Strategic delegation in a sequential model with multiple stages

Paraskevas Lekeas∗ Giorgos Stamatopoulos†

Abstract
We analyze strategic delegation in a Stackelberg model with an arbitrary number, \( n \), of firms. We show that the \( n - 1 \) last movers delegate their production decisions to managers whereas the first mover does not. Equilibrium incentive rates are increasing in the order with which managers select quantities. Letting \( u^*_i \) denote the equilibrium payoff of the firm whose manager moves in the \( i \)-th place, we show that \( u^*_n > u^*_{n-1} > ... > u^*_2 > u^*_1 \). We also compare the delegation outcome of our game with that of a Cournot oligopoly and show that the late (early) moving firms choose higher (lower) incentive rates than the Cournot firms.

Keywords: Sequential competition; late-movers’ advantage; delegation

1 Introduction

The Stackelberg model of market competition is a benchmark model of industrial economics. In this model, firms select their market strategies (quantities or prices) sequentially. One of the most important issues in this framework has to do with the relation between order of play and relative profitability of firms. For the case of two players, Gal-Or (1985) showed that if the players’ reaction functions are downwards-sloping then the first-mover achieves a higher payoff than his opponent. On the other hand, in the case of upwards-sloping reaction functions the advantage is with the second-mover. Further studies showed that this result is not robust to

∗Department of Applied Mathematics, University of Crete, Heraklion, Crete, Greece; email: plekeas@tem.uoc.gr

†Department of Economics, University of Crete, 74100 Rethymno, Crete, Greece; email: gstamato@econ.soc.uoc.gr

The result of Gal-Or (1985) is obtained in a set-up which includes the Stackelberg model as a special case.
variations of the model. Gal-Or (1987) studied a Stackelberg model where firms compete under private information about market demand. In this model the first-mover might earn a lower profit than his opponent, as he produces a relatively low quantity in order to send a signal for low demand. Liu (2005) analyzed a model where only the first-mover has incomplete information about the demand and showed that for some cases the first-mover loses the advantage. Vardy (2004) analyzed a sequential game where observing the first-mover’s choice is costly. It is shown that being the leader has no value, no matter how small the observation cost is.

For the case of \( n \geq 2 \) symmetric firms, Boyer and Moreau (1986) and Anderson and Engers (1992) showed that the \( i \)-th mover obtains a higher profit than the \( i+1 \)-st mover, for \( i = 1, 2, ..., n-1 \). Pal and Sarkar (2001) analyzed a model with \( n \geq 2 \) cost-asymmetric firms under the assumption that the later a firm moves in the market, the lower its marginal cost. They showed that if cost differentials are sufficiently low, the firm that moves in stage \( i \) obtains a higher payoff than its successor \( i + 1 \); otherwise, the ranking of profits is reversed.

Recently, an integration of the Stackelberg model with the theory of endogenous objectives of oligopolistic firms has taken place. The latter theory was launched with the works of Fershtman and Judd (1985), Vickers (1985) and Sklivas (1987). These works endogenized the objective functions of firms in a context of management/ownership separation by postulating that firms maximize a combination of revenue and profit or quantity and profit. This framework was applied by Kopel and Loffler (2008) to a Stackelberg duopoly with homogeneous commodities (which give rise to downwards sloping reaction functions). Their paper analyzed the effect of delegation on the structure of leader versus follower advantage. The authors showed that only the follower delegates the production decision to a manager. As a result, the follower produces a higher quantity than the leader and thus achieves higher profit.

Our paper analyzes strategic delegation in a Stackelberg model with an arbitrary number of firms. Our model is an extension of the strategic delegation setup presented in Kopel and Loffler (2008). Our aim is to determine the relations between: (i) the timing of commitment in the market; (ii) the equilibrium delegation decisions and (iii) the relative performance of firms. Moreover, we are interested in comparing the equilibrium of the sequential market with the equilibrium of a corresponding Cournot market.

Our results are as follows: First, we show that all firms delegate, except for the first-mover. Moreover, the incentive rate is increasing function of the order of play. Namely, the later a firm’s manager selects a quantity for his firm, the higher his incentive rate. More importantly, letting \( u^*_n \) denote the equilibrium payoff of the firm whose manager selects in stage \( i \), we show that \( u^*_n > u^*_n-1 > \ldots > u^*_2 > u^*_1 \). This is due to the aggressiveness of the managers of the late-moving firms.

\(^2\)Kopel and Loffler also considered investment in R&D, but the present paper focuses on their delegation setup in an \( n \)-firm oligopoly.
Delegation in a Cournot model leads to an equilibrium where all firms end-up with a lower payoff compared to the case of non-delegation. This is not true though for the Stackelberg model: we show that there is a stage of play such that all firms moving after this stage prefer the delegation regime over non-delegation. However, we show that if the number of firms is \( n \geq 3 \), each firm in the Stackelberg market earns a lower payoff than a Cournot firm.

The rest of the paper is organized as follows. Section 2 presents the model and section 3 presents the results. Section 4 concludes.

2 The framework

Consider an \( n \)-firm sequential oligopoly. Firms face the inverse demand function \( P(Q) = \max\{a - Q, 0\} \), where \( P(.) \) is the market price and \( Q \) is the total market quantity given by \( Q = q_1 + q_2 + \ldots + q_n \), \( q_i \) being the quantity of firm \( i = 1, 2, \ldots, n \).

The production technology of firm \( i \) is represented by the cost function \( C(q_i) = cq_i \), \( i = 1, 2, \ldots, n \). Firms are characterized by separation of ownership-management. Firm \( i \)'s manager selects a quantity via the maximization of an objective function delegated to him by the owners of the firm. We assume that the objective function is a combination of profit and quantity (Vickers 1985)\(^3\).

\[
T_i(Q) = (P(Q) - c)q_i + a_iq_i, \quad a_i \geq 0, \quad i = 1, 2, \ldots, n
\]

The time structure of the interaction among firms and managers is as follows. In stage 0, the owners of the firms decide simultaneously on the incentive scheme of their managers. In particular, firm \( i \) chooses the parameter \( a_i \) so as to maximize its profit function

\[
u_i = (a - Q - c)q_i, \quad i = 1, 2, \ldots, n
\]

The choices of firms are made publicly known. Then managers commit to quantities in a sequential manner: in stage \( i \) the manager of firm \( i \) selects a quantity for his firm given the quantity choices of managers of firms 1, 2, \ldots, \( i - 1 \) and the choices of incentive rates in stage 0.

We denote the above interaction by \( G_S \). In the next section we identify its sub-game perfect Nash equilibrium (SPNE) outcome.

3 Results

3.1 Quantity stages

Working backwards, we first analyze the quantity competition stages. Notice first that the quantity stages form a subgame which is equivalent to a Stackelberg

\(^3\)The results of this paper do not change if we assume that the objective function of each firm is a convex combination of profit and revenue (Fershtman and Judd 1985, Sklivas 1987).
game with \( n \) marginal cost-asymmetric firms having marginal cost parameters \((c_1, c_2, ..., c_n) = (c-a_1, c-a_2, ..., c-a_n)\). Depending on the choices of \((a_1, a_2, ..., a_n)\) in stage 0 we can have, a priori, some managers producing zero quantities in the Stackelberg game. We will show though that any such configuration cannot be part of any SPNE outcome of \(G_S\).

Define \(Q^i = q_1 + q_2 + ... + q_{i-2} + q_i\). Consider first stage \( n \). We will denote by \(f^1_n(q_1, ..., q_{n-1})\) the step 1 reaction function (or simply reaction function) of manager \( n \), defined by

\[
f^1_n(q_1, ..., q_{n-1}) = \arg\max_{q_n \geq 0} (a - Q + c)q_n
\]

For the moment we do not give the conditions under which a manager selects positive or zero quantity; we turn to this later on in the analysis. Moving to stage \( n - 1 \), the (step 1) reaction of manager \( n - 1 \) is

\[
f^1_{n-1}(q_1, ..., q_{n-2}) = \arg\max_{q_{n-1} \geq 0} (a - Q^{n-2} - q_{n-1} - f^1_n - c + a_{n-1})q_{n-1}
\]

Then the step 2 reaction of manager \( n \) is derived by \(f^1_n\) when \( q_{n-1} \) is replaced by \(f^1_{n-1}\), i.e.,

\[
f^2_n(q_1, ..., q_{n-2}) = f^1_n|_{q_{n-1}=f^1_{n-1}}
\]

Moving on to stage \( n - 2 \), the step 1 reaction function of manager \( n - 2 \) is defined by

\[
f^1_{n-2}(q_1, ..., q_{n-3}) = \arg\max_{q_{n-2} \geq 0} (a - Q^{n-2} - q_{n-2} - f^1_{n-1} - f^2_n - c + a_{n-2})q_{n-2}
\]

Plugging \(f^1_{n-2}\) into \(f^1_{n-1}\) will give us the step 2 reaction of firm \( n - 1 \). Namely,

\[
f^2_{n-1}(q_1, ..., q_{n-3}) = f^1_{n-1}|_{q_{n-2}=f^1_{n-2}}
\]

Moreover plugging \(f^1_{n-2}\) into \(f^2_n\) (for \( q_{n-2} \)) will give us the step 3 reaction function of manager \( n \), i.e.,

\[
f^3_n(q_1, ..., q_{n-3}) = f^2_n|_{q_{n-2}=f^1_{n-2}}
\]

We can iteratively continue this way and define the reaction functions up to stage 2.\(^4\) Then, in stage 1, manager 1 solves

\[
\max_{q_1 \geq 0} (a - q_1 - \sum_{k=2}^{n} f^k_{k-1} - c + a_1)q_1
\]

Let \(q^*_1\) denote manager 1’s choice. Then \(q^*_2 = f^2_2(q^*_1), q^*_3 = f^2_3(q^*_1) = f^3_3(q^*_1, q^*_2), \ldots,\)

up to \(q^*_n = f^2_{n-1}(q^*_1) = ... = f^1_n(q^*_1, ..., q^*_1)\).

\(^4\)Whenever there is no confusion, we will drop the variables \(q_1, q_2, \ldots \) from the definitions of the various reaction functions.

\(^5\)Recall that the first-moving manager does not have a reaction function.
Given the above we can now look more closely on what quantity configurations can support an SPNE outcome of $G_S$. In particular we need to examine conditions under which a manager selects positive or zero quantity. To this end, consider the generic stage $i$. Using our description, manager $i$’s (step 1) reaction function is given by

$$f^1_i(q_1, ..., q_{i-1}) = \arg\max_{q_i \geq 0} (a - Q^i - q_i - \sum_{k=i+1}^{n} f^k_{i-k} - c + a_i)q_i$$

Let $T_i(q_1, ..., q_i) = (a - Q^i - q_i - \sum_{k=i+1}^{n} f^k_{i-k} - c + a_i)q_i$ \[\text{Notice that} \]

$$\frac{\partial T_i(q_1, ..., q_i)}{\partial q_i} = a - Q^i - 2q_i - \sum_{k=i+1}^{n} (f^k_{i-k}(q_1, ..., q_i) - \frac{\partial f^k_{i-k}}{\partial q_i})q_i - c + a_i$$

By the concavity of $T_i$ in $q_i$, if $\frac{\partial T_i(q_1, ..., 0)}{\partial q_i} \leq 0$ then $f^1_i = 0$ whereas if $\frac{\partial T_i(q_1, ..., 0)}{\partial q_i} > 0$ then $f^1_i > 0$. These conditions imply that: (i) if $Q^i + f^1_{i+1}(q_1, ..., 0) + ... + f^{n-i}_{n}(q_1, ..., 0) \geq a - c + a_i$ then $f^1_i = 0$ and (ii) if $Q^i + f^1_{i+1}(q_1, ..., 0) + ... + f^{n-i}_{n}(q_1, ..., 0) < a - c + a_i$ then $f^1_i > 0$.

We argue that case (i) cannot be part of any SPNE outcome. To this end, consider a vector $(a_1, a_2, ..., a_n)$ of 0-stage choices. Assume that these choices are such that all managers produce positive quantities except for one, manager $i$. Let $(q^*_1, ..., q^*_{i-1}, 0, q^*_{i+1}, ..., q^*_n)$ denote this market outcome. Since $q^*_{i+1} = f^1_{i+1}(q^*_1, ..., q^*_{i-1}, 0), ..., q^*_n = f^{n-i}_{n}(q^*_1, ..., q^*_{i-1}, 0)$ and since we are in case (i), we have $q^*_1 + ... + q^*_n \geq a - c + a_i$.

But then the profit of firm $j, j \neq i$, in stage 0 is

$$u_j = (a - q^*_j - ... - q^*_n - c)q^*_j \leq (a - (a - c + a_i) - c)q^*_j = -a_iq^*_j \leq 0$$

Hence a configuration of the form $(q^*_1, ..., q^*_{i-1}, 0, q^*_{i+1}, ..., q^*_n)$ cannot support an SPNE outcome. A similar argument holds for the case where more than one managers select zero quantities. Hence in what follows we can restrict attention to the case where all firms produce positive quantities in the market.

Since in any SPNE outcome all managers produce positive quantities (by the above argument), we can use the results of Pal and Sarkar (2001) who computed the equilibrium quantities in a $n$-stage Stackelberg with cost-asymmetric firms (but without delegation) under the assumption that all firms are active. By adjusting their analysis to ours, the manager of firm $i$ chooses the quantity

$$q^*_i = (P^* - c + a_i)2^{n-i}, \quad i = 1, 2, ..., n$$  \hspace{1cm} (1)

\[\text{To be consistent, when dealing with } T_n \text{ we need to set } f^0_n = 0; \text{ and when dealing with } T_1 \text{ we set } Q^1 = 0.\]
where

\[ P^* = \frac{a}{2^n} + \sum_{i=1}^{n} \frac{c - a_i}{2^i} \]  

(1')

is the market price.

### 3.2 Delegation stage

Armed with the above, we can move to stage 0 (the delegation stage). Let \((a_1, a_2, ..., a_n) = (a_i, a_{-i})\). Using (1), the payoff of firm \(i\) in stage 0 is

\[ u_i(a_i, a_{-i}) = 2^n - i \left( \frac{a}{2^n} + \sum_{j=1}^{n} \frac{c - a_j}{2^j} - c + a_i \right) \]

The maximization problem facing firm \(i\) is

\[ \max a_i u_i(a_i, a_{-i}), \quad i = 1, 2, ..., n \]

Define \(D_i = \frac{2i}{n+1}, \sigma(i) = \frac{2i+1}{2i-2}\) and \(h(n) = -2 + 2n + 2^{2-n}\).

**Lemma 1.** The following hold in \(G_S\).

(i) Equilibrium delegation schemes are \(a_1^* = 0, \quad a_i^* = D_i \frac{a_i - c}{2^n h(n)} > 0, \quad i = 2, 3, ..., n\)

(ii) \(a_n^* > a_{n-1}^* > ... > a_2^* > a_1^*\)

**Proof.** Appears in the Appendix.

By Lemma 1, all firms, except for the first-mover, delegate in equilibrium. Moreover, the later a firm’s manager moves in the product market, the more aggressive he is. To comprehend this, notice that the effect of a marginal change of \(a_i\) on the profit of firm \(i\) consists of two conflicting components: as \(a_i\) increases, the quantity of firm \(i\) increases whereas the market price decreases. The later a manager decides on his firm’s quantity, (i) the higher the magnitude of the first effect, i.e., \(\left| \frac{\partial q_i^*}{\partial a_i} \right| > \left| \frac{\partial q_{i+j}^*}{\partial a_{i+j}} \right|, j > 0\) and (ii) the smaller the magnitude of the second effect, i.e., \(\left| \frac{\partial P^*}{\partial a_i} \right| < \left| \frac{\partial P^*}{\partial a_{i+j}} \right|, j > 0\). As a result, the owner of firm \(i+j\) has a higher incentive to make his manager aggressive in comparison to the owner of firm \(i\). Therefore, \(a_{i+j}^* > a_i^*\).

Using Lemma 1, market price, individual and total market quantities are given respectively by

\[ P^*_S = \frac{a}{2^{n-1} h(n)} + \frac{c(1 + n2^n - 2^{-n})}{2^{n-1} h(n)} \]  

(2)

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7To see this, recall that quantities and market price, before the choices of \((a_1, a_2, ..., a_n)\) are made, are given by (1) and (1').
\[ q_i^* = (2 - 2^{1-i}) \frac{a-c}{h(n)}, \quad i = 1, 2, \ldots, n \] (3)

\[ Q^*_S = (a-c)[1 - 2^{-n} + \frac{2n - 4 + 2^{2-n}}{2^nh(n)}] \] (4)

Let \( u_i^* \) denote the equilibrium profit of firm \( i, i = 1, 2, \ldots, n \), in \( G_S \). Our next result ranks these profits.

**Proposition 1.** The inequalities \( u_n^* > u_{n-1}^* > \ldots > u_2^* > u_1^* \) hold in \( G_S \).

**Proof.** Since firms face the same price and they are cost-symmetric, \( u_{i+1}^* > u_i^* \) if and only if \( q_{i+1}^* > q_i^* \), \( i = 1, 2, \ldots, n-1 \), which holds due to (3).

One question raised at this point is how does the performance of firms in \( G_S \) compare with their performance in a sequential market without any delegation activities. To address this issue, let \( \bar{u}_i \) denote the equilibrium profit of the \( i \)-th firm in a Stackelberg market without delegation. We have the following.

**Corollary 0.** There exists a function \( i'(n) \) such that \( u_i^* > \bar{u}_i \) if and only if \( i > i'(n) \).

**Proof.** The equilibrium profit of the \( i \)-th mover in \( G_S \) is \( u_i^* = (a-c)^2(1 - 2^{-i})/[2^{n-2}[h(n)]^2] \) whereas the profit of the same firm in a market without delegation activities is \( \bar{u}_i = 2^{-i}(a-c)^2/2^n \). Then, \( u_i^* > \bar{u}_i \) if and only if \( 2^{2+i} > 4 + |h(n)|^2 \). Let \( r(i) = 2^{2+i} \). It is easy to show that \( r(1) < 4 + |h(n)|^2 < r(n) \); further, \( r(i) \) is increasing in \( i \). Hence there exists a unique \( i'(n) < n \) such that \( u_i^* > \bar{u}_i \) if and only if \( i > i'(n) \).

Therefore the firms that move after the \( i'(n) \)-th stage prefer the delegation regime over non-delegation, whereas the remaining ones prefer the non-delegation regime.

### 3.3 Comparison with Cournot competition

In this section we compare the market outcome of \( G_S \) with the outcome of the corresponding Cournot market. In the latter framework, the \( n \) firms compete in a two-stage interaction as follows: in stage 0, firms select the incentive schemes of their managers. These choices are made publicly known. Then in stage 1, the managers of the \( n \) firms select simultaneously quantities for their firms, using the incentive schemes decided upon in stage 0. Let \( G_C \) denote this game.

It is known that in the absence of delegation, the Stackelberg market is more efficient than the Cournot market, as it produces a higher market quantity. Under

\[ \text{\footnote{This result is explained again by the fact that the late-moving firms have incentive to make their managers relatively aggressive at the expense of the early-movers.}} \]
not all firms delegate, unlike $G_C$. Hence a comparison between Stackelberg and Cournot competition under delegation is not obvious. In what follows we make this comparison and we also address the issue of relative profitability in $G_S$ and $G_C$.

**Corollary 1.** Consider the games $G_S$ and $G_C$.

(i) Total market quantity is higher in $G_S$.

(ii) The inequalities $a_i^* > a_C^*$ and $a_C^* > a_i^*$, $i = 1, 2, ... n - 1$, hold.

(iii) If $n = 2$, then $u_2^* > u_C^*$; if $n \geq 3$, then $u_C^* > u_i^*$, for all $i$.

**Proof.** (i) Consider the last stage of $G_C$. The quantity that the manager of firm $i$ chooses is

$$q_i(a) = \max\{(a - n(c - a_i) + \sum_{i \neq j}(c - a_j))/(n + 1), 0\}, \quad i = 1, 2, ..., n$$

Equilibrium delegation schemes are

$$a_i^* = a_C^* = \frac{n - 1}{n^2 + 1}(a - c), \quad i = 1, 2, ..., n$$

Hence, individual and total market quantities are given respectively by

$$q_i^* = q_C^* = \frac{n(a - c)}{n^2 + 1}, \quad Q^n_i = \frac{n^2(a - c)}{(n^2 + 1)}, \quad i = 1, 2, ..., n$$

Recall that market quantity in $G_S$ is

$$Q^*_S = (a - c)[1 - 2^{-n} + \frac{2n - 4 + 2^{2-n}}{2^n h(n)}]$$

It is then easy to show that $Q^*_S > Q^n_C$ if and only if $(n - 1)2^{1+n} + 2 - 2n^2 > 0$ which holds.

(ii) Notice that $a_i^* > a_C^*$ if $2^i+1 > 4 + [(n - 1)2^n h(n)]/(n^2 + 1)$. Define the function $w(i) = 2^{i+1}$ and notice that $w(n - 1) < 4 + [(n - 1)2^n h(n)]/(n^2 + 1) < w(n)$. Since $w(i)$ is strictly increasing in $i$, we conclude that $a_i^* < a_C^*$ for $i = 1, 2, ..., n - 1$ and $a_n^* > a_C^*$.

(iii) The equilibrium profit of the $i$-th mover in $G_S$ is $u^*_i = \frac{4(1 - 2^{-i})}{2^n h(n)}(a - c)^2$ whereas the profit of each firm in $G_C$ is $u^*_C = \frac{n}{(n^2 + 1)^2}(a - c)^2$. Notice that

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$^9$Vickers (1985) was the first to analyze a Cournot delegation game where firms maximize combinations of profits and quantities.

$^{10}$In the absence of delegation, each Cournot firm earns $(a - c)^2/(n + 1)^2$. Thus the non-delegation regime is preferable by all Cournot firms over the delegation regime.
$u_i^* > u_C^*$ if and only if $4 - 2^{2-i} > \frac{n2^n(h(n))^2}{(n^2+1)^2}$. Let $y(n)$ denote the right part of the last inequality. For $n \geq 3$, $y(n) > 4 > 4 - 2^{2-i}$. On the other hand, if $n = 2$, $u_1^* = (a-c)^2/18 < u_C^* = 2(a-c)^2/25 < u_2^* = (a-c)^2/12$.

Anderson and Engers (1992) compared the outcomes of Stackelberg and Cournot models with $n \geq 2$ firms but without considering the possibility of delegation. They showed that when $n = 2$, the first (second)-mover in the sequential market earns a higher (lower) profit than each of the Cournot duopolists. In our model, the second (first)-mover earns a higher (lower) profit than each of the Cournot duopolists. On the other hand, for $n \geq 3$, all Stackelberg players earn a lower profit than the Cournot players in both Anderson and Engers (1992) and in our model.

Finally, Corollary 1(ii) shows an interesting relation: all firms in $G_S$, except for the last mover, have a lower incentive to delegate than firms in $G_C$.

4 Conclusions

We analyzed strategic delegation in a Stackelberg model with an arbitrary number of firms. We showed that the later a firm’s manager commits to a quantity decision, the higher the firm’s profit. Delegation improves the payoff of the late-movers in the market and hurts the early-movers. Namely, the firms whose managers commit late (early) in the market end-up with a higher (lower) payoff compared to the non-delegation regime. This is different from the case of delegation under Cournot competition, where all firms are hurt by delegation.

Our paper has analyzed a framework with downwards-sloping reaction functions. Introducing a delegation framework with upwards-sloping reaction functions (as in a market with complementary goods) will allow us to examine whether early-movers can "steal" the advantage from late-movers.

Appendix

Proof of Lemma 1. (i) Notice that

$$\frac{\partial u_i(a_i, a_{-i})}{\partial a_i} > 0 \iff \frac{1}{2^i}(\frac{a}{2^n} + \sum_{j=1}^{n} \frac{c - a_j}{2^j} - c + a_i) + (\frac{a}{2^n} + \sum_{j=1}^{n} \frac{c - a_j}{2^j} - c)(1 - \frac{1}{2^n}) > 0$$

or iff

$$a_i < \left(\frac{a - c}{2^n} - \sum_{j \neq i} \frac{a_j}{2^j}\right) \frac{2^i(2^i - 2)}{2^{i+1} - 2}$$

Clearly, for $i = 1$, the derivative is negative and hence the equilibrium incentive rate that firm 1 chooses is $a_1^* = 0$. Let now $i \geq 2$. Then the reaction function of firm $i$ is given by
\[
a_i = \begin{cases} 
0, & \text{if } \sum_{j \neq i} a_j / 2^j \geq (a - c) / 2^n, \\
(a - c) / 2^n - \sum_{j \neq i} a_j / 2^j, & \text{if } \sum_{j \neq i} a_j / 2^j < (a - c) / 2^n,
\end{cases}
\]

where \(\sigma(i) = (2^{i+1} - 2)/(2^i - 2)\), \(i \geq 2\). We first notice that in any equilibrium of the delegation game only the first-mover chooses a zero incentive rate. All the remaining firms will choose positive incentive rates. To see this, consider an outcome \(a_i = 0\) and \(a_j > 0\), \(j = 2, \ldots, i - 1, i + 1, \ldots, n\), \(j \neq i\). The market price is \(P = a - c / 2^n + \sum_{i=1}^{n} c - a_i / 2^i\). Since \(a_i = 0\), we have \(\sum_{j \neq i} a_j / 2^j \geq (a - c) / 2^n\). But then it is easy to show the last inequality implies that the price would fall below the marginal cost \(c\). Hence we restrict attention to the case where \(\sum_{j \neq i} a_j / 2^j < (a - c) / 2^n\) for all \(i\). Then we have the system

\[
\frac{1}{2^2} \alpha_2 + \frac{1}{2^3} \alpha_3 + \cdots + \sigma(i) \frac{1}{2^i} \alpha_i + \cdots + \frac{1}{2^n} \alpha_n = \frac{a - c}{2^n}, \quad i = 2, 3, \ldots, n \quad (5)
\]

\[
\sigma(i) = \frac{2^{i+1} - 2}{2^i - 2}.
\]

Using (5) the equations for firms \(i = 2\) and 3 we get,

\[
\alpha_i = \frac{\sigma(2)}{\sigma(i) - 1} \cdot 2^{i-2} \alpha_2 \quad (6)
\]

and hence

\[
\alpha_2 = \frac{a - c}{2n-2} \sigma(2) + \sum_{i=3}^{n} \sigma(2) - \frac{1}{\sigma(i) - 1})^{-1} \quad (7)
\]

It is straightforward to show that \(\sigma(2) + \sum_{i=3}^{n} \sigma(2) - \frac{1}{\sigma(i) - 1})^{-1} = -2 + 2n + 2^{2-n} \equiv (h(n))\).

Using then (6) and (7), the solution for \(a_i\), \(i \geq 2\), is \(a_i^* = D_i \cdot \frac{a - c}{2^n} \cdot \frac{1}{h(n)}\), where \(D_i = \frac{2^{i+1}}{\sigma(i) - 1}\).

(ii) Notice that \(a_{i+1}^* > a_i^*\) if and only if \(D_{i+1} > D_i\) or \(2^{i+2}/(\sigma(i + 1) - 1) > 2^{i+1}/(\sigma(i) - 1)\), which holds because \(\sigma(i + 1) < \sigma(i)\).

**References**


7. Liu Z. (2005), Stackelberg leadership with demand uncertainty, Managerial and Decision Economics, 26, 345350.


