Inviscid and viscous aerodynamics of dense gases

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A numerical investigation of transonic and low-supersonic flows of dense gases of the Bethe–Zel’dovich–Thompson (BZT) type is presented. BZT gases exhibit, in a range of thermodynamic conditions close to the liquid/vapour coexistence curve, negative values of the Fundamental Derivative of Gasdynamics. This can lead, in the transonic and supersonic regime, to non-classical gasdynamic behaviours, such as rarefaction shock waves, mixed shock/fan waves, shock splitting and other. In the present work, inviscid and viscous flows of a BZT fluid past an airfoil are investigated using accurate thermo-physical models for gases close to saturation conditions and a third-order centred numerical method. The influence of the upstream kinematic and thermodynamic conditions on the flow patterns and the airfoil aerodynamic performance is analyzed, and possible advantages deriving from the use of a non-conventional working fluid are pointed out.

1. Introduction

Dense gases are defined as single-phase vapours operating at temperatures and pressures of the order of magnitude of those of their thermodynamic critical point. At these conditions, real gas effects play a crucial role in the dynamic behaviour of the fluid. The
study of the complicated dynamics of compressible flows of dense gases is strongly moti-
vated by the potential technological advantages of their use as working fluids in energy-
conversion cycles. Specifically, such fluids possess large heat capacities compared to their 
molecular weight, which makes them excellent heat transfer fluids in Organic Rankine 
Cycles (ORCs). Specific interest has been developed in a particular class of dense gases, 
known as the Bethe–Zel’dovich–Thompson fluids (Thompson 1971), which exhibit non-
classical gasdynamic behaviours, such as expansion shock waves, mixed shock/fan waves, 
and splitting shocks, in a range of thermodynamic conditions above the liquid/vapour 
coexistence curve, such that the fundamental derivative of gasdynamics:

\[ \Gamma := 1 + \frac{\rho}{a} \left( \frac{\partial a}{\partial \rho} \right)_s, \]  

with \( \rho \) the fluid density, \( a \) the sound speed, and \( s \) the entropy, becomes negative. \( \Gamma \) 
measures the rate of change of the sound speed in isentropic perturbations: if \( \Gamma < 1 \), 
the flow exhibits a reversed sound speed variation: \( a \) grows in isentropic expansions and 
falls in isentropic compressions, contrarily to what happens in ”common” fluids, e.g. 
perfect gases, for which \( \Gamma = (\gamma + 1)/2 \) is strictly greater than one for thermodynamic 
stability reasons, \( \gamma \) being the specific heat ratio of the fluid. The thermodynamic region 
where \( \Gamma < 0 \) is referred to as the inversion zone, and the \( \Gamma = 0 \) contour is said the 
transition line (Cramer & Kluwick 1984). In flow regions with negative \( \Gamma \), the second law 
of thermodynamics requires that compression shocks cannot form, whereas expansion 
shocks are physically admissible (see Cramer & Kluwick 1984). It is possible to show 
that the entropy change across a weak shock can be written as:

\[ \Delta s = -\frac{a^2 \Gamma}{v^3} \frac{(\Delta v)^3}{6T} + O((\Delta v)^4), \]  

where \( \Delta \) represents a change in a given fluid property through the shock, \( v = 1/\rho \) is 
the specific volume, and \( T \) is the absolute temperature. As a result, in order to satisfy
the second law of thermodynamics, a negative change in the specific volume, i.e. a compression, is required if \( \Gamma > 0 \), whereas a positive change, i.e. an expansion, is the only physically admissible solution when \( \Gamma < 0 \). In practice, \( \Gamma \) rarely has constant negative sign throughout the flow, because of the finite extent of the inversion zone. At points where \( \Gamma \) vanishes, the genuine nonlinearity of the flow characteristic fields is lost, and non-classical waves can be generated, such as mixed shock/fan waves and splitting shocks (Cramer 1991, 1989b). For example, it can happen that a compressive wave would start in the positive \( \Gamma \) region as a shock, and then split into a compression fan as the flow enters the inversion zone. This also results in discontinuities of limited strength for thermodynamic conditions close to the transition line (Cramer & Kluwick 1984). Specifically, the shock strength is reduced up to an order of magnitude from that predicted by equation (1.2) for thermodynamic conditions where \( \Gamma \approx 0 \). A steady, inviscid dense gas flow at transonic speeds undergoing small disturbances of the speed, pressure, density, etc. is then characterized by much smaller changes of the specific entropy, and the flow can be considered, in the leading orders of the disturbances, as isentropic.

Past research efforts toward demonstrating the existence of BZT fluids (see Lambrakis & Thompson 1972; Thompson & Lambrakis 1973; Cramer 1989a) indicate that several heavy compounds employed for heat transfer applications and as working fluids in Organic Rankine power cycles (ORCs) do possess BZT properties. Now, ORC turbines typically work in the transonic/low supersonic regime and their major loss mechanism is related to the generation of shock waves and their interactions with the blade boundary layers. Therefore, on the one hand a detailed study of turbomachinery flows of dense gases is necessary to correctly predict the system behaviour; on the other hand, non-classical dense gas phenomena could be exploited to improve efficiency: namely, shock formation and subsequent losses could be ideally suppressed, if the turbine expansion could happen
entirely within or in the immediate neighbourhood of the inversion zone. Previous works about BZT transonic flows past airfoils (Cramer & Tarkenton 1992) and through turbine cascades (Monaco, Cramer & Watson 1997; Brown & Argrow 2000) show that, properly operating the turbine in the very neighbourhood of the \( \Gamma = 0 \) curve, the flow field evolves almost entirely within the inversion zone and is shock-free: as a result, except for viscous drag, the flow remains almost isentropic through the entire cascade. Unfortunately, the inversion zone has a quite limited extent; therefore, a reduction in the temperature jump between the heater and condenser stages is generally required in order to completely operate the turbine within the inversion zone. Now, it is well-known from thermodynamic theory that a too small temperature jump implies low thermal cycle efficiency. Moreover, a small temperature (i.e. enthalpy) jump also means low cycle power output. This important drawback has been the stumbling block to the development of real-world BZT ORCs. In practice, BZT gas effects can find application in ORC turbomachinery only if a reasonable trade-off between the above-mentioned opposite requirements is found.

Previous works on transonic dense gas flows past airfoils and through turbine cascades generally consider operating conditions in the very vicinity of the transition line. Cramer & Tarkenton (1992) studied transonic flows past thin airfoils by solving an extended transonic small disturbance equation derived for flows with \( \Gamma \approx 0 \), \( M \approx 1 \) (with \( M \) the Mach number), and also characterized by small values of the second nonlinearity parameter \( \Lambda := \rho (\partial \Gamma / \partial \rho)_s \) (Cramer & Kluwick 1984). They found a significant increase of the critical Mach number in flows of BZT fluids over profiles. Numerical solutions of the small disturbance equation completed by the Martin–Hou gas model revealed substantial reductions in the strength of compression shocks. Morren (1991) performed numerical simulations using the Euler equations and the van der Waals equation of state, and observed an evident decrease in the pressure drag over the airfoil. Transonic flows
of a dense gas around the leading edge of a thin airfoil with a parabolic nose have been
studied by Rusak & Wang (1997). Once again, the oncoming flow is supposed to be almost
sonic, and characterized by small values of $\Gamma$ and $\Lambda$. Wang & Rusak (1999) also provided
numerical studies of transonic BZT flows past a NACA0012 airfoil at zero angle of attack
(non lifting case) using the numerical code of Morren (1991), and provided a classification
of possible flow patterns for oncoming flow conditions such that $\Gamma_{\infty}, \Lambda_{\infty} \approx 0$. Numerical
solutions of the Euler equations for isenthalpic flows through turbine cascades by using
the Martin–Hou gas model have been provided by Monaco et al. (1997) for incoming flow
conditions characterized by small values of $\Gamma$. Finally, results concerning flows through
realistic impulse turbine cascades have been presented by Brown & Argrow (2000), who
solved the Euler equations closed by the Martin-Hou gas model.

Recently, Cinnella & Congedo (2005a) have investigated the influence of BZT effects
on the system performance of inviscid transonic lifting flows past a NACA0012 airfoil.
Numerical simulations were performed by solving the Euler equations discretized by a
third-order-accurate numerical scheme on very fine meshes. The gas response was mod-
elled by the van der Waals equation of state for polytropic gases. In contrast with previous
studies, the investigation was not restricted to flows with small free-stream $\Gamma$. On the
contrary, the objective of the research was to explore the possibility of keeping part of
the benefits deriving from BZT behaviour while enlarging the operation range. To this
purpose, a detailed parametric investigation of the airfoil aerodynamic performance was
undertaken, with specific interest in configurations providing the best trade-off between
high lift and low drag. Figure 1 summarizes the results obtained for a BZT van der
Waals gas flowing at $M_{\infty} = 0.85, \alpha = 1^0$ over the NACA0012 airfoil for various free-
stream thermodynamic conditions, corresponding to a series of operation points selected
along constant-entropy lines (isentropes) crossing the inversion zone (Fig. 1a); for each
isentrope, the lift coefficient and the lift-to-drag ratio are plotted against the free-stream value of $\Gamma$ (Fig. 1b). Note that the fundamental derivative does not vary monotonically along an isentrope. Nevertheless, the operating region of interest for ORCs is located at the high-pressure side of the inversion zone, where $\Gamma$ monotonically increases with pressure. For the smallest values of $\Gamma_{\infty}$, results presented in previous studies are recovered, namely, the flow is subcritical and characterized by zero drag (in the limit of vanishing mesh size). Consequently, the lift-to-drag ratio tends to infinity. The price to pay is decreased lift with respect to a system using a perfect gas as the working fluid: this is the counterpart, for an airfoil, of the aforementioned trade-off between high turbine efficiency and high cycle power output for energy-conversion systems. When $\Gamma$ is sufficiently high (approximately in the range $2 \div 3$), a significant growth in both lift and drag is observed: the increase in lift is produced by the formation of an expansion shock, close to the leading edge, which strongly enhances the suction peak at the airfoil upper surface. The increase in drag is due to the occurrence of shocks on the airfoil surface. Nevertheless, losses introduced by flow discontinuities are very low, due to the smallness of entropy changes across weak shocks in the vicinity of the transition line. In fact, the lift-to-drag ratio remains one order of magnitude greater than in perfect gas flow. Finally, when $\Gamma_{\infty}$ reaches higher values, far from the inversion zone, the flow becomes qualitatively similar to that of a perfect gas, with even poorer performance, and the benefits due to BZT effects progressively disappear. In summary, the results presented in Cinnella & Congedo (2005a) suggest that the choice of upstream conditions within or very close to the transition line is not only not mandatory in order to improve airfoil performance, as suggested in previous studies, but also not optimal. Specifically, optimal aerodynamic performance (i.e. the best trade-off between high lift and low drag) is obtained for $\Gamma = O(1)$, more precisely $\Gamma \approx 3$. This is of great importance, in light of the design of BZT Organic
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Rankine Cycles, since it suggests the possibility of enlarging the operation range of the expansion stage without losing the benefits of dense gas effects.

Results presented in Cinnella & Congedo (2005a) have been obtained using the van der Waals equation of state, also used in many former studies, as the simplest gas model accounting for BZT effects: it is computationally inexpensive, and allows capturing qualitative features of BZT fluid flows. On the other hand, this model is not very accurate for thermodynamic conditions close to saturation, i.e. the region of interest in the present study, and largely over-predicts the extent of the inversion zone (Thompson & Lambrakis 1973). The Amagat \( (p - v) \) diagrams reported in Fig. 2 give an idea of the different behaviour of a van der Waals gas with \( \gamma = 1.0125 \), representative of a generic heavy fluorocarbon, and of a real gas modelled through the more realistic Martin–Hou equation of state, namely, the heavy fluorocarbon pf-perhydrofluorene (commercial name PP10). The van der Waals gas exhibits a very large inversion zone. Outside this zone, the fundamental derivative quickly increases, reaching values close to 3 already at a short distance from the transition line, and tends to the perfect gas value when the specific volume tends to infinity. On the contrary, the inversion zone for PP10 is much more reduced. Nevertheless, the increase of \( \Gamma \) outside the inversion zone for increasing pressure is much slower. In the following, it will be shown that this particular variation of \( \Gamma \) counterbalances to some extent the reduction in the inversion zone size encountered in real-world gases, and favourably affects the system performance.

Another limitation of the study reported in Cinnella & Congedo (2005a) is related to the fact that thermo-viscous effects are completely neglected, as also done in almost all previous studies. Indeed, viscous effects in flows of dense gases have remained largely unexplored. One of the most important differences between dense gases and perfect gases is the downward curvature and nearly horizontal character of the isotherms in the neigh-
bourhood of the critical point and upper saturation curve in the $p - v$ plane: the region of downward curvature of the isotherms is associated with the aforementioned reversed behaviour of the sound speed in isentropic perturbations. In the same region, the specific heat at constant pressure, $c_p$, can become quite large: this strongly influences the development of the thermal boundary layer and its coupling with the viscous boundary layer in high-speed flows. A second consideration is that in the dense gas regime the dynamic viscosity $\mu$ and the thermal conductivity $\kappa$ cannot be longer considered independent of the temperature and pressure, even in flows with relatively small temperature variations. On the other hand, the well-known Sutherland law, commonly used to represent viscosity variation with temperature, becomes invalid, as it is based on the hypothesis that the gas molecules act as non-interacting rigid spheres, and intermolecular forces are neglected. The complexity of the behaviour of $\mu$ in the dense regime can be anticipated by recalling that the viscosity of liquids tends to decrease with increasing temperature, whereas that of gases tends to increase: the dense gas regime is a transition between these two qualitatively different behaviours. Similarly, the classical approximation of nearly constant Prandtl number ($Pr = \mu c_p / \kappa \approx \text{const}$) cannot be used any more. As the thermal conductivity has roughly the same variation as viscosity with temperature and pressure, the behaviour of $Pr$ tends to be controlled by variations of $c_p$. In regions where $c_p$ becomes large, strong variations of $Pr$ can be observed, contrarily to what happens in perfect gases. Nevertheless, if the immediate vicinity of the thermodynamic critical point is excluded from considerations, the Prandtl number remains of order one, similar to perfect gases. In contrast, the Eckert number ($Ec = U_0^2 / (c_p T_0)$, where $U_0$ and $T_0$ refer to a suitable reference state) decreases significantly. Small flow Eckert number implies reduced sensitivity of the boundary layer to friction heating, which remains negligible even at moderately large supersonic Mach numbers. Moreover, for flows past adiabatic walls the
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Temperature, and thus also the density, is almost constant across the boundary layer. In the past, investigations of the viscous structure of one-dimensional non classical shocks have been presented in Cramer & Crickenberg (1991). Cramer, Whitlock & Tarkenton (1996) have presented a numerical investigation of laminar flows of dense gases over a flat plate. The results indicate a failure of classical scaling laws for compressible boundary layers (Chapman–Rubesin scaling), and a reduction of the boundary layer friction heating in complex gases with large heat capacities. Numerical results showing a suppression of shock induced separation in supersonic Bethe–Zel’dovich–Thompson flows past sharp compression corners have been provided by Cramer & Park (1999). More recently, Kluwick (2000) has discussed a new form of marginal boundary layer separation in laminar flows of dense gases using asymptotic methods: the non monotonous Mach-number variation with pressure leads to non conventional distributions of the shear stress and displacement body in boundary layers subjected to adverse pressure gradients, which contributes to delay separation. Kluwick & Wrabel (2004) investigated shock/boundary layer interaction in dense gases via the triple deck theory. Their results show that it is possible to reduce the size of the separation bubble or even to avoid the occurrence of flow separation by choosing an optimal operation thermodynamic state.

In the present work, a numerical investigation of two-dimensional inviscid and viscous dense gas flows past an isolated airfoil is provided. Dense gas effects are modelled through the realistic equation of state of Martin & Hou (1955), involving five virial expansion terms. This equation is widely accepted as the reference thermodynamic model for dense gases (see for example Emanuel 1994; Guardone et al. 2004). The equation is implemented within a numerical code for dense gas flow simulations, based on a simple and efficient third-order accurate centred solver (Cinnella & Congedo 2005b). The code also includes proper thermophysical models for the variation of the dynamic viscosity.
and thermal conductivity of dense gases (Chung et al. 1988). The aim of the present investigations is twofold: (i) further investigate, using more predictive thermodynamic models, mechanisms affecting aerodynamic performance (lift, drag, lift-to-drag ratio) for inviscid dense gas flows past an airfoil, providing a more quantitative evaluation of the possible gains, and extending the study to a larger range of upstream conditions, including different values of the Mach number and incidence; (ii) to investigate for the first time, to the authors’ knowledge, how the peculiar behaviour of BZT gases affects the viscous-flow performance of an airfoil. A parametric study of the influence of free-stream thermodynamic conditions on the aerodynamic performance of a BZT inviscid and viscous transonic flows past a NACA0012 airfoil has been undertaken. The results are critically analyzed and compared to those obtained for a perfect diatomic gas flowing past the same configuration, pointing out peculiarities related to the use of non conventional working fluids.

2. Governing equations, thermodynamic models, and flow solver

We consider the Navier–Stokes equations for single-phase gases, written in integral form for a control volume $\Omega$ with boundary $\partial\Omega$:

$$\frac{d}{dt}\int_{\Omega} w \, d\Omega + \oint_{\partial\Omega} \left( f - f_v \right) \cdot n \, dS = 0. \quad (2.1)$$

In Eq. (2.1), $w$ is the conservative variable vector, $n$ is the unit outer normal to $\Omega$, and $f, f_v$ are the inviscid and the viscous part of the flux density, respectively. The equations are completed by a thermal and a caloric equation of state, and by thermodynamic models relating the dynamic viscosity, $\mu$, and thermal conductivity, $\kappa$, to the gas temperature and pressure. In the present work, the Martin–Hou equation (Martin & Hou 1955) of state is used, which provides a realistic description of the gas behaviour and of the inversion zone size. Such equation, involving five virial terms and satisfying ten
thermodynamic constraints, ensures high accuracy with a minimum amount of experimental information. A power law is used to model variations of the low-density specific heat with temperature. The fluid viscosity and thermal conductivity are evaluated using the method proposed in Chung et al. (1988).

The governing equations are discretized using a cell-centred finite volume scheme of third-order accuracy, extended to the computation of flows with an arbitrary equation of state (Cinnella & Congedo 2005b). The scheme is constructed by correcting the dispersive error term of second-order-accurate Jameson’s scheme (Jameson, Schmidt & Turkel 1981). The use of a scalar dissipation term simplifies the scheme implementation with highly complex equations of state and greatly reduces computational costs. In order to preserve the high accuracy of the scheme on non Cartesian grids, the numerical fluxes are evaluated using suitably weighted discretization formulas, which take into account the stretching and the skewness of the mesh: this allows to ensure a truly third-order accuracy on moderately deformed meshes and at least second-order accuracy on highly distorted meshes (see Rezgui, Cinnella & Lerat 2001, for details). The governing equations are integrated in time using a four-stage Runge–Kutta scheme. Local time-stepping, implicit residual smoothing and multigrid are used to efficiently drive the solution to the steady state. The method has been successfully validated for a variety of perfect and real gas flows (see Cinnella & Congedo 2005a, b, and references cited therein).

3. Dense gas flows past an airfoil

The preceding numerical method is used to investigate inviscid and viscous transonic flows of a BZT dense gas (DG) past a NACA0012 airfoil. The working fluid considered in the following computations is PP10. The objective is to explore the influence of dense-gas
effects on the airfoil aerodynamic performance, also in comparison with reference results for a perfect gas (PFG) flowing at the same free-stream conditions.

3.1. Choice of the operating conditions

For a dense gas, the parameters governing the flow are, in addition to the free-stream Mach number and angle of attack, the free stream thermodynamic conditions, i.e. the thermodynamic operation point. Since typical ORC turbine blades work in the high-subsonic/low-supersonic regimes, flows past the NACA0012 airfoil are investigated for three different free-stream conditions: high-subsonic, sonic, and low-supersonic. The angle of attack is taken fixed and equal to $10^\circ$. For BZT inviscid steady flows, the flow adiabat in the $p - v$ plane is roughly superposed to the isentrope corresponding to free-stream conditions. In the $p - v$ diagram, the locus of possible thermodynamic states of a flow field is then approximately superposed with the arch of the isentrope included between the minimum and maximum pressures in the flow. If the locus crosses the inversion zone, the flow field exhibits a region of BZT effects. The operation points chosen for the present study are picked on five different isentropes of the $p - v$ plane. Figure 3 shows the five isentropes, the operation points, the inversion zone, and the dense gas region ($\Gamma < 1$) for PP10. Moving right to left along an isentrope the free-stream fundamental derivative $\Gamma_\infty$, initially positive, decreases, changes its sign where the isentrope crosses the inversion zone, reaches a minimum, then increases again (see figure 4). For high-pressure operation points, $\Gamma_\infty$ is greater than one. For these points, asymptotic theories based on the assumption $|\Gamma| << 1$ are no longer valid. Isentropes S1 to S3 cross the inversion zone. Isentrope S4 is approximately tangent to the transition line and represents a limiting case. Finally, isentrope S5 lies completely outside the inversion zone, but crosses the extended thermodynamic region where $\Gamma < 1$. For flows with free-stream entropy S5, BZT effects
cannot appear, but significant DG effects related to reverse sound speed behaviour are expected.

3.2. Inviscid flow behaviour

Inviscid flow computations are performed using three C-grids, formed by $136 \times 20$, $272 \times 40$ and $544 \times 80$ cells, respectively. The finest and the coarsest grids are generated by doubling or halving, respectively, the number of cells of the medium one in each direction. The outer boundary is about 20 chords away from the airfoil and the mean height of the first cell closest to the wall is about $5 \times 10^{-2}$ chords on the medium grid. For most of the computations presented in the following, grid convergence for the wall pressure and Mach number has been obtained on the medium grid. However, the results presented have been obtained on the finest grid. For a more uniform reporting of the grid convergence, the scheme’s order of convergence is estimated following Roache’s method (Roache 1998), based on Richardson extrapolation. Given three numerical solutions computed on grids of increasing spacing, with constant grid refinement ratio $r$, the actual order of convergence is:

$$q = \frac{\ln \left( \frac{f_3 - f_2}{f_2 - f_1} \right)}{\ln(r)},$$

where $f$ is a solution functional and indices 1 and 3 are referred to the finest and the coarsest grid solution, respectively. A computed order of convergence of 2.2 or higher, based on the lift coefficient $C_L$, has been found for the present computations, against a theoretical order of convergence of 3. The estimated order of convergence may be used to compute Roache’s Grid Convergence Index (GCI) on the finest and medium grid, which represents an estimate of how far the numerical solution is from its asymptotic value. GCIs of 0.08% and 0.39% have been found for the finer and the medium grid,
respectively, indicating that the solution is well within the asymptotic range. Such values are likely to be conservative.

3.2.1. Dense gas flows in high-subsonic free-stream

The reference solution for a perfect diatomic gas (specific heat ratio $\gamma = 1.4$) flowing at $M_{\infty} = 0.85$, $\alpha = 1^\circ$ is first considered. This is a well-known test case, which has been often utilized in the literature for the validation of numerical schemes for the Euler equations (see for instance Dervieux et al. 1989; Cinnella & Congedo 2005b): therefore, detailed results are not reported for brevity. The flow is characterized by two shocks at about 85% of the chord at the suction side, and 63% at the pressure side. The lift coefficient, drag coefficient, and lift-to-drag ratio computed with the present numerical method on the finest grid are:

$$C_L = 0.373; \quad C_D = 5.74 \times 10^{-2}; \quad C_L/C_D = 6.51.$$ 

These values fairly agree with other results reported in the literature for the same case (Dervieux et al. 1989). Then, flows of the dense gas PP10 past the same airfoil are computed. Results obtained for lift, drag, and lift-to-drag ratio for different choices of the free-stream fundamental derivative and free-stream entropy are summarized in figure 5. For the lowest values of $\Gamma_{\infty}$, the drag is almost equal to zero (order $10^{-4}$), then increases monotonically. The lift coefficient initially grows, reaches a maximum, and then drops dramatically. The lift-to-drag ratio is very poor for high $\Gamma_{\infty}$ flows, but tends to infinity as the free-stream value of the fundamental derivative approaches unity. The best aerodynamic performance, offering a satisfactory trade-off between high lift and low drag is obtained for $\Gamma_{\infty}$ approximately in the range $1/3:1.3$: in such conditions, the flow displays higher lift and significantly reduced wave drag compared to PFG results. Note that the curves exhibit quite sudden changes in slope, which are related to corresponding changes
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In the flow patterns. In order to explain the computed behaviour of the aerodynamic performance, a detailed analysis of the flow fields obtained for each operating condition is undertaken, which allows identifying three typical flow regimes, described in the following.

For flows characterized by relatively low free-stream pressures and small values of the free-stream fundamental derivative ($\Gamma_\infty$ less than about 1), the computed lift-to-drag ratio is extremely high, due to the very low values taken by the drag coefficient, although the lift coefficient is lower than in the perfect gas case. Inspection of the Mach number field shows that such flows remain entirely subsonic. Since the free-stream is uniform and steady and no viscous effects are taken into account, the flow should also be isentropic, with drag coefficient exactly equal to zero. In practice, small entropy gradients are generated close to the wall, because of numerical errors introduced by the numerical scheme and boundary conditions, which lead to small nonzero values, $O(10^{-4})$, for the computed drag. As a consequence, the computed lift-to-drag ratio is not unbounded, but $O(10^3)$. An estimate for the critical Mach number, $M_c$, in BZT transonic flows with $\Gamma_\infty \approx 0$, $\Lambda_\infty \approx 0$, has been provided by Cramer & Tarkenton (1992), using an extended transonic small disturbance theory:

$$M_c \approx 1 - \frac{\Gamma_\infty^2}{2\Lambda_\infty},$$

(3.1)

where $\Lambda$ is the previously defined second nonlinearity parameter, representing the rate of change of $\Gamma$ along an isentrope. Figure 6 (a) shows the distribution of $\Lambda_\infty$ associated to the chosen operation points. $\Lambda_\infty$ is evaluated numerically using second-order weighted central differences. Figure 6 (b) displays the predicted critical Mach numbers for the considered cases, versus the maximum Mach number actually attained in the flow field: there is a quite good agreement between estimate (3.1) and the present numerical results. The estimate just fails for operating condition $p_\infty/p_c = 1.07$, $\rho_\infty/\rho_c = 0.850$ ($\Gamma_\infty = 1.15$), where the flow is found to be supercritical although $M_\infty$ is slightly below $M_c$, and for
condition \(\rho_\infty/\rho = 0.821, \rho_\infty/\rho = 0.821\) (\(\Gamma_\infty = 0.335\)), where supercritical flow is also found in spite of free-stream conditions below the critical value. In the first case, the estimated critical Mach number (equal to 0.858), is very close to the free-stream value, and the disagreement may be due to numerical inaccuracies in the evaluation of \(\Lambda\). On the other hand, the operation point corresponding to the latter condition lies quite far outside the inversion zone, and the use of approximation (3.1) is not really justified. It is important to remark that subcritical (shock-free) flow can be obtained even for operating conditions outside the inversion zone! This leads to conclude that beneficial effects on the aerodynamic performance can also be obtained using non-BZT working fluids, provided they display a sufficiently large region of very small, albeit positive \(\Gamma\) values. A typical pressure contour plot for subcritical flow cases is displayed in figure 7 along with \(\Gamma = 0\) contours. Typical distributions of the Mach number, pressure coefficient, fundamental derivative, and sound speed at the wall are presented in figure 8. When a fluid particle from the free-stream approaches the airfoil along the wall streamline, it undergoes a compression and the local fundamental derivative grows, reaching a maximum at the stagnation point where \(\Gamma_{max} \approx 1.5 \div 2\). Then, \(\Gamma\) suddenly drops when the flow begins to expand accelerating over the top of the airfoil. Both pressure coefficient and \(\Gamma\) variations in the neighbourhood of the stagnation point are very large, \(O(1)\). If \(\Gamma_\infty\) is sufficiently small, roughly \(\Gamma_\infty < 1\), the local fundamental derivative becomes smaller than 1, or even negative, less than 0.01 chords downstream the leading edge: consequently, the speed of sound grows sharply enough to counterbalance the increase of velocity and the flow remains subsonic. The smaller \(\Gamma\), the steeper is sound speed growth. As high values of the sound speed are associated to low values of the compressibility parameter, flows with low \(\Gamma_\infty\) conserve a behaviour closer to the incompressible one. This can be better seen using results from the small transonic disturbance theory (Cramer & Tarkenton 1992).
In fact, writing the classical transonic similarity parameter in terms of $\Gamma_\infty$ (Hayes 1966):

$$\kappa = \frac{1 - M^2_\infty}{(\Gamma_\infty \varepsilon)^{2/3}},$$

with $\varepsilon$ the airfoil thickness, and using the well-known Prandtl–Glauert similarity law:

$$C_p = \frac{C^{inc}_p}{\varepsilon \kappa^{1/2}},$$

where $C^{inc}_p$ is the pressure coefficient for the same airfoil, albeit in incompressible flow, we find that:

$$C_p = \frac{C^{inc}_p \Gamma_\infty^{1/3}}{\varepsilon^{2/3} \sqrt{1 - M^2_\infty}},$$

that is, the pressure coefficient approximately grows as $\Gamma_\infty^{1/3}$, and a similar behaviour can be expected for the lift. For $\Gamma_\infty = 0$, equation (3.2) predicts $C_p = 0$. Of course, this does not occur in practice, because of higher-order effects in the airfoil nose region. An estimate of the pressure coefficient variation for flows with $\Gamma_\infty = 0$, $\Lambda_\infty = 0$ has been provided by (Wang & Rusak 1999). Results plotted in figure 5 (a) show how, for flow conditions characterized by low $\Gamma_\infty$ the lift coefficient actually follows quite well a law of the form $C_p = C_{p0}(1 + \Gamma_\infty)^{1/3}$, where the coefficient $C_{p0}$ has been computed from numerical results obtained for $\Gamma_\infty \approx 0$. In practice, thanks to nonlinear effects and strong gradients in the leading edge region, subcritical flow is also obtained for free-stream conditions characterized by values of $\Gamma_\infty$ not really "small", but $O(1)$. In this sense, predictions from the small disturbance theory are quite conservative. Pressure coefficient distributions for different values of $\Gamma_\infty$ (at fixed entropy) are shown for completeness in figure 9, which illustrates well the growth of $C_p$ with $\Gamma_\infty$. In summary, for "sufficiently low" $\Gamma_\infty (< 1)$:

(a) the flow past the airfoil is subcritical;

(b) the drag coefficient vanishes;

(c) lift is lower than in the perfect gas case, because of the reduced flow compressibility;
(d) Lift tends to increase with $\Gamma_\infty$.

When $\Gamma_\infty$ is approximately in the range $1 \div 1.5$ the flow patterns change dramatically. In this range, a significant growth in both lift and drag is observed with respect to the previous case (see figure 5 a). Nevertheless, the lift-to-drag ratio is still about one order of magnitude greater than in the perfect gas case due, on the one hand, to high values obtained for the lift and, on the other, to very low wave drag. In this regime, the flow becomes supercritical, in agreement with estimate (3.1). For operation points lying on S1 to S4, the flow-field displays significant BZT effects, which are responsible for the high aerodynamic performance. On the other hand, no BZT effects appear for operating conditions S5: however, since the fundamental derivatives takes very small albeit positive values, the overall flow behaviour does not differ very much from the other two cases: the flow patterns are similar to those obtained at operating conditions S1-S4 at slightly higher pressures. A typical view of the pressure contours for this flow regime is presented in figure 10, whereas figure 11 shows the wall distributions of the Mach number, pressure coefficient, fundamental derivative, and sound speed. For this kind of flows, characterized by higher free-stream $\Gamma$, the reversed behaviour of the sound speed associated to flow regions with $\Gamma < 1$ is delayed, and the flow expands to supersonic conditions downstream the leading edge. Along the upper surface, if the free-stream values of $\Gamma$ and of the entropy are sufficiently low, $\Gamma$ may become negative just downstream the stagnation point, where the pressure is still steeply falling, so that an expansion shock is generated. Downstream of this shock, the pressure coefficient drops to values much lower than in the perfect gas case. The expansion shock is followed firstly by a continuous expansion and then by a gradual compression, which terminates in a compression shock as soon as the flow exits the inversion zone. Increasing $\Gamma_\infty$ and/or the free-stream entropy, the change of sign of
the fundamental derivative is delayed, or never happens (this is the case of conditions S5), and no expansion shock appears, the flow being always recompressed through a classical shock at the rear part of the upper surface. Along the lower surface, only a weak compression shock forms. Both expansion and compression shocks have jump conditions in the vicinity of the transition line: the entropy jump across such shocks (normalized with the free-stream entropy) is $O(10^{-5})$, whereas it is $O(10^{-2})$ for perfect gas flow. Accordingly, the wave drag is approximately one order of magnitude lower with respect to the PFG value. In summary, flows in the second regime (called hereafter the low-pressure transonic BZT regime) are supercritical and characterized by high lift and very low wave drag, due to the fact that shock waves occurring in the vicinity of the transition line are much weaker than usual. For operation points characterized by sufficiently low values of $\Gamma_\infty$ and $s_\infty$, the aerodynamic performance is even further improved by the formation of an expansion shock close to the leading edge, which strongly enhances the suction peak, and consequently the lift, at the airfoil upper surface. This mechanism is similar to that observed by Cinnella & Congedo (2005a) for BZT flows of a van der Waals gas.

When $\Gamma_\infty$ and/or $s_\infty$ are even higher, the flow becomes qualitatively similar to that of a perfect gas. The flow accelerates from the stagnation point to supersonic velocities and then recompresses at the rear part of the airfoil by means of compression shocks. As the free-stream fundamental derivative is increased, the region of flow characterized by $\Gamma < 0$ becomes smaller and finally disappears. At the same time, the lift coefficient decreases, and the drag increases, due to the stronger entropy gradients generated across the shocks. This progressively reduces the airfoil aerodynamic performance, which finally becomes very poor. Figure 12 shows typical pressure contours for this kind of flow. Figs 13 illustrate the wall Mach number, pressure coefficient, fundamental derivative and sound
speed at operating conditions $p_\infty/p_c = 1.17$, $\rho_\infty/\rho_c = 1.11$ on isentrope S4 ($\Gamma_\infty = 1.91$).

At these extreme conditions, the fundamental derivative remains positive everywhere, and no BZT effects appear.

3.2.2. Near-sonic and low-supersonic free-stream

Since ORC turbomachinery frequently works in the high-transonic or low-supersonic regime, dense flows at $M_\infty = 0.9999$ and $M_\infty = 1.1$ ($\alpha = 1.0$) past the NACA0012 airfoil are also investigated. Only isentropes S1, S4, and S5 are retained for the study.

The reference solution (not shown for brevity) for a diatomic perfect gas flow at $M_\infty = 0.9999$ and $\alpha = 1.0$ is characterized by oblique shocks attached to the airfoil trailing edge and displays a very poor aerodynamic performance:

$$C_L = 8.98 \times 10^{-2}; \quad C_D = 1.06 \times 10^{-1}; \quad C_L/C_D = 8.47 \times 10^{-1}. $$

Dense gas flows at $M_\infty = 0.9999$ are always beyond critical conditions: very different flow patterns are found according to the operation point considered. The more complex patterns are found for conditions S1, where non classical structures due to BZT effects appear. Conversely, only classical behaviours are found for flow conditions S5, where the flow field is always qualitatively similar to that characterizing perfect gas flow with the same free-stream conditions. An intermediate behaviour is displayed by flows at conditions S4. Plots of the lift and drag coefficients and their ratio versus free-stream fundamental derivative are shown in figure 14. Markedly different curves are obtained for different free-stream entropies, due to dramatic changes in the flow patterns. Similarly to the preceding transonic case, the lift coefficient initially increases with operating pressure, reaches an optimum, and finally decreases while remaining above the reference PFG value. Specifically, for $\Gamma_\infty$ less than approximately $0.8 \div 1$, the lift coefficient is twice or more the PFG value. On the other hand, since the flow is supercritical at all operating
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At operating conditions, the drag coefficient remains approximately of the same order of magnitude as in PFG flow, due to the appearance of quite strong discontinuities. At operating conditions $p_\infty/p_c = 0.863$, $\rho = 0.455$ ($\Gamma = 0.064$) and $p_\infty/p_c = 0.911$, $\rho = 0.526$ ($\Gamma = -0.101$) (isentrope S1), the drag coefficient is specially large, due to the occurrence of weak bow-shocks upstream the airfoil nose in spite of subsonic free-stream conditions (see pressure coefficient and $\Gamma = 0$ contour plots on figure 15 a,b). This non classical effect, related to the non-monotone variation of the Mach number with pressure in dense gases, has been previously described by Cramer & Tarkenton (1992) for flows of the fluorocarbon PP11 over a circular arc airfoil, and by Wang & Rusak (1999) and Cinnella & Congedo (2005a) for flows of van der Waals gases over a NACA0012 airfoil, and will not be discussed further. At operating conditions $p_\infty/p_c = 0.966$, $\rho/\rho_c = 0.653$ ($\Gamma = -0.121$) on isentrope S1, the flow field radically changes. When the flow approaches the stagnation point, $\Gamma$ grows beyond 1, and the Mach number behaves in a classical way. Therefore, no bow-shock is formed. Then, the flow over-expands downstream the leading edge and recompresses through a classical shock wave located about mid-chord at the upper surface. Nevertheless, the shock has jump conditions in the vicinity of the transition line, and the associated losses and wave drag remain largely below the reference value: in fact, the drag coefficient displays a minimum. Pressure coefficient contour plots for this case are shown in figure 15 (c), where $\Gamma = 0$ contours are also reported. Increasing the free-stream pressure, the compression shock strengthens and the drag increases to a maximum for operating conditions $p_\infty/p_c = 1.02$, $\rho/\rho_c = 0.813$ ($\Gamma = 0.886$). The flow expansion downstream of the stagnation point is sufficiently strong to drive the fundamental derivative below zero in a region characterized by steep positive pressure gradients: thus, expansion shocks form close to the leading edge at both airfoil surfaces; then, the flow recompresses across classical shock waves (see figure 15 d). Moving the
free-stream state further upwards along the operating isentrope, the compression shocks move toward the trailing edge, while expansion shocks disappear from the flow (figure 15 e). Finally, for high operating pressures the flow returns similar to that of a PFG (figure 15 f). Note that just classical PFG-like behaviours are obtained for operating conditions S5. In the sonic case, improvements of the aerodynamic performance deriving from the use of a BZT working fluid are not as impressive as in the previous transonic case. Nevertheless, the lift-to-drag ratio is above the PFG value for the whole range of operating conditions considered in the study, and it is more than 5 times greater at peak performance conditions. Peak efficiency tends to decrease when increasing the operating entropy, becoming closer and closer to the PFG value.

The last inviscid case concerns supersonic flow, with $M_\infty = 1.1$ and $\alpha = 1^\circ$. Perfect gas flow displays a bow-shock upstream the airfoil nose and oblique shocks attached to the trailing edge. The computed lift and drag coefficients, and their ratio, are:

\[ C_L = 9.11 \times 10^{-2}; \quad C_D = 1.06 \times 10^{-1}; \quad C_L/C_D = 8.63 \times 10^{-1}. \]

A bow-shock also forms in dense gas flows at all operating conditions, including operation points with negative free-stream $\Gamma$. In fact, the strong compression the flow experiences when approaching the stagnation point quickly drives the thermodynamic state outside the (small sized) inversion zone, and the formation of a detached compression shock can not be avoided, at least for the working fluid and the airfoil shape considered in the present study (of course the NACA0012 airfoil is not suitable at all for supersonic flow conditions). Flow patterns observed for this supersonic case are much simpler and do not change very much with operating conditions. The flow always displays a bow shock, upstream the airfoil nose, and compressive waves attached to the trailing edge. These can be either classical oblique shocks, as in the PFG case, or mixed compression waves, according to the choice of the upstream state. Mixed waves are associated with operating
conditions characterized by lower values of $\Gamma_\infty$, which progressively change into oblique shocks when this parameter increases. Pressure coefficient contours for typical dense gas flows are shown in figure 16. The lift and drag coefficients and the lift-to-drag ratio are shown in figure 17: they increase with operating pressure for flow conditions such that $\Lambda_\infty < 0$, and reverse their behaviour when $\Lambda_\infty > 0$. The lift and lift-to-drag ratio are greater than in the reference PFG case for almost all of the investigated operation points albeit the drag exerted on the airfoil is somewhat higher.

3.3. Viscous behaviour

3.3.1. Laminar flow over a flat plate

In order to preliminarily investigate the influence of DG effects on the development of a laminar boundary layer, the classical problem of laminar flow over a flat plate with no pressure gradient is first considered. The incoming flow has $M_\infty = 0.2$ and $Re = 500$, based on inlet density and velocity, and unit plate length. Results are computed in the rectangular domain $[0, 14] \times [0, 5]$, with the plate leading and trailing edges located in $(1,0)$ and $(13,0)$, respectively. A Cartesian grid of $140 \times 50$ cells, stretched in the direction normal to the wall, with first-cell height about $10^{-2}$, is used. The plate wall is adiabatic. Working fluid is PP10. The numerical results are compared to Blasius’ solution for incompressible boundary layers. Two operation points are considered: the first one lies in the dilute gas region ($p_\infty/p_c = 0.6$, $\rho_\infty/\rho_c = 0.2$), the second in the dense gas region, close to the transition line ($p_\infty/p_c = 0.976$, $\rho_\infty/\rho_c = 0.571$). In both cases, numerical results agree with theory, compressibility and dense gas effects being negligible (see figure 18). In order to explore dense gas effects, the flow is also computed at $M_\infty = 0.9$ and $M_\infty = 2.0$. Note the fluid viscosity has been rescaled in order to conserve both Mach and Reynolds numbers. At $M_\infty = 0.9$, results for both operating conditions are still close to Blasius’ solution, the coupling between the viscous and the thermal boundary
layer remaining weak. At $M_\infty = 2$, friction heating becomes more significant, leading to a growth of boundary-layer thickness for the perfect gas, in agreement with theory and results available in the literature (see for example Schlichting & Gersten 2003); conversely, for the dense gas, the higher specific heat limits such growth and the velocity profile remains close to the incompressible one (figure 19); in fact, the flow Eckert number of DG flows is found to be one order of magnitude lower than in PFG flows at all flow conditions.

3.3.2. Laminar transonic flows past an airfoil

The next series of results concerns the symmetric (zero incidence) laminar flow past the NACA0012 airfoil, at $M_\infty = 0.85$ and $Re = 1000$, where $Re$ refers to the Reynolds number based on the free-stream conditions and the airfoil chord. The airfoil wall is adiabatic. Computations are performed using a half-C grid made by $134 \times 68$ cells, with mean height of the first cell closest to the wall equal to about 0.0001 chords, and outer boundary located approximately 20 chords away from the airfoil. Computations performed on a finer grid of $268 \times 136$ cells show that the GCI based on the drag coefficient is less than 3%.

Mach number contours for the flow of a perfect gas are depicted in figure 20 (a). Along the upper surface the flow over-expands to supersonic conditions, then recompresses through a shock wave located approximately 0.8 chords downstream the leading edge; the boundary layer separates at about 95% the chord. The computed drag coefficient is equal to 0.1848.

Let us compare this solution with that obtained for PP10 flowing at the same conditions of Mach and Reynolds number and free-stream thermodynamic conditions $p_\infty/p_c = 1.01$, $\rho_\infty/\rho_c = 0.676$. This operation point lies on isentrope S4, close to the inversion zone. With the preceding choice of operating conditions, the flow is subcritical. Except in the
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stagnation-point region, Mach number variations through the flow are very small and the
Mach number remains less than one and close to the free-stream value almost everywhere
in the flow (see Mach contours in figure 20 b).

Figure 21 shows the pressure and the skin friction coefficients at the wall for the
two cases. Note that the dense gas flow remains attached. When the flow expands from
stagnation conditions, the boundary layer begins to develop. For a perfect gas, the flow
accelerates to supersonic conditions, which promotes boundary layer growth. On the
contrary, for the dense gas, the flow remains subsonic during the whole expansion;
moreover, the boundary layer is subject to a much stronger favourable pressure gradient.
As a consequence, boundary layer growth is slower. Accordingly, the skin friction coeffi-
cient in the leading edge region takes higher values for the dense gas with respect to the
perfect gas. When the flow begins to recompress, skin friction drops in both cases. Nev-
ertheless, it remains slightly higher for the dense gas, and no separation point appears.
The computed drag coefficient for DG flow, $C_D = 0.1652$, is about 10% lower than the
PFG value.

A supersonic flow ($M_\infty = 2, Re = 1000$) over the same airfoil has also been computed.
In this case, a bow shock forms ahead of the airfoil nose in both perfect and dense
gas flow (thermodynamic conditions $p_\infty/p_c = 1.01, \rho_\infty/\rho_c = 0.676$). Iso-Mach contours
for both flows are shown in figure 22. This time, the outer flow is supersonic for both
the perfect and the dense gas. Nevertheless, friction heating is much lower in the second
case: this leads to a thinner boundary layer and to lower skin friction (see velocity profiles
and skin friction distributions in figure 23 b,c; the pressure coefficient is also shown for
completeness). Here again, the computed drag coefficient for the dense gas, $C_D = 0.1929$,
is much lower (less 16%) with respect to the perfect gas ($C_D = 0.2309$).
3.3.3. *Turbulent transonic flows past an airfoil*

The last series of results is intended to provide for the first time a data set about the aerodynamic performance of dense gases flowing at realistic Reynolds number conditions. Since typical Reynolds numbers for flows past turbine blades is in the range $10^5 - 10^6$, turbulence effects shall be necessarily taken into account. Given the very high flow Reynolds numbers, direct simulations are unfortunately not viable. On the other hand, at this stage the main objective is to collect some information, even if approximate, about how the presence of a thin turbulent boundary layer affects the aerodynamic performance of the system in comparison with results obtained for the previously investigated inviscid case. To this purpose, a parametric investigation of turbulent DG flows past an airfoil is undertaken, based on the following working hypotheses:

(a) flow conditions are supposed to be sufficiently far from the thermodynamic critical point, so that DG effects such as dramatic variations of the fluid specific heat and compressibility can be neglected; in these conditions, density fluctuations will not be as huge as in near-critical conditions and subsequently the turbulence structure will not be affected significantly;

(b) at least for equilibrium boundary layers, the mean flow behaviour can be predicted adequately using the compressible Reynolds-averaged Navier–Stokes equations (RANS) completed by an eddy viscosity turbulence model; similarly, the turbulent heat transfer can be modelled through a "turbulent Fourier law", as usual for PFG flows, where the turbulent thermal conductivity is computed in a classical way by introducing a turbulent Prandtl number, assumed to be roughly constant and $O(1)$ throughout the flow.

Hypothesis (a) is justified by the fact that the flows of interest for this study actually do not evolve in the immediate neighbourhood of the critical point; and in fact, if inviscid analyses and computations show an uncommon variation of the fluid speed af
sound (and hence compressibility) with pressure perturbations, nevertheless the magnitude of these variations is approximately of the order of those occurring in perfect gases. Moreover, peculiar DG phenomena related to flow heating or cooling are excluded from considerations, since the airfoil wall is supposed to be adiabatic. On the contrary, Hypothesis (b) should be considered with some caution. On the one hand, if Hypothesis (a) is verified, it seems quite reasonable to apply to compressible DG flows turbulence models initially developed for incompressible flows of perfect gases and currently extended in the common practice to compressible PFG flows; on the other hand, more or less strong pressure gradients and shock waves characterizing the outer inviscid flow are likely to affect the boundary layer, which can no longer be considered an "equilibrium" one; this is also true for the reference PFG flows considered in the study, characterized by strong shock waves and shock/boundary layer interactions. Thus, aerodynamic performance predictions will necessarily be affected by deficiencies inherent with the chosen turbulence model. Nevertheless, since investigations are intended to provide trends of behaviour more than accurate values of the computed aerodynamic coefficients, use of hypothesis (b) represents a means of obtaining preliminary information about realistic DG flows with a reasonable computational expense. Specifically, present results have been obtained using the simple algebraic model of (Baldwin & Lomax 1978), whose deficiencies in nonequilibrium boundary layer are well known (see for example Wilcox 1998, for a wider discussion): for example, flow features such as the location of shock waves and the length of separation bubbles will not be predicted accurately. Nevertheless, it is expected that the model will roughly be able to predict the main trends and qualitative features of the flow field.
As in previous inviscid flow computations, the working fluid is PP10, flowing at $M_\infty=0.85$, $\alpha=10^\circ$, and $Re = 9 \times 10^6$. A parametric study is performed for a series of operation points lying on isentrope S4 (see figure 3). Solutions are computed using C-grids of $256 \times 64$ and $256 \times 128$ cells, respectively, with mean $y^+ \approx 5$ on the coarser grid, and $y^+ \approx 1$ on the finer one. In both cases the outer boundary is located about 20 chords away from the airfoil. Here again, assuming a (conservative) value of the convergence order equal to 1.8 for the scheme extended to viscous flows, the computed GCI turns out to be about 0.5% on the finer grid.

The reference solution for a diatomic perfect gas flowing at the same conditions is represented in figure 24 (a). The flow is characterized by strong shock waves at both airfoil surfaces, which interact with the turbulent boundary layer. At the upper surface, an extended post-shock separation bubble appears. The aerodynamic coefficients are $C_D = 5.28e-2$ and $C_L = -0.012$, the negative sign being due to significant upstream displacement of the upper shock due to flow separation (shock stall). Also note that the drag coefficient is very close (actually, slightly lower) than the value given in Section 3.2.1 for an inviscid computation. In practice, reduced strength of shock waves due to their interactions with the boundary layer leads to lower wave drag with respect to the inviscid case, and this counterbalances the effect of viscous drag. Wall distributions of the pressure coefficient for viscous and inviscid flow are displayed in figure 24 (b). The computed values of the aerodynamic coefficients for PP10 at various operating conditions are reported in figure 26. If the free-stream state is taken close enough to the inversion zone, the flow remains subsonic: no shock waves are formed and flow separation is suppressed. In this regime (subcritical BZT regime), the drag coefficient drops from its PFG value as wave drag disappears, whereas the lift coefficient is considerably higher. For operation points at higher free-stream $\Gamma$, in the regime previously christened as "low-pressure tran-
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sonic BZT”, a supersonic region forms. This enhances lift, whereas wave drag remains quite low with respect to the perfect gas case. Two mechanisms contribute to this effect: the first one is of inviscid nature, and is related to the fact that shock waves have jump conditions in the neighbourhood of the transition line, and are therefore less dissipative than normal; on the other hand, such weak shock waves do not cause flow separation, so that pressure drag is further reduced. Further increasing the free-stream pressure (high-pressure transonic BZT regime) leads to increase the strength of shock waves: thus, wave drag grows and the flow finally separates because of shock/boundary layer interactions. As a consequence, both the lift coefficient and the lift-to-drag ratio drop. Figure 24 (c,e,g) shows typical pressure coefficient contours and flow streamlines in the three regimes; wall distributions of the pressure coefficient for inviscid and viscous flow are shown in figure 24 (d,f,h). Figure 25 compares skin friction distributions for a perfect gas and for PP10 at different operating conditions. Note that the extended separated regions characterizing the perfect gas flow at both airfoil surfaces are absent in dense gas flows insofar as the operating conditions are chosen sufficiently close to the inversion zone. In the subcritical case, for flows at high Reynolds number, the pressure distribution remains essentially similar to the inviscid one, with just only some smoothing of the suction peaks at both surfaces downstream the leading edge. The lift coefficient is slightly below the inviscid value, whereas the lift-to-drag ratio now takes of course finite, although high values. In the low-pressure transonic BZT regime the differences become more significant. Namely, the suction peak at the airfoil upper surface is dramatically smoothed out because of viscous effects, and the location of the upper shock wave moves upstream because of interactions with the boundary layer: nevertheless, the flow remains attached. Also note that in this regime the skin friction (see figure 25) significantly grows with chordwise distance up to $x/c \approx 0.25$, due to the very strong favourable pressure gradient acting
on the boundary layer. Finally, in the third regime strong shock/boundary layer interactions lead to flow separation at both airfoil surfaces: nonetheless, separation is delayed and separated regions are smaller than in perfect gas flow. In summary, the preceding results suggest that DG effects mainly affect the inviscid flow behaviour, whereas the viscous behaviour is influenced indirectly according to the distributions of the external pressure and Mach number characterizing flows at different operating conditions. Concerning system efficiency, the use of dense gases working at proper operating conditions has a definitely beneficial effect on system efficiency, not only because of significant reductions in wave drag, but also because losses due to shock/boundary layer interaction are completely suppressed or strongly attenuated.

4. Conclusions

In the present work a careful numerical investigation of a number of inviscid, viscous laminar, and viscous turbulent flows of a dense gas over an airfoil at transonic speeds has been undertaken. The numerical results provide a complete picture of the complex aerodynamics of an airfoil immersed in a dense gas stream. Inviscid mechanisms include non-monotone variation of the Mach number with density, leading to increases in the critical Mach number and in a delay of the transonic drag rise, and the appearance of weak non-classical waves with jump conditions in the vicinity of the transition line, which may lead to significant improvements in the airfoil aerodynamic performance over a classical working fluid. For flows with high-subsonic free-stream, analyses of the variation of aerodynamic coefficients with free-stream thermodynamic conditions allow identifying three flow regimes in the range of thermodynamic conditions swept in present calculations, representative of the operating range of an Organic Rankine Cycle turbine. While the drag always increases with free-stream pressure and fundamental derivative $\Gamma$, the lift co-
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efficient displays an optimum in the second regime, referred-to as low-pressure transonic BZT regime in the present study, and then drops dramatically. The lift-to-drag ratio is very poor for high $\Gamma_\infty$ flows, but tends to infinity as the free-stream value of the $\Gamma$ approaches unity from the high-pressure side of the inversion zone. The best compromise solution between high lift and low drag is obtained in the middle regime (the low-pressure BZT regime), for $\Gamma_\infty$ approximately in the range $1 \div 1.3$: in these conditions, higher lift and significantly reduced wave drag compared to perfect gas results is observed.

When low supersonic conditions are considered, a bow shock appears ahead the airfoil nose in perfect gas flows. Even in dense gas flows, because of the limited extent of the inversion zone compared to pressure variations experienced by the flow upstream the airfoil, the formation of such a shock can not be avoided for any choice of the thermodynamic conditions. Consequently, efficiency gains over classical working fluids are less impressive with respect to the previous case. It is noteworthy that, for all inviscid computations performed in this study, the best trade-off between high lift and aerodynamic efficiency is obtained for operation points relatively far from the inversion zone, that is, for flows with $\Gamma_\infty$ of the order of unity. Such results are of outstanding importance for practical applications; in particular, it seems possible to overcome one of the major difficulties for the development of BZT Organic Rankine Cycles, namely, the necessity of operating the turbine inside the inversion zone. Present results indicate that in practice, significant performance enhancement can be achieved operating the system within the thermodynamic region where $\Gamma < 1$. In order to get the maximum benefit from the use of BZT working fluids, it is nevertheless important to operate the system at transonic speeds, as the aerodynamic performance drops quite quickly to values proper of classical flows as the free-stream Mach number increases.

Numerical studies of the aerodynamic behaviour of viscous dense gas flows past an
airfoil have also been provided for the first time, to the authors’ knowledge. For laminar flows, dense gas effects in the outer inviscid flow region allow delaying boundary layer separation. Moreover, at sufficiently large Mach numbers, further benefits derive from reduced friction heating in flows of gases with large specific heats, as BZT gases are. Beneficial effects deriving from the use of a dense working fluid are also observed when the aerodynamic performance of viscous turbulent airfoil flows at large Reynolds number and transonic speeds is considered. The nonclassical variation of the Mach number with density favourably affects the boundary layer development, contributes to reducing friction drag and to avoiding boundary layer separation due to large adverse pressure gradients. Specifically, post-shock separations due to shock/boundary layer interaction are suppressed or greatly reduced, which ensures satisfactory lift and aerodynamic efficiency at flow conditions where the aerodynamic performance of perfect gas flows suffers from shock stall.

As a final consideration, note that present results have been obtained for a specific airfoil shape, and that efficiency gains are only due to the peculiar properties of the working fluid: it is possible to imagine that the use modern multi-point optimization techniques would allow selecting an airfoil shape that ensures even higher aerodynamic performances over a larger operation range. For example, the study presented in Congedo (2005) shows that shape optimization allows simultaneously increasing the lift in the subcritical BZT regime and minimizing the drag for a given lift in the supercritical regimes, further delaying the transonic drag rise and enlarging the system operation range. Further details will be provided in a forthcoming publication.

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Figure 1. Aerodynamic performance of BZT transonic flow of a van der Waals gas ($M_\infty = 0.85$, $\alpha = 1^\circ$) past the NACA0012 airfoil. (a) Choice of the thermodynamic conditions; (b) lift coefficient and lift-to-drag ratio versus free-stream fundamental derivative.
Figure 2. Amagat diagrams for (a) a BZT van der Waals gas ($\gamma = 1.0125$) and (b) for PP10.
**Figure 3.** Location of the operation points in the $p - v$ diagram.
Figure 4. Fundamental derivative versus pressure along selected isentropes.
Figure 5. Aerodynamic coefficients versus free-stream fundamental derivative for flow at $M_{\infty} = 0.85$, $\alpha = 1^\circ$ past a NACA0012.
Figure 6. Second nonlinearity parameter (a) and estimated critical Mach number versus maximum flow Mach number (b) for DG flows past the NACA0012 airfoil at $M_\infty = 0.85$, $\alpha = 1^\circ$ at different free-stream thermodynamic conditions.
Figure 7. Pressure coefficient contours and \( \Gamma = 0 \) contours for operating conditions \( p_\infty/p_c = 0.986, \rho_\infty/\rho_c = 0.658, \Gamma_\infty = 0.017 \) (BZT subcritical regime).
Figure 8. Wall distributions of the Mach number, pressure coefficient, fundamental derivative and sound speed for operating conditions $p_\infty/p_c = 0.986$, $p_\infty/\rho_c = 0.658$, $\Gamma_\infty = 0.017$ (BZT subcritical regime).
Figure 9. Wall distributions of the pressure coefficient for different operating conditions in the BZT subcritical regime.
Figure 10. Pressure coefficient contours and $\Gamma = 0$ contours for operating conditions

$p_{\infty}/p_e = 1.03$, $\rho_{\infty}/\rho_e = 0.877$, $\Gamma_{\infty} = 1.33$ (low-pressure transonic BZT regime).
Figure 11. Wall distributions of the Mach number, pressure coefficient, fundamental derivative and sound speed for operating conditions $p_{\infty}/p_e = 1.03$, $\rho_{\infty}/\rho_e = 0.877$, $\Gamma_\infty = 1.33$ (low-pressure transonic BZT regime).
Figure 12. Pressure coefficient contours and $\Gamma = 0$ contours for operating conditions $p_\infty/p_\infty = 1.17$, $p_\infty/p_\infty = 1.11$, $\Gamma_\infty = 1.91$ (high-pressure transonic BZT regime).
Figure 13. Wall distributions of the Mach number, pressure coefficient, fundamental derivative and sound speed for operating conditions $p_\infty/p_c = 1.17$, $\rho_\infty/\rho_c = 1.11$, $\Gamma_\infty = 1.91$ (high-pressure transonic BZT regime).
Figure 14. Aerodynamic coefficients versus free-stream fundamental derivative for flows at $M_\infty = 0.9999$, $\alpha = 1^\circ$ past a NACA0012.
Figure 15. Pressure coefficient contours for flow conditions $p_\infty/p_c = 0.863$, $p_\infty/p_c = 0.455$, $\Gamma_\infty = 0.064$ (a), $p_\infty/p_c = 0.911$, $p_\infty/p_c = 0.526$, $\Gamma_\infty = -0.101$ (b), $p_\infty/p_c = 0.966$, $\rho_\infty/\rho_c = 0.653$, $\Gamma_\infty = -0.121$ (c), $p_\infty/p_c = 1.00$, $\rho_\infty/\rho_c = 0.752$, $\Gamma_\infty = 0.416$ (d), $p_\infty/p_c = 1.02$, $\rho_\infty/\rho_c = 0.813$, $\Gamma_\infty = 0.886$ (e), $p_\infty/p_c = 1.05$, $\rho_\infty/\rho_c = 0.944$, $\Gamma_\infty = 1.62$ (f). $M_\infty = 0.9999$, $\alpha = 1^\circ$. 
Figure 16. Pressure coefficient contours for flow conditions $p_\infty/p_c = 0.863, \rho_\infty/\rho_c = 0.455, \Gamma_\infty = 0.064$ (a), $p_\infty/p_c = 0.911, \rho_\infty/\rho_c = 0.526, \Gamma_\infty = -0.101$ (b). $M_\infty = 1.1, \alpha = 1^\circ$. 
Figure 17. Aerodynamic coefficients versus free-stream fundamental derivative for flows at $M_\infty = 1.1, \alpha = 1^0$ past a NACA0012.
Figure 18. Laminar flow over a flat plate ($Re = 500$, $M_\infty = 0.2$). (a) Skin friction, (b) velocity profile.
Figure 19. Laminar flow over a flat plate. Velocity profiles at different Mach numbers.
Figure 20. Iso-Mach contours and streamlines for laminar flow over a NACA0012 airfoil ($Re = 1000, M_\infty = 0.85$). (a) Perfect gas, (b) dense gas at thermodynamic conditions $p_\infty / p_c = 1.01$, $\rho_\infty / \rho_c = 0.676$ (b).
Figure 21. Wall distributions of the pressure coefficient and skin friction for laminar flow over a NACA0012 airfoil ($Re = 1000$, $M_{\infty} = 0.85$).
Figure 22. Iso-Mach contours and streamlines for laminar flow over a NACA0012 airfoil ($Re = 1000$, $M_\infty = 2.0$). (a) Perfect gas, (b) dense gas at thermodynamic conditions $p_\infty/p_c = 1.01$, $\rho_\infty/\rho_c = 0.676$ (b).
Figure 23. Wall distributions of the pressure coefficient and skin friction for laminar flow over a NACA0012 airfoil ($Re = 1000$, $M_\infty = 2.0$).
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Figure 24. Turbulent flow over the NACA0012 airfoil, $M_\infty = 0.85$, $\alpha = 1^\circ$, $Re = 9 \times 10^6$.
Pressure coefficient contours and streamlines (left) and wall pressure coefficient (right) for a perfect gas flow (a,b) and for PP10 at operating conditions $p_\infty/p_c = 1.01$, $\rho_\infty/\rho_c = 0.676$ (c,d), $p_\infty/p_c = 1.08$, $\rho_\infty/\rho_c = 0.850$ (e,f), $p_\infty/p_c = 1.17$, $\rho_\infty/\rho_c = 1.11$ (g,h).
Figure 25. Skin friction for a perfect gas flow and dense gas flows at different operating conditions.
Figure 26. Aerodynamic coefficients versus free-stream fundamental derivative for turbulent flows at $M_\infty = 0.85$, $\alpha = 1^\circ$ and $Re = 9 \times 10^6$ past a NACA0012.