Analysis of seismic earth pressures: some recent developments

Analyse de la pousée des terres sismique: quelques dévelopements récents

P. Kloukinas, G. Mylonakis
Department of Civil Engineering, University of Patras, Greece

ABSTRACT

An exact limit-stress solution is presented for the problem of seismic earth pressures on L-shaped cantilever walls retaining dry cohesionless soil. It is shown that the problem can be analyzed by means of a Rankine stress field in the backfill for an infinite set of geometric and material parameters obeying a transcendental equation in friction angle ϕ. Key to the proposed approach is that the stress characteristics in the soil mass do not intersect the stem of the wall. Consequently, the soil-wall interface is not part of the sliding wedge and the roughness of the wall does not influence the response, as the interface remains bonded. In light of the above, the solution can be obtained analytically, in the realm of plane strain conditions and pseudo-dynamic seismic action in the backfill. The suitability of the popular "virtual vertical back" approach for evaluating active thrusts is confirmed and the corresponding roughness angle $\delta(0)$ is derived in closed form as function of friction angle, backfill inclination and seismic acceleration. It is shown that existing recommendations for $\delta(0)$ in seismic codes are often erroneous – yet may yield predictions on the safe side. Optimization aspects referring to improvement of system stability and corresponding safety factors are investigated. Issues related to the use (or misuse) of the safety factor against overturning are discussed.

RÉSUMÉ

Une solution exacte des contraintes-limite est présentée au problème de la pouseé des terres sismique du terrain sur les murs cantilever en forme de L retenant des sols secs et sans cohésion. Il est montré que le problème peut être analysé au moyen d'un champ de contraintes dans le remblai de Rankine obéissant à une équation transcendantale de l'angle de frottement interne. La clé de l'approche proposée est que les caractéristiques des contraintes dans la masse du terrain n'ont pas d' intersection avec la mur. Par conséquent, l'interface sol-mur ne fait pas partie de la tranche de glissement et la rugosité de la paroi n'a pas d'influence sur la réponse tant que l'interface reste soudée. A la lumière de ce qui a précède, la solution peut être obtenue de façon analytique, dans des conditions de déformation plane et d'action sismique pseudo-dynamique dans le remblai. L'adéquation de l'approche commune "retour vertical virtuel" pour évaluer des poussées actives est confirmée et l'angle de la rugosité correspondant $\delta(0)$ est dérivé sous forme fermée en tant que fonction de l'angle de frottement, de l'inclinaison du remblai et de l'accélération sismique. Il est montré que les recommandations existantes pour $\delta(0)$ dans les codes sismiques sont souvent erronées - mais elles peuvent donner des prédictions concernant le coté sécuritaire. Des aspects de l'optimisation concernant l'amélioration de la stabilité du système et des facteurs de sécurité correspondants sont étudiés. Les questions liées à l'usage (ou abus) du facteur de sécurité contre le renversement (basculement) sont discutées.

Keywords: L-shaped Retaining Wall, Stress Plasticity, Limit Analysis, Rankine , Seismic earth pressures

1 INTRODUCTION

Cantilever walls of L or inverted T-shape represent a popular type of retaining system, which is widely considered as advantageous over conventional gravity walls for it combines economy and ease in construction and installation. Furthermore, the specific design is deemed particularly rational, as it exploits the stabilizing action of soil weight over the footing slab against both sliding and overturning. A contradictory issue in the literature relates to the calculation of active thrust, assumed to be acting on a virtual wall back, which is usually taken as the vertical plane passing through the heel of the wall, under an certain mobilized roughness on that plane (Trenter, 2004; O'Sullivan and Creed, 2007). A variety of virtual back "roughnesses" have been reported ranging from 0 (a perfectly smooth plane) to φ (a perfectly rough plane). These values are often recommended with little justification as explained in the ensuing. Investigating the significance of these assumptions in retaining wall design provided the initial motivation for the herein-reported work.

The problem under consideration is presented in Fig. 1: a sloping backfill of dry cohesionless soil is retained by an L - shaped cantilever wall. The system is subjected to plane-strain conditions under the combined action of gravity (g) and pseudo-static seismic body forces (a_h x g) and (a_v x g) in the horizontal and vertical direction, respectively. The problem parameters are: wall height H, heel width b, footing width B, wall thickness t, wall unit weight γ_w, wall roughness δ_w, soil unit weight γ and soil friction angle ϕ. The inclination of the overall body force vector resulting from the gravitational and inertial actions, shown in Fig. 1, is $\psi_e = \tan^{-1}[a_h/(1-a_v)]$. The problem is treated by means of a stress limit analysis method in the realm of plasticity theory. Contrary to the general earth-pressure problem, the solution at hand can be derived by means of a Rankine stress field, which allows simple closed-form solutions to be obtained, which offer considerable insight to the physics of soil thrust

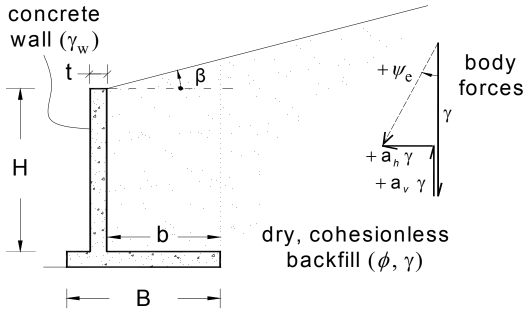

Figure 1. The problem under consideration

2 STRESS LIMIT ANALYSIS

2.1 General approximate solution

The herein-reported solution is derived based on the stress limit analysis approach of Mylonakis et al (2007) and Lancelotta (2007) for the problem of earth pressures on a rough wall retaining an inclined backfill, which makes use of discontinuous stress fields like the one shown in Fig. 2.

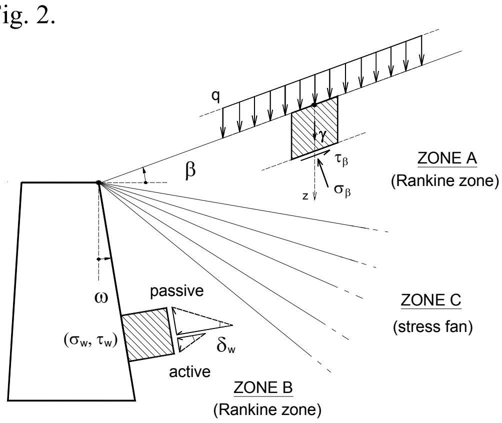

Figure 2. Discontinuous stress field in the case of a gravity wall (after Mylonakis et al., 2007)

In this generic problem the soil mass is divided into three regions: Zone A, a Rankine Zone located near the free surface of the semi-finite slope; Zone B, a Rankine Zone which satisfies the stress boundary condition at the soil-wall interface; Zone C, a transition region which satisfies the stress boundary conditions at the boundaries of the particular zone with regions A and B.

With reference to the above problem, the boundary condition on the wall (Zone B) imposes two restrictions: *First* it enforces the failure criterion at the soil-wall interface (i.e., $\tau_w = \sigma_w \tan\delta_w$) as the particular surface is a failure plane; *Second* it prescribes the direction of the shear traction on the wall surface, that points in the same direction as the velocity vector of the retained soil mass (which is obviously different in the active and passive case).

As to the transition Zone C, a logarithmic stress fan is adopted, which is an exact solution for a weightless material, yet only an approximate for a material with weight. The expression providing the ultimate thrust for active and passive conditions is given by the well known equation (Ebeling & Morison, 1992; Kramer, 1996):

$$P_E = K_{qE}(1-a_v)qH + \frac{1}{2}K_{\gamma E}(1-a_v)\gamma H^2 \tag{1}$$

where

$$K_{\gamma E} = \frac{\cos(\omega-\beta)\cos(\beta+\psi_e)}{\cos\psi_e \cos\delta_w \cos^2\omega} \times \left[\frac{1-\sin\phi\cos(\Delta_2-\delta_w)}{1+\sin\phi\cos[\Delta_{1e}+\beta+\psi_e]}\right]\exp(-2\theta_E\tan\phi) \tag{2}$$

$$K_{qE} = K_{\gamma E}\cos\omega / \cos(\omega-\beta) \tag{3}$$

$$\sin\Delta_{1e} = \frac{\sin(\beta+\psi_e)}{\sin\phi}, \quad \sin\Delta_2 = \frac{\sin\delta}{\sin\phi} \tag{4}$$

$$2\theta_E = (\Delta_2-\delta_w) - (\Delta_{1e}-\beta) - 2\omega - \psi_e \tag{5}$$

In the above equations, $K_{\gamma E}$ and K_{qE} are earth pressure coefficients pertaining to self-weight and surcharge actions, respectively; Δ_{1e} and Δ_2 are the corresponding Caquot angles (measured in radians), used also in EC7 provisions and θ_E is the rotation angle of the principal planes (and, accordingly, stress characteristics), which is equal to the opening angle of the transition zone. Equations (2) to (5) provide both the active thrust and the passive resistance, provided that a proper sign is used for the friction angle ϕ and the roughness angle δ_w. This merely requires positive δ_w and ϕ values for active conditions and negative for passive.

2.2 Generalized Rankine theory

In the special case of a vanishing angle θ_E, the three stress fields in Fig. 2 collapse into a single Rankine zone, which is depicted by the Mohr circle of Fig. 3. In this special case, the exponential term in Eq. (2) vanishes and the equation provides an exact plasticity solution to the generalized problem with earthquake loading. The necessary condition for the validity of this solution is (see Eq. 5):

$$(\Delta_2-\delta_w) - (\Delta_{1e}-\beta) - 2\omega - \psi_e = 0 \tag{6}$$

which, naturally, is satisfied by an infinite number of combinations of the five governing parameters ϕ, δ_w, ω, β and ψ_e. Among them, as special cases, one can identify the classical solutions $\delta_w = \omega = \beta = \psi_e = 0$ and $\delta_w = \beta$, $\omega = \psi_e = 0$, both of which were first obtained by Rankine (1857). Under these conditions, the solution attains, respectively, the special forms (Rankine 1857, Terzaghi 1943).

$$K_\gamma = (1-\sin\phi)/(1+\sin\phi) \tag{7}$$

$$K_\gamma = \cos\beta\frac{1-\sin\phi\cos(\Delta_1-\beta)}{1+\sin\phi\cos(\Delta_1+\beta)} \tag{8}$$

From Eq. (6) and the Mohr circle of Fig. 3, it is straightforward to derive the critical values of each of the above parameters as function of the others, to satisfy the generalized Rankine condition. These solutions are given in Eqs. (9) to (12). Equation (9) results directly from condition (6), whereas derivation of Eqs. (10) to

(12) requires additional geometric considerations from the stress tensor of Fig. 3.

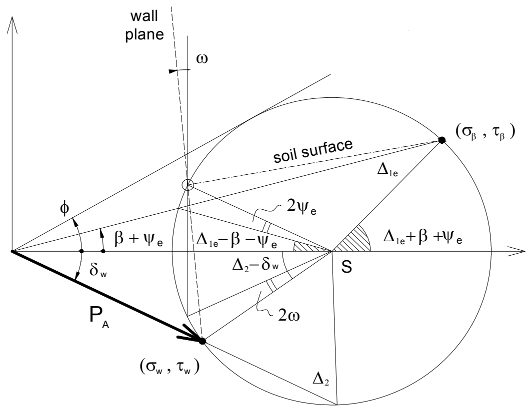

Figure 3 Generalized Rankine stress tensor for earthquake loading

$$\omega_R = \frac{1}{2}\left[(\Delta_2 - \delta_w) - (\Delta_{1e} - \beta) - \psi_e\right] \tag{9}$$

$$\beta_R = tan^{-1}\left[\frac{\sin\phi\sin(\Delta_2 - \delta_w - 2\omega - 2\psi_e)}{1 - \sin\phi\cos(\Delta_2 - \delta_w - 2\omega - 2\psi_e)}\right] - \psi_e \tag{10}$$

$$\psi_{eR} = tan^{-1}\left[\frac{\sin\phi\sin(\Delta_2 - \delta_w - 2\omega + 2\beta)}{1 + \sin\phi\cos(\Delta_2 - \delta_w - 2\omega + 2\beta)}\right] - \beta \tag{11}$$

$$\delta_R = tan^{-1}\left[\frac{\sin\phi\sin(\Delta_{1e} - \beta + \psi_e + 2\omega)}{1 - \sin\phi\cos(\Delta_{1e} - \beta + \psi_e + 2\omega)}\right] \tag{12}$$

For the simpler case of gravitational loading, the counterparts of Eqs. (9) and (12) can be found in the work of Costet & Sanglerat (1979), whereas an alternative form of equation (12) is given in Chu (1991). The latter solution seems to make use of the erroneous assumption that the soil thrust inclination δ_R in Eq. (12) (for $\psi_e = 0$) can actually develop on the wall plane regardless of the true interface roughness δ_w. As a result, the analysis reported in that work violates the failure criterion at the interface and may be incompatible with the kinematics of the problem (Budhu, 2007). A more complete treatment has been recently provided by Evangelista et al (2010).

It should be noticed that for every combination of parameters ϕ, δ_w, ω, β and ψ_e satisfying Eqs. (9) – (12), the predictions of Eq. (2) coincide with those of the Mononobe-Okabe equation and other approximate solutions (Chu, 1991; Greco, 1999). This stems from the properties of the Rankine stress field and the associated straight stress characteristics, which are intrinsically compatible with a planar failure surface. As a result, the predictions of all these methods not only coincide, but they are also exact in the context of classical limit analysis theory. As pointed out by Heyman (1973), the origins of these solutions can be traced back in the pioneering studies of Rankine (1857), Levy (1874) and Boussinesq (1876).

3 ANALYSIS OF L – SHAPED WALLS

Unlike the case of gravity walls where the generalized Rankine condition arises only for specific combinations of the five governing parameters specified by Eq. (6), in the case of L-shaped cantilever walls the theory has much wider applicability. Indeed, when the heel of the wall is sufficiently long, the α and β stress characteristics (i.e., the conjugate failure planes on the Mohr circle of stresses) in the backfill do not intersect the stem of the wall. Then, the sliding prism is formed entirely in the backfill, as shown in Fig. 4a.

Accordingly, the soil-wall interface is not part of the Rankine zone and, thereby, does not influence the response as the interface remains bonded and lies outside the failure mechanism.

From Figure 4a, the necessary geometric condition for the validity of the above Rankine solution is:

$$\omega_\beta \le \omega_{wall} \text{ or, equivalently, } \omega_\beta \le tan^{-1}\left(\frac{b}{H}\right) \tag{13}$$

which is tantamount to the criterion provided by Clayton et al. (1993) for the corresponding static problem. The inclination of the β – characteristic can be determined either graphically, from the corresponding Mohr circle (Figure 4b), or from Eq. (9) using $\delta = \phi$; this yields the solution:

$$\omega_\beta = \frac{\pi}{4} - \frac{\phi}{2} - \frac{(\Delta_{1e} - \beta)}{2} - \frac{\psi_e}{2} \tag{14}$$

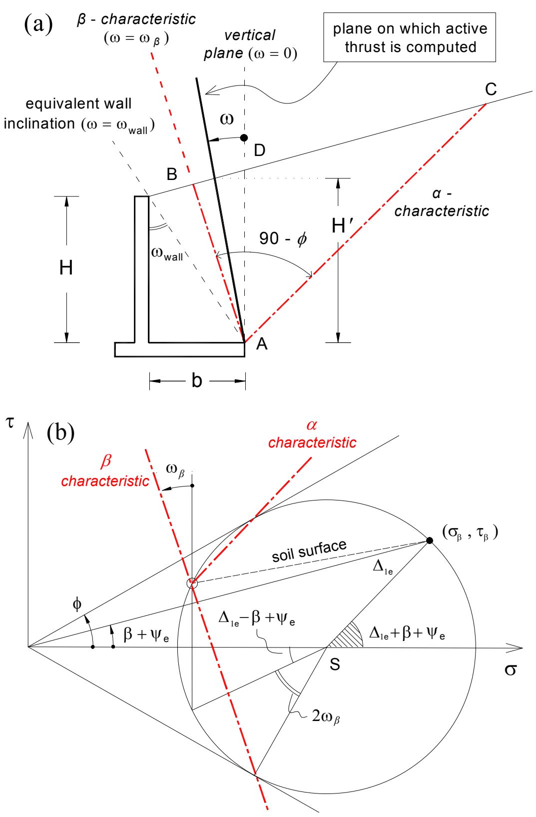

Figure 4. (a) Rankine wedge in the backfill and applicability condition of Rankine solution, (b) Stress tensor and stress characteristics in the retained soil.

For the case of gravitational loading ($\psi_e = 0$), Eq. (14) can be found in a number of publications (Costet & Sanglerat, 1979; Chu, 1991; Clayton et al, 1993). With minor exception (Evangelista et al 2010) the case of earthquake loading has not been investigated in the past.

In Fig. 5, the minimum required heel length to ensure the validity of the Rankine condition is presented as function of the horizontal seismic acceleration a_h and the backfill inclination, β. It can be clearly seen that the minimum required len gth is not constant but decreases with increasing acceleration level. [This is in contrast to the constant value (b > H/3) proposed by a number of seismic provisions including the Greek Seismic Code, EAK2000]. This decrease in heel length suggests that condition (13) can be satisfied in many cases involving earthquake loading, even if it is not satisfied for the pure gravitational case. As a result, the condition proposed by the Greek code covers a wide range of common cases, except for those associated with small values of slope angle, β.

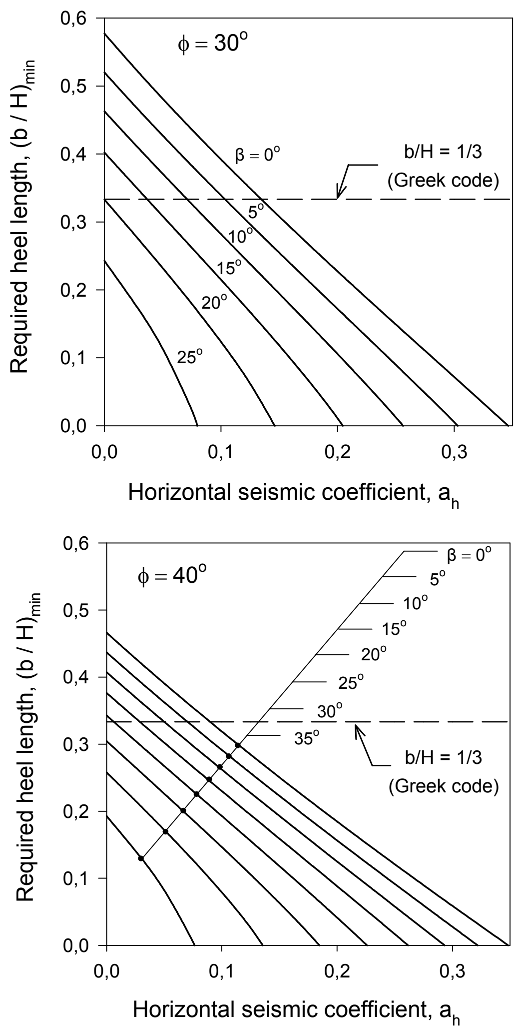

Figure 5. Variation of minimum required heel length with horizontal acceleration coefficient and backfill angle.

For the cases where the Rankine condition is valid, the active earth pressure can be determined from Eqs. (1) – (4) in conjunction with Eq. (12), on any plane (to be referred to hereafter as "virtual back") inclined at an angle ω from vertical (Fig.4), resulting to:

$$K_\gamma = \frac{\cos(\omega-\beta)\cos(\beta+\psi_e)}{\cos\delta(\omega)\cos^2\omega\cos\psi_e} \times \left[\frac{1-\sin\phi\cos(\Delta_2-\delta(\omega))}{1+\sin\phi\cos(\Delta_{1e}+\beta+\psi_e)}\right] \quad (15)$$

On this arbitrary plane, the inclination of active thrust, $\delta(\omega)$, is given by Eq. (12) as function of ω. This elucidates that the frequently employed assumption of a "soil-to-soil" friction angle ϕ is not justified (BS8002, 1994; Trenter, 2004, O'Sullivan and Creed, 2007). The exception is the case where $\omega = \omega_\beta$, i.e., when the plane under consideration is parallel to a β-characteristic (Fig. 4), generating a mobilized friction angle $\delta(\omega)$ equal to ϕ. Naturally, this is the maximum value that the interface friction can mobilize.

In light of Fiq. 4, an effective wall height H' has to be introduced according to Eq. (16), which, evidently varies with the angle ω of the virtual back.

$$H' = H\left[1+\left(\frac{b}{H}-\tan\omega\right)\frac{\sin\beta\cos\omega}{\cos(\omega-\beta)}\right] \quad (16)$$

In Figure 6, two extreme cases are presented, corresponding to cases where $\omega = \omega_\beta$ and $\omega = 0$, which have been widely used in the literature (Clayton et al., 1993; Greco, 1999; Trenter, 2004). It can be easily shown that the above choices are equivalent, as they lead to the same resultant force on the wall (if the body forces in the corresponding hatched soil prisms in Fig. 6a, b are accounted for).

This property holds despite the fact that the active thrust P_A, calculated for each case on plane ω is different (Fig. 6a, b). Notwithstanding the validity of these comments, it is preferable to adopt the conventional vertical plane AD, corresponding to ω = 0, as virtual back, for it leads to a simpler geometry and facilitates calculations (Fig. 6b). Accordingly, Eqs. (17) and (18) are derived for the determination of the magnitude of the thrust and its inclination δ(0):

$$K_\gamma = \frac{\cos\beta\cos(\beta+\psi_e)}{\cos\delta(0)\cos\psi_e}\left[\frac{1-\sin\phi\cos(\Delta_{1e}-\beta+\psi_e)}{1+\sin\phi\cos(\Delta_{1e}+\beta+\psi_e)}\right] \quad (17)$$

$$\delta(0) = tan^{-1}\left[\frac{\sin\phi\sin(\Delta_{1e}-\beta+\psi_e)}{1-\sin\phi\cos(\Delta_{1e}-\beta+\psi_e)}\right] \quad (18)$$

In Figure 7 results for the mobilized friction angle $\delta(0)$ on the vertical virtual back are presented, as function of horizontal seismic acceleration and slope inclination for soil friction angles $\phi = 30^o$ and 40^o. It can be observed that the virtual back roughness is not always equal to the slope inclination β; this holds only for the case of gravitational loading ($\psi_e = a_h = 0$).

In presence of seismic action, the "virtual roughness" increases significantly up to the maximum value $\delta = \phi$, when the β-characteristic becomes vertical ($\omega_\beta = 0$), that is for the same earthquake level for which the required heel length in Fig. 5 vanishes. [Note in this regard that all curves are plotted up to the peak value; the ensuing decreasing branch is not shown for simplicity.] This suggests that the common assumption $\delta = \beta$ in the seismic codes, is precise only for gravitational loading and generally underestimates δ.

Accordingly this assumption yields results on the safety side, as the increase in roughness, forces an increase in stability (through a higher vertical and a lower horizontal component of soil thrust). Other inexact assumptions for the mobilized δ value can be found in the literature (i.e. BS:8002, $\delta=\phi$ based on the aforementioned assumption of a "soil-to-soil" friction parameter; AASHTO LRFD, $\delta=\delta_w$ by an association of the actual soil-wall interface with the vertical virtual back).

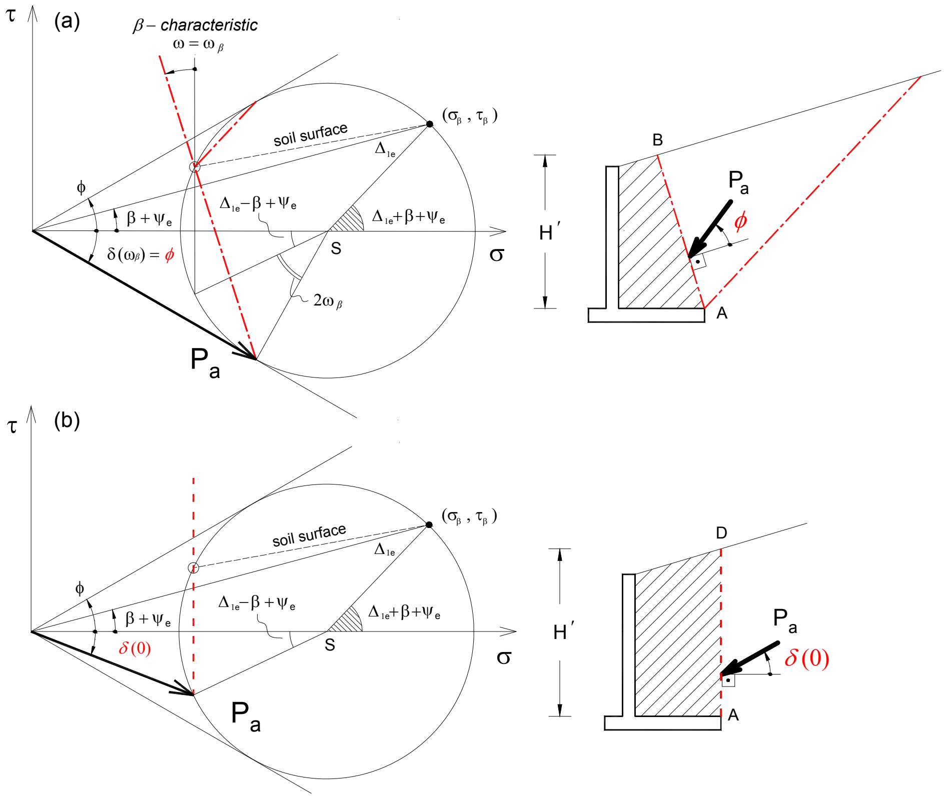

Figure 6. (a) Active thrust on the actual slip line AB (*β*-characteristic) and (b) Active thrust on the vertical virtual back AD.

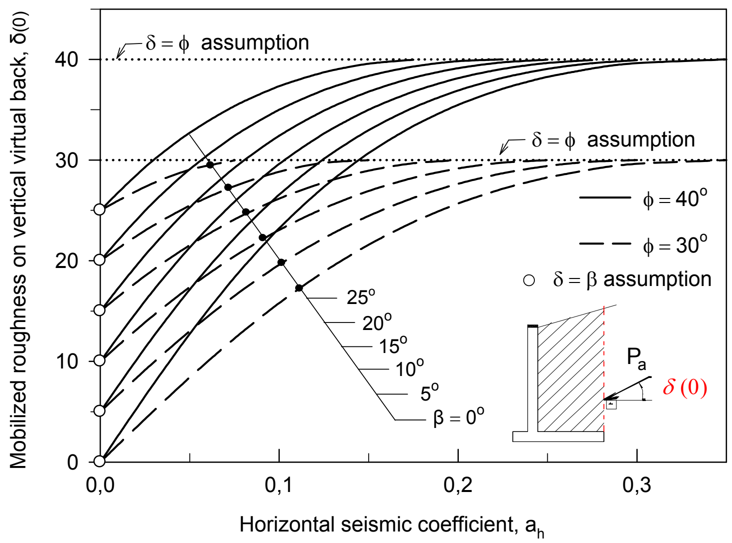

Figure 7. Variation of inclination of active thrust on vertical plane as function of horizontal seismic acceleration and backfill inclination

3 STABILITY SAFETY FACTORS

Traditionally, stability control of retaining walls is based on safety factors against bearing capacity, sliding and overturning. Of these, only the first two are rationally defined, whereas the safety factor against overturning is known to be problematic and of uncertain usefulness (Greco, 1997). In Figure 8, the equilibrium of forces acting on the retaining wall is presented. Evidently, the total vertical and horizontal forces acting on the wall are compensated by the external reactions N and T acting on the footing. The combination of these two actions, together with the resulting eccentricity e, determines the bearing capacity of the wall based on classical limit analysis procedures for a strip footing subjected an eccentric inclined load (e.g. EC7).

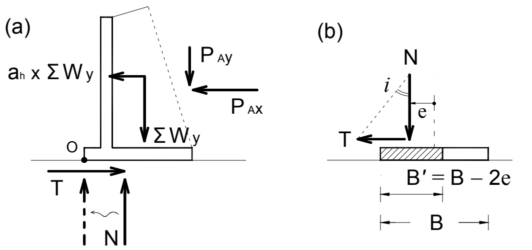

Figure 8. (a) Equilibrium of forces; (b) equivalent centrically loaded footing according to Meyerhof

The safety factors against bearing capacity and sliding are well defined as the ratio of forces N and T over the corresponding ultimate loads. In contrast, the safety factor against overturning is not defined on a rational basis. Its calculation assumes a limit state of rotation about the toe O of the wall, which is the point where the vertical reaction N acts (so it generates zero moment with respect to O), since the wall base is not considered to be in contact with the ground. Then, the moments of the remaining forces acting on the wall are compared upon classification (in an arbitrary manner), into stabilizing and overturning components. The spurious nature of this analysis can be proven analytically, since the safety factor is not invariant with respect to the arbitrary choice of the virtual back ω (Fig. 4) (Greco, 1997). It can also be proven that the assumed limit state does not represent the most critical failure mechanism, as the bearing capacity of the footing, or even the structural integrity of the wall, will be exhausted before the wall starts rotating around O. The last point is recognized in recent codes; however the conventional safety factor against overturning is either preserved (EC7), or replaced by a check of eccentricity of the vertical reaction N on the base of the wall (e.g. AASHTO LRFD, 1994). The above issues are highlighted with the help of following numerical example.

3.1 Numerical example

The case of an L-shaped wall with stem width t/H=0.05 and interface roughness $\delta_w=2\phi/3$ retaining soil of unit weight ratio $\gamma/\gamma_w=0.8$ with respect to the unit weight of the wall, friction angle $\phi=35^o$, and slope inclination $\beta=10^o$, is examined. The soil under the wall is assumed to be the same as the retained one. No passive resistance is considered.

In Figures (9a – c), the safety factors against overturning, bearing capacity and sliding are compared for variable heel length and seismic acceleration. In Figure (9d) the corresponding eccentricity of base reaction is presented for the same cases. It is evident that the safety factor against overturning is always higher than 1, even for cases where the safety factors against sliding and bearing capacity are not satisfied. The significant overtopping of bearing capacity shown in Fig. 9d, is due to the high inclination and high eccentricity of the reaction on the footing, which exceeds the limit of B/6 for a wide range of seismic acceleration coefficients and wall base widths. It can also be noticed that the bearing capacity is overtopped in cases where the eccentricity does not exceed the limit B/3 for dynamic loading according to seismic codes (EC7, EC8).

In Table 1 the corresponding safety factors obtained for different virtual backs ω, are presented. It is evident that the safety factors against sliding and bearing capacity coincide regardless of the arbitrary choice of virtual back, as all possible ω's generate the same reaction at the base of the wall. In contrast, the safety factor against overturning about point O depends on the arbitrary choice of virtual back (angle ω) – a behavior which is obviously unacceptable.

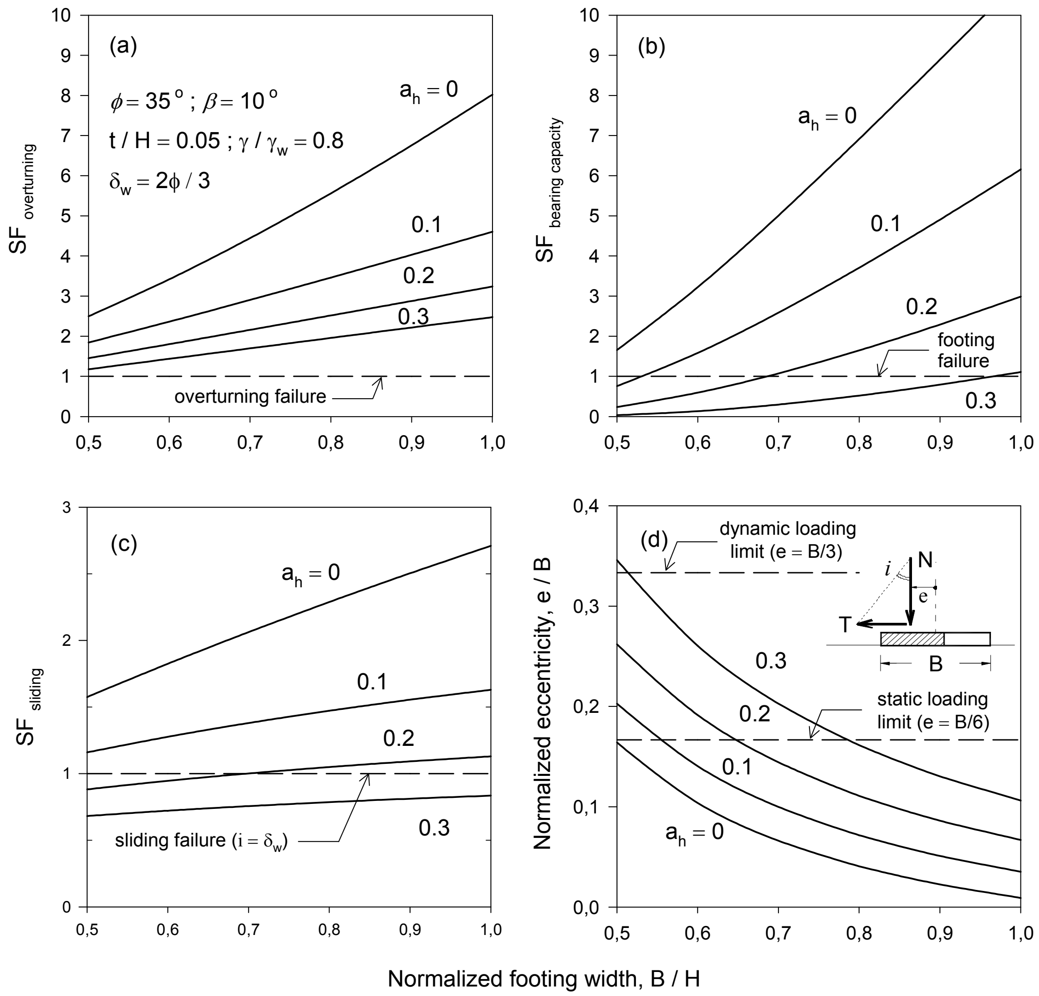

Figure 9. Safety factors against: (a) bearing capacity, (b) overturning, (c) sliding and (d) eccentricity of the contact force, as function of base width and seismic acceleration.

Table 1. Safety factors against sliding, overturning and bearing capacity (ϕ=35°, δ_w=2ϕ/3, a_h=0.2, B/H=0.8, t/H=0.05, γ/γ_w=0.8).

Arbitrary angle of virtual back, ω (°)	$SF_{Sliding}$	$SF_{Overturning}$	$SF_{Bear. Capacity}$
-30		2,27	
-20		2,35	
-10	1,05	2,44	1,65
0	(all ω's)	2,52	(all ω's)
10		2,60	
20		2,70	
30		2,81	

3 CONCLUSIONS

An exact analytical solution of the Rankine type was presented for the gravitational and earthquake-induced earth pressures on L-shaped cantilever retaining walls. The proposed analysis leads to the following conclusions:

1) The classical Coulomb and Mononobe-Okabe solutions can be substituted with more accurate and conservative methods, such as stress limit analysis solutions.

2) L-shaped walls should be treated as special wall types by EC-8, as done in some national codes (EAK 2000), since Rankine conditions prevail. However, the geometric condition proposed by some seismic codes for the validity of the Rankine condition is not accurate, yet it may be satisfactory for a wide range of cases encountered in practice.

3) The active thrust on the wall and the corresponding inclination of soil thrust can be determined for any arbitrary virtual back in the backfill. However, the use of the vertical virtual back is advantageous as it leads to simpler geometry. Equations (17) and (18) were derived to determine the active thrust, which are recommended for practical use.

4) The inclination of soil thrust on the vertical virtual back is equal to the slope inclination β only for the case of the gravitational loading (a_h= 0), contrary to the recommendations of a number of seismic codes. However, this spurious assumption would yield results on the safe side, with an increasing margin of safety under increasing seismic acceleration. This may result in an over-sizing of the retaining walls, beyond the desirable safety factors suggested by EC7 and EC8. The exact value for $\delta(0)$ could be employed to this end.

5) The retaining wall stability check may be viewed as a footing stability problem subjected to an eccentric, inclined load. Compared to isolated footings, the eccentricity can be much higher than the limits specified by EC7 and EC8 for spread footings. On the other hand, given the small duration of this loading and the compliance of these systems, these exceedance can be used to prevent collapse at the expense of accumulating some plastic deformation.

6) The conventional safety factor against overturning is defined arbitrarily and does not represent the most critical failure mechanism. It is the opinion of the authors that its use in practice is misleading and should be discontinued.

REFERENCES

AASHTO LRFD (1994), “Bridge Design Specifications”, American Association of State Highway and Transportation Officials (AASHTO), Washington, D.C., USA.

Boussinesq, J. (1876).”Essai théoritique sur l’équilibre d’élastité des massifs pulveruients”, Mem. Savante etrangere, Acad. Belgique, 40, pp. 1-80.

Budhu, M. (2007), “Soil Mechanics & Foundations”, 2nd Edition, J. Wiley & Sons Inc, New York.

BS 8002:1994, Code of Practice for Earth Retaining Structures, B.S. Institution.

Chu, S.C. (1991), “Rankine analysis of active and passive pressures in dry sands”, Soils & Foundations, Vol. 31, No. 4, pp.115 – 120.

Clayton, C.R.I., Militisky, J., and Woods, R.I. (1993). Earth Pressure and Earth Retaining Structures, 2nd Edition, Blackie Acad.& Prof.

Costet, J. and Sanglerat, G. (1975). Cours pratique de mechanique des sols, plasticité et calcul des tassements. 2nd edition, Dunod Technique Press, Paris.

EAK 2000 (2003), Greek Seismic Code, OASP, Athens.

Ebeling, R.M., Morrison, E.E., Whitman, R.V., Liam Finn, W.D. (1992). A Manual for Seismic Design of Waterfront Retaining Strutures, US Army Corps of Engineers, Technical Report ITL-92-11.

EN 1997-1 (2004), Eurocode 7, Geotechnical Design, Part 1: General Rules.

EN 1998-5 (2004), Eurocode 8, Design provisions for earthquake resistance of structures, Part 5: Foundations, retaining structures and geotechnical aspects.

Evangelista A., Scotto di Santolo A., Simonelli A.L. (2010). “Evaluation of pseudostatic active earth pressure coefficient of cantilever retaining walls”, Soil Dynamics & Eathquake Engineering, Vol. 30, Issue 11, pp 1119-1128

Greco, V.R. (1997), “Stability of Retaining Walls Against Overturning”, Journal of Geotechnical & Geoenviromental Engineering, ASCE, Vol. 123, No. 8, pp. 778 – 780.

Greco, V.R. (1999). “Active Earth Thrust on Cantilever Walls in General Conditions”, Soils and Foundations, Vol. 39, No. 6, pp. 65–78.

Heyman, J. (1973). “Coulomb’s Memoir on Statics; an essay in the history of civil engineering”, Cambridge University Press.

Kloukinas, P., Mylonakis, G.E, (2010). “Generalized Rankine Solution for seismic earth pressures” – Submitted for publication.

Kramer, SL (1996). Geotechnical Earthquake Engineering, Prentice Hall.

Levy (1874). “La Statique Graphique et Ses Applications à l'Art des Constructions”.

Mylonakis, G.E, Kloukinas, P. and Papantonopoulos, C. (2007). "An alternative to the Mononobe–Okabe equations for seismic earth pressures", Soil Dynamics and Earthquake Engineering, Vol. 27, No. 10, pp. 957-969.

O’Sullivan, C. and Creed, M. (2007). “Using a virtual back in retaining wall design”, Geotechnical Engineering, Vol. 160, No. GE3, pp. 147 – 151.

Rankine, W.J.M. (1857). On the stability of loose earth. Philosophical Transactions of the Royal Society of London, Vol. 147, pp. 9 – 27.

Terzaghi, K. (1943). Theoretical soil mechanics, John Wiley & Sons Inc., New York.

Trenter, N.A (2004). “Approaches to the design of cantilever retaining walls”, Geotechnical Engineering, Vol. 157, No. 1, pp. 27 – 35.