless situation.

**Table 1:** Coordinates recovered from the approximated model, without noise

| $x\,(\lambda)$ | $y\,(\lambda)$ | $\widetilde{x}\,(\lambda)$ | $\widetilde{y}\,(\lambda)$ |
|---|---|---|---|
| $-1.25$ | $1.25$ | $-1.2421$ | $1.3009$ |
| $-0.5$ | $1.5$ | $-0.4946$ | $1.5908$ |
| $0.5$ | $1.25$ | $-0.5388$ | $1.3253$ |

The approximated model method presents ambiguity when the inter-element spacing is greater than $\lambda/4$. But for the exact model, we have no such problem, which is shown in Fig. 3 with $K = 100$, $d = 2\,\lambda$ and $d_t = d_r = \lambda/2$.

The simulation results show that the proposed method has high precision due to the use of exact model and works as well for $\lambda/4$ and $\lambda/2$ inter-element spacing unlike conventional approximated model methods, for which $\lambda/4$ is the upper limit of the inter-element spacing.

*Conclusion:* This letter proposes an accurate exact model based method to estimate the position of the targets in near field region with a bistatic MIMO radar system. The proposed method uses factor analysis to estimate the components and then precisely locates the targets in terms

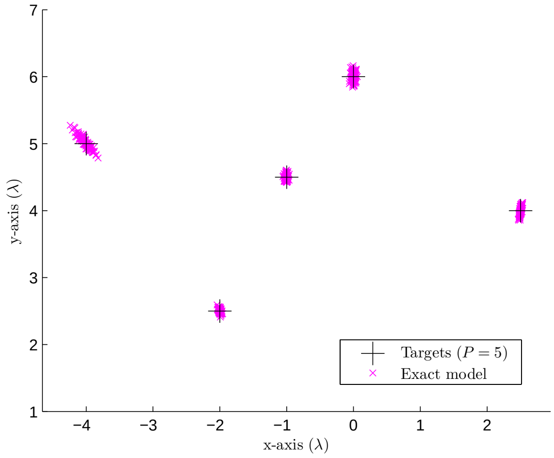


**Fig. 3** *Estimated positions when $d = 2\,\lambda$, $d_t = d_r = \lambda/2$, $M = 2$, $N = 3$, $L = 100$, $K = 100$ , $P = 5$ and SNR = 5 dB*

of Cartesian coordinates. Thanks to the automatic pairing property of PARAFAC, no extra efforts are required to pair the directional vectors of emitter and receiver, which is often an issue with the methods suggested for a bistatic MIMO system. Another benefit of the proposed method over other approximated model methods is that it works even for inter-sensor spacing of $\lambda/2$.


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