Abstract: One of the important extensions of the classical resource allocation problem is integrated resource allocation and routing problem with time window (IRARPTW). IRARPTW problem focuses on the time window for the service at the demanding node with the consideration of travel time of vehicle for a varying demand-oriented multi echelon supply chain with the consideration of limitation on the number of supply catalyst resource. We have developed a unified heuristic named clustering inherent genetic algorithm (CLING) to solve vehicle routing problem with time windows and IRARPTW. Heuristic CLING was tested for benchmark datasets of VRPTW and derived datasets of IRARPTW and yielded encouraging results.
Keywords: integrated resource allocation and routing problem with time window; IRARPTW; vehicle routing problem with time windows; VRPTWs; clustering inherent genetic algorithm; CLING.


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1 Introduction

Resource allocation (RA) involves the distribution and utilisation of available resources in the system. Because resource availability is usually scarce and expensive, it becomes important to find optimal solutions to such problems. Thus, RA problems represent an
important class of problems faced by mathematical programmers. Conventionally, such RA problems have been modelled and solved for allocation in single echelon supply chain (SC), single objective allocation, allocation with certainty of static input data, single performance measure driven allocation, disintegrated allocation and routing both in strategic and operational level. Such models that consider the above assumptions/constraints are nominal models and their solutions are denoted nominal solutions. However, in practice, these assumptions are rarely, if ever, true, which raises questions regarding the practicability and validity of the problems and solutions obtained under these assumptions. The allocation problems focusing bi or multiple objectives, dynamic allocation bases on dynamic input data and constraints, multiple performance driven allocation and integrated allocation and routing context are complex combinatorial problems which demand high computational time and effort for deriving compromised near-optimal/optimal solutions. In fact, Mulvey et al. (1981), and Ben-Tal and Nemirovski (2000) showed that such nominal solutions shall become irrelevant in the presence of real-world uncertainty. Edmondson and Schmidt (2010) presented the multi-agent distributed adaptive resource allocation (MADARA) toolkit, which is designed to address grid and cloud allocation and deployment needs and present a heuristic called the comparison-based iteration by degree (CID) heuristic, which they used to approximate optimal deployments in MADARA and also they analysed the performance of applying the CID heuristic to approximate common grid and cloud operations, such as broadcast, gather and reduce. Lin (2011) addressed a pick up and delivery problem with time window constraints, using two types of delivery resources, which allows for coordination. They formulated the problem as a mixed integer programme, assuming a reasonable coordination strategy and also they designed a two-stage heuristic by employing both exact approaches and an ejection chain based on modifying the shortest path algorithm implemented in a dynamic network. Park and Kim (2010) reviewed the school bus routing problem (SBRP) to plan an efficient schedule for a fleet of school buses where each bus picks up students from various bus stops and delivers them to their designated schools while satisfying various constraints such as the maximum capacity of a bus, the maximum riding time of a student in a bus, and the time window of a school. In this research, we study RA problem involving flow of resources over a typically, large-scale multi-echelon SC network in an optimal manner. This research focuses on development of model and heuristic for a new and complex sub-class of RA problem in SC network focusing integrated allocation and routing with complex constraints. This sub-class has some applications that are of special interest, including those that arise in the areas of warehousing, transportation, logistics, and distribution. These application domains have important economic value, and high importance has been linked to achieve efficient solutions. The diagrammatic representation of RA problem in SC is indicated in Figure 1.

The basic elements and position involved in RA problem with basic formulation is detailed:

- set of elements (e.g., personnel, facilities, tasks): \( A = \{a_1, \ldots, a_n\} \)
- set of positions (e.g., locations, processors): \( B = \{b_1, \ldots, b_m\} \)

(now let \( n = m \))
Effectiveness of pair $a_i$ and $b_j$ is: $c(a_i, b_j) \quad (1)$

$$x_{ij} = 1 \text{ if } a_i \text{ is located into position } b_j \text{ and } 0 \text{ otherwise } \left( x_{ij} \in \{0,1\} \right)$$

The problem is: $\max \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$ \quad (1)$

Subject to: $\max \sum_{i=1}^{n} x_{ij} = 0 \quad \forall j \quad (2)$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i \quad (3)$$

Time is of the utmost importance in logistics. Time can even be the decisive factor for efficiency and effectiveness of the SC. The just-in-time concept, for example, successfully streamlined the inbound logistics for production companies. Service lead times are another well-known element in logistics. Agreed service lead times can force choices in the SC processes, which are not necessarily the most cost effective or environmentally friendly. The integrated decision with allocation and routing in a time window driven scenario in the capacitated SC network is a challenging problem. The integrated allocation and routing is purely based on the time window for the service at the demanding node with the consideration of travel time of vehicle for a varying demand-oriented multi echelon SC with the consideration of limitation on the number of supply catalyst resource. This RA variant is termed as integrated resource allocation and routing problem with time window (IRARPTW). Applications of IRARPTW are in manufacturing and service industry which include urban solid waste management, taxi cab routing, postal distribution and school bus routing.

Figure 1 Diagrammatic representation of RA in SC (see online version for colours)
2 Literature review

Employee routing problem (ERP) can be defined as the facility of an enterprise or organisation transportation from their living location to their working locations or ERP can be defined as the provision of public transportation from their residences to and from their organisations. It consists of finding out of a series of bus routes that ensure that the service is provided equitably to all employees. The solution to this problem is the creation of an efficient schedule for a fleet of buses where each bus picks up employee from various bus stops and delivers them to their designated organisations while satisfying various constraints such as the maximum capacity of a bus, the maximum riding time of an employee in a bus, and the time window of an organisation. This includes finding out of a series of Routes that ensure that the service is provided equally to all employees.

The current literature deals primarily with single-objective problems and the models with multiple objectives typically employ a weighted function to combine the objectives into a single one. The ERP falls into a larger class of problem that is called the vehicle routing problem (VRP). VRP focuses on the efficient use of a fleet of vehicles (e.g., trucks, buses and cars) that must make a number of stops to pick up and/or deliver passengers or products. A survey of the literature may be found in Fisher, Desrosiers, Federgruen, Laporte and Bodin. Because VRP is a well-known hard problem, it is futile to search for an algorithm that gives the optimal solution in every instance. Therefore, most of the research in this area concentrates on the development of heuristic algorithms.

Newton and Thomas (1969) developed a practical method for generating school bus routes and schedules on a digital computer. Routing is accomplished by a two step procedure. First, the shortest route a bus of infinite capacity traverses in order to visit all of the stops is determined. This route, the solution of the travelling-salesman problem associated with the set of bus stops, is obtained by an efficient heuristic procedure which yields near-optimal solutions to problems of a realistic size. This single route is then partitioned to provide individual bus routes and schedules which satisfy bus capacity, bus loading policy, and passenger riding time constraints.

Bennett and Gazis (1972) described a procedure for the designing of school bus routes by computer. The procedure is an extension of the Clarke and Wright algorithm for scheduling delivery vehicles.

Newton and Thomas (1974) describes and evaluates a practical computer-based method for translating data concerning the location of each school in a multi-school system to be serviced by a bus fleet, the location of each student to be transported to each school, the time period during which students assigned to each school are to be transported, and the available bus facilities into a set of bus routes which specify school-to-school sequencing of each bus and the stop-to-stop route to be followed in travelling to every school. Each route is designed so that bus capacity and student riding time constraints are satisfied while attempting to minimise both total bus travel time and the number of routes required to service all the stops associated with the school. The mathematical models developed were programmed in FORTRAN IV for use on a CDC 6,400 computer and were applied to four schools in a Western New York school district.

Russell and Igo (1979) examined a routing design problem in which the objective is to assign customer demand points to days of the week in order to solve the resulting node routing problems over the entire week most effectively. The emphasis is on obtaining
approximate solutions for this type of combinatorial problem. They developed and tested several heuristics on a large-scale refuse collection problem.

Bowerman et al. (1995) presented a multi-objective mathematical formulation for the urban SBRP. They developed a heuristic algorithm and it was tested with data from a sample school board location in Wellington County in Ontario, Canada. This report reviews traffic modelling and bus-routing optimisation for urban cities by means of an entropy-based formulation of their vehicular movements within the domain under examination. The perceived level of disorder caused by the numerous vehicle-student-trips in the domain under examination is subsequently used for the formulation of a policy and a bus-routing scheme in order to minimise the entropy in the system.

Corberan et al. (2002) addressed the problem of routing school buses in a rural area with a node routing model with multiple objectives that arise from conflicting viewpoints. From the point of view of cost, it is desirable to minimise the number of buses used to transport students from their homes to school and back. From the point of view of service, it is desirable to minimise the time that a given student spends en route. They developed a solution procedure that considers each objective separately and search for a set of efficient solutions instead of a single optimum and their solution procedure is based on constructing, improving and then combining solutions within the framework of the evolutionary approach known as scatter search. Experimental testing with real data is used to assess the merit of their proposed procedure.

Li and Fu (2002) describe a case study of the SBRP. It is formulated as a multi-objective combinatorial optimisation problem. It also aims at balancing the loads and travel times between buses. They proposed a heuristic algorithm for its solution. The algorithm has been programmed and run efficiently on a PC. Numerical results are reported using test data from a kindergarten in Hong Kong.

The SBRP (Schittekat et al., 2006), is similar to the standard VRP, but has several interesting additional features. They develop an integer programming formulation for this problem, as well as a problem instance generator. They then show how the problem can be solved using a commercial integer programming solver and discuss some of their results on small instances.

Bektas and Elmastas (2007) described an exact solution approach for solving a real-life SBRP for transporting the students of an elementary school throughout central Ankara, Turkey. The problem is modelled as a capacitated and distance constrained open VRP and an associated integer linear programme is presented. The integer programme borrows some well-known inequalities from the VRP, which are also shown to be valid for the SBRP under consideration.

Park and Kim (2010) aims to provide a comprehensive review of the SBRP, the various assumptions, constraints, and solution methods used in the literature on SBRP are summarised. They also presented a list of issues requiring further research.

Christodoulou (2010) presented a method by which traffic flow estimation between known origins and destinations can be evaluated based on a modified entropy model, and by which bus-routing optimisation can be performed. The traffic flow analysis is performed by use of an entropy-based formulation of the vehicular movements of students within the domain under examination, while the perceived level of disorder caused by the numerous vehicle-student-trips in the domain under examination is subsequently used for the formulation of a policy and a bus-routing scheme in order to minimise the original entropy in the system. The entropy metric used in the scheduling
optimisation is related to the probability of student-trips by origin and destination, and an application of the method is illustrated via a case study of an urban university initiating bus service for its students.

Souza and Siquerira (2010) discussed similar to the VRP, however, a heuristic algorithm is proposed to determine the set of the bus stops. They proposed to construct digital maps containing the roads where the vehicles will be able to travel, since there are no digital maps of these regions. The real distances between the points are calculated and the heuristics location-based heuristic with some additional features was used to propose the new routes. The algorithm was named by adapted location-based heuristic. The school transportation problem was implemented in the State of Parana for 399 cities.

Raut et al. (2011) developed a model for supplier selection problem for a real world application in India. They proposed a combined MCDM methodology and utilised fuzzy analytic hierarchy process (AHP) and linear programming (LP) for assigning weights of the criteria for supplier selection and LP (technique for order preference by similarity to ideal solution) is used to determine the most suitable alternative using these criteria weights. Sandeep et al. (2011) used fuzzy strategy-aligned simple multi-attributes rating technique (SMART) approach for supplier selection process. The proposed methodology is exemplified with a case study.

Liao (2011) discussed the wireless communications and human-system interactions of the taxi dispatch process and explored taxi service operations management based on the automatic vehicle location and dispatch system and wireless communication technology. In particular, they are able to offer timely and quality service to many different customers who request for taxi services at different locations.

Lee and Baker (2011) utilised a non-linear integer programming model of ABC’s combined forward-reverse logistics system that enables them to enhance their strategic planning in three ways:

1. improve their pricing on both outbound and inbound items for clients
2. systemically select better transportation options
3. systemically select better outsourcing options.

They compare the recommendations from this model against the most representative model for ABC’s system in the extant literature.

Rajesh et al. (2011) developed a three-phase analytical model with the integration of AHP and quality function deployment (QFD). In the current competitive scenario, the consideration of business metrics and voice of customer for evaluating the selection criteria of 3PL service provider benchmarks the agility of an organisation. The analytical model was validated and illustrated with a real-life case study.

Anbuudayasankar et al. (2009) proposed a composite genetic algorithm (GA) combined with different local search methods to solve the MVRPB to obtain approximate solutions. This was the first research paper for the application of meta-heuristics with local search methods to solve the MVRPB.

Kim et al. (2009) developed a fair and equitable mechanism of sharing the profits achieved due to cooperation in a SC between a single manufacturer and a single retailer. At first, they take into account the bargaining dynamics between them and they assume that the current SC is not coordinated such that the leader (the dominating partner)
determines the lot size unilaterally. The follower (the weak partner) proposes to pay a compensatory payment according to the leader’s modified lot size.

Biswas and Mahapatra (2009) proposed a meta-heuristic approach based on particle swarm optimisation (PSO) to solve the machine loading problem. It deals with the assignment of the necessary operations and tools among various machines in an optimal manner to minimise system unbalance under technological constraints.

Sachan et al. (2008) developed a RA plan based on customer preferences and organisational strategy. They chose the automatic teller machine (ATM) service of the bank as the service in the current research and conjoint experiments were used to identify choice patterns of customers.

Raja et al. (2008) proposed a genetic algorithm-fuzzy (GA-fuzzy) logic approach to select the optimal weighted earliness-tardiness combinations in a non-identical parallel-machine environment. They compared their performance of the combined objective function obtained by the proposed GA-fuzzy technique has been compared with the solutions yielded by the GA techniques available in literature, known as genetic algorithm with partially mapped crossover operator (GAPMX) and genetic algorithm with multi-component uniform order-based crossover generator (GA-MCUOX). The comparison shows that the proposed GA-fuzzy technique outperforms both the GA-PMX and GA-MCUOX.

Ganesh and Narendran (2007) present a multistage heuristic to solve a VRP with delivery, pick up and time windows that is applicable to the public healthcare system and they proposed a methodology that exploits the inherent features of the problem. Further they developed a GA with innovative operators for crossover and mutation and conduct a rapid search to find a near-optimal solution.

Sivakumar et al. (2008) proposed a three-phase composite analytical model (CAM) to solve IAR problem for public sector blood banks and five-phase CAM for private sector blood banks. They addressed the integrated problem of allocating and routing blood for a public and private healthcare system and modelled the problem as multiple-vehicle, multidepot, multi-criteria allocation-routing problem which is termed as integrated allocation-routing (IAR) problem. Models are developed based on the integration of AHP and mixed-integer LP model.

Ledesma and Gonzalez (2012) introduced a generalisation of the VRP called the multi-vehicle travelling purchaser problem, modelling a family of routing problems combining stop selection and bus route generation. It discusses a mixed integer programming formulation extending previous studies on the classical single vehicle travelling purchaser problem. The proposed model is based on a single commodity flow formulation combining continuous variables with binary variables by means of coupling constraints. Additional valid inequalities are proposed with the purpose of strengthening its LP relaxation. These valid inequalities are obtained by projecting out the flow variables. They develop a branch-and-cut algorithm that makes use of the proposed model and valid inequalities. This cutting plane algorithm is implemented and tested on a large family of symmetric and asymmetric instances derived from randomly generated problems, showing the usefulness of the proposed valid inequalities.

Based on the literature, it is evident that IRARPTW is not addressed by the researchers and there is a need to develop methodology to solve this variant.
3 Problem description

One of the important extensions of the classical RA problems in the context of integrated decision and time driven capacitated multi echelon SC network is vehicle routing problem with time window (VRPTW). VRPTW can be described as follows (Homberger et al., 1999): \( n \) nodes are to be served by a depot with vehicles of the same capacity \( Q \). The demand \( q_i \) of each node \( i, i = 1, \ldots, n \), is to be covered by exactly one service within a given time window. The locations of the depot and of the nodes and the shortest distance with the corresponding travel time between every pair of locations are given. The objective is to determine a feasible route schedule which minimises the total distance travelled. This is addressed by various researchers and numerous solution methodologies are proposed for this variant.

IRARPTW. The variant considered in this chapter is in the context of integrated decision and time driven capacitated multi echelon SC network and one of the classical RA problems and it is an extension of VRPTW with the consideration of capacity constraint in the vehicle and within the stipulated time windows. This is a new variant which is not addressed by previous researchers in the literature and the variant is termed as IRARPTW. The base variant of IRARPTW is VRPTW. The IRARPTW can be stated as follows: The objective of the problem is to find the optimal allocation and routing of vehicles to pick up the customers with capacity constraint in the vehicle and within the stipulated time windows. Applications of IRARPTW are in manufacturing and service industry which include urban solid waste management, taxi cab routing, postal distribution and school bus routing. Research by Figliozzi (2009) is considered as the base paper for the variant of IRARPTW. The objective of this variant is to find an optimal route for each vehicle (bus), starting and ending at the office, such that each employee are served by exactly one bus, and the cost of each route are minimised. Also each bus should carry only \( N \) number of employees. The problem for this project is defined as follows: Total \( N \) numbers of employees of an organisation are to be picked up daily using a fleet of \( M \) seater mini-buses from their home to the office.

The following assumptions apply.

- all routes start and end at the node of origin, also known as depot
- each node in \( N \) is visited exactly once and served within its time window
- demand at any node shall never exceed the vehicle capacity \( Q \)
- all vehicles have the same capacity and are stationed at the node of origin
- split delivery is not permitted
- each vehicle makes exactly one trip
- all delivery quantities are loaded at the depot; all quantities picked up must be unloaded at the depot
- if a node requires both delivery and pick up, delivery precedes pick up
- every route should start with a pure delivery node, cover a mix of delivery and delivery and pick up nodes and finally include pure pick up nodes, if required
a vehicle may arrive at a node ahead of its earliest time but must not reach a node after its latest time.

4 Solution methodology to solve IRARPTW

We have developed a unified heuristic named clustering inherent genetic algorithm (CLING) to solve both VRPTW and IRARPTW. This heuristic has devised based on clustering concept, which is inherent in the meta-heuristic GA. The emergence of meta-heuristics for solving difficult combinatorial optimisation problems is one of the most notable achievements of the last two decades. Notable among them are the emergence of VRP. The most promising and effective solution methods for VRP are meta-heuristics, which are general-purpose mechanisms for solving hard optimisation problems. In meta-heuristics, the emphasis is on performing a deep exploration of the most promising regions of the solution space. These methods typically combine sophisticated neighbourhood search rules, memory structures, and recombination of solutions. The quality of solutions produced by them is usually much higher than those obtained by classical heuristics. The only problem is increased computing time. The procedures are usually context-dependent and require finely tuned parameters for effective search. Each meta-heuristic has one or more adjustable parameters. This permits flexibility, but for any application to a specific class of problems, requires careful calibration on a set of numerical instances as well as testing on an independent set of instances. Meta-heuristics are classified as memory-less and memory-based, according to the use of previously exploited areas of the solution space. GAs are population-based algorithms that simulate the evolutionary process of species that reproduce. A GA causes the evolution of a population of individuals encoded as chromosomes by creating new generations of offspring through an iterative process that continues until some convergence criteria are met. At the end of this process, it is expected that an initial population of randomly generated chromosomes will improve and be replaced by better offspring. The best chromosome obtained by this process is then decoded to obtain the solution. The probability of selecting a chromosome from the population is usually proportional to its fitness in order to emphasise genetic quality while maintaining genetic diversity. Fitness refers to the value of an objective function that has to be maximised or minimised while exploring the solution space. The highest fitness value over $Q$ generations is the final result of a GA.

4.1 CLING for VRPTW and IRARPTW

We develop a unified heuristic CLING to solve both VRPTW and IRARPTW. The basic principle of clustering inherent is explained: All customer locations are specified in Cartesian coordinates. A transformation factor relating distance and time is defined. All the Cartesian coordinates are converted into polar coordinates. The service pick up points shall be numbered according to the angle the ray makes with the depot. The ranking should be performed in ascending order. The pick up points are divided into clusters by rotating the ray through 360 degrees. Two point crossover and single point mutation is used in GA.

The brief phase-wise description of CLING is described as follows.
Phase 1  Construction of initial solution using clustering

Objects in same cluster are similar in some sense and those indifferent clusters are dissimilar in the same sense. Four proximity measures such as single linkage, complete linkage, group average and centroid are used. The inputs required for clustering phase are set of customer nodes, set of vehicles, travel times and time windows data. The clustering algorithm works on similar lines as the Dondo Cerda algorithm. This algorithm is used by employing the sweep strategy to form clusters. Proposed clustering algorithm is a modified form of the existing sweep algorithm such that time window constraints are incorporated. There are total four constraints considered in this phase 1. Those are: total time of travel; maximum capacity of each vehicle; maximum number of vehicles in a cluster and minimum number of vehicles. Each cluster is assigned to a single vehicle. There exists a route connecting the nodes of a cluster. Average length per node travelled by the assigned vehicle should be minimal. There are total six sub-steps in Phase 1 which is detailed below:

Step 1  All customer locations must be specified in Cartesian coordinates.
Step 2  Define a transformation factor relating distance and time.
Step 3  Convert the Cartesian coordinates into polar coordinates.
Step 4  Employee pick up points shall be numbered according to the angle the ray makes with the depot.
Step 5  The ranking shall be in ascending order.
Step 6  Divide the pick up points into clusters by rotating the ray through 360 degrees.

Phase 2  Improvement of initial solution using GA

A GA is employed for tour improvement. The initial population is checked for the fitness. Fitness corresponds to the objective function. Travel time of each route is taken as the objective function.

Step 1  Obtain initial solution from Phase 1.

The initial solutions are obtained from Phase 1. Set of initial solutions are generated based on the clustering construction heuristics and equal amount of solutions are taken as initial solutions for GA.

Step 2  Specify the population size.

The initial population (pop size) is generated by clustering construction heuristic based on the fitness and yield up to the required population size by satisfying the chromosome feasibility. Disallow strings that are exact replicas of the existing members of the population.

Step 3  Chromosome representation.

The chromosome contains a permutation of index of edges having non-zero demand. Distance between two neighbouring edges in chromosome is assumed to be shortest distance existing between them. A sample chromosome is shown in Figure 2.
Figure 2  Chromosome representation for VRPTW and IRARPTW problems

<table>
<thead>
<tr>
<th>Vehicle 1 &amp; 2</th>
<th>Vehicle 1</th>
<th>Vehicle 1</th>
<th>Vehicle 1</th>
<th>Vehicle 2</th>
<th>Vehicle 2</th>
<th>Vehicle 2</th>
<th>Vehicle 1 &amp; 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depot</td>
<td>Node 1</td>
<td>Node 2</td>
<td>Node 3</td>
<td>Node 4</td>
<td>Node 5</td>
<td>Node 6</td>
<td>Node 7</td>
</tr>
</tbody>
</table>

Step 4  Generate population using Or-opt.

Or-opt, a well-known node exchange heuristic (Or, 1976; Taillard et al., 1997) removes a maximum of three consecutive nodes from a route and inserts them, in the same sequence, at another stretch of the same route. Or-opt can be considered as a special case of 3-opt in which a chain of two or three consecutive nodes is shifted to a different part of the route. The preceding and succeeding nodes of the earlier route are now directly linked by an arc. The chain is inserted between some other pair of nodes, replacing the arc that linked them earlier. Solomon and Desrosiers (1988) showed that Or-opt produces good solutions despite considering fewer exchanges than the 3-opt procedure; it also requires less computational time. After each set of exchanges, we check for feasibility, compute the total distance travelled, compare it with the current best route and update the solution.

The Or-opt algorithm is given below:

Step 4.1  Consider an initial route and set \( t = 1 \) and \( s = 3 \).

Step 4.2  From the route, remove a chain of \( s \) consecutive vertices from position \( t \) to \( t + 2 \) and tentatively insert it between all remaining pairs of consecutive vertices on the route.

Step 4.2.1  If one or more insertions bring about a decrease in the cost of the route, choose the new route, based on the maximum reduction in cost.

Step 4.2.2  If no insertion decreases the cost of the route, set \( t = t + 1 \). If \( t < n + 1 \), repeat Step 2.

Step 4.3  Set \( t = 1 \) and \( s = s - 1 \). If \( s > 0 \), go to Step 2; else, stop.

Step 4.4  Specify the number of iterations.

Step 4.5  Define a fitness function.

A selection criterion is used for choosing two parents to apply the crossover operator. A fitness value reflects the goodness of an individual, compared with the other individuals in the population. In this study, a fitness value corresponds to a value of the objective function, namely, the total distance travelled. We adopt two selection methods: an elitist-preserve strategy and a roulette-wheel selection (Bentall and Stent, 2001). The former method is used when two higher rank individuals are unconditionally preserved to the next generation and the latter is employed to randomly pick up a superior individual from the remainder.

Step 5  Calculate the fitness value of each chromosome.

Step 6  Choose a method for crossover.
Step 7  Choose a crossover probability.

Step 8  Perform crossover.

Two point crossovers in which selection is based on Roulette wheel selection is used for GA. Apply the crossover operator to each of the $N/2$ pairs of strings in the mating pool with a crossover probability $p_c$. Use a two-point crossover; select a pair of crossing points randomly, along the length of the first parent string. Based on the string length, generate two crossover points randomly to select a segment in another parent between these crossover points. Generate an offspring by arranging the elements of the selected segment in this parent according to the order in which they appear in the other parent, with the order of the remaining elements being the same as in the first node; check it for feasibility. Exchange the role of these parents to generate another offspring. We implement two versions of an operator, labelled C1 and C2. In the first version (Figure 2), the nodes outside the two selected points are copied into the proto child and the remaining nodes are copied from the second parent in the order of their appearance. In Figure 2, the two crossing points are the nodes 5 and 7. In the second version of the operator (Figure 3), we copy the sequence of the nodes between the selected crossing points into the child and take the remaining nodes from the second parent in the order of their appearance. Here is an illustration of the crossover operator:

Select two strings randomly from the population and denote them as parent 1 and parent 2. Consider two random crossover points: 4 and 7. Exchange the selected segment between two parents to produce offspring as shown in Figures 3 and 4.

**Figure 3** Two point crossovers (C1)
Step 9 Mutate with a low probability. 
Single point mutation is used as mutation operator.

Step 10 Termination.
Terminate based on number of iterations.

4.2 Parameter settings for CLING

The values of the parameters used in CLING are detailed below. These parameters are evolved based on various runs with different settings. The population size is number of chromosomes considered for the total iteration of the heuristic. The probability of crossover and mutation are critical inputs, which has arrived based on various trail runs. The maximum number of generations will terminate the iteration to obtain the final feasible solutions.

- population size \( (\text{pop size}) = 30 \)
- probability of crossover \( (P_c) = 0.85 \)
- probability of mutation \( (P_m) = 0.15 \)
- maximum number of generations \( (\text{max gen}) = 100 \).
5 Computational experiments and results

CLING is coded in C++ and both run on a PC Pentium IV 1.70 GHz processor for benchmark datasets of VRPTW and derived datasets of IRARPTW. We have used 25 standard benchmark datasets of VRPTW from Solomon (1987) and ten derived IRARPTW datasets from VRPTW datasets of Solomon (1987) to evaluate the proposed CLING. An average of the RD is then calculated for the best solutions and presented. We also report the computing times but do not use them for comparison owing to possible variations in the configurations of hardware and software employed.

5.1 Performance of solution methodology

The absolute deviation is simply the difference between an experimentally determined value and the accepted value. The absolute deviation is a measure of the accuracy, and not the error itself. The relative percentage deviation is a more meaningful value than the absolute deviation because it accounts for the relative size of the error. The relative percentage deviation is given by the absolute deviation divided by the accepted value and multiplied by 100%. Relative percentage difference (RPD) is a way of measuring the variation in a set of data that looks at the variation as a proportion of the average or target value. For example, it may be used for quality or portion control in a factory that makes 1 kilogram (kg) and 10 kg bags of widgets. A 1 kg bag that was 100 g too light would have a greater RPD than a 10 kg bag that was 100 g too light; 10% and 1%, respectively.

The number of nodes, the best known solution (BKS) value reported in the literature, the solution obtained by solution methodologies and the computational unit (CPU) time for all solution methodologies are reported for all datasets. A statistic called relative percentage deviation (RD) is calculated for each solution as follows:

$$RD = \left[ \frac{\text{Solution of CLING} - \text{Optimal or best known solution of lower bound}}{\text{Optimal or best known solution of lower bound}} \right] \times 100$$

An average of the RD’s is then calculated for the BKS and presented in the last row of each table.

5.1.1 Twenty-five standard benchmark datasets of VRPTW from Solomon (1987)

The problems reported by Solomon (1987) have long existed as benchmarks among datasets for VRPTW. The original networks contain 100 nodes with different settings. Table 1 shows the result of the heuristic CLING for the datasets along the best-known. It is observed that the heuristic CLING performed equally well on all datasets. The RD for the solution obtained by DIALING for MCARP datasets when compared to BKS is 0%. On an average, CLING performs equally better in terms of computational time when compared to computational time of BKS heuristic for VRPTW datasets. It is inferred that the proposed heuristic proves to be competitive with the known best solution. The computational times are indicated only to show that it is ‘implementably low’.
Table 1  Comparison of CLING for 25 standard benchmark VRPTW datasets of Solomon (1987)

<table>
<thead>
<tr>
<th>Dataset</th>
<th>N</th>
<th>BKS</th>
<th>CLING</th>
<th>RD</th>
<th>CPU by CLING (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c101</td>
<td>100</td>
<td>827.3</td>
<td>827.3</td>
<td>0</td>
<td>11.23</td>
</tr>
<tr>
<td>c102</td>
<td>100</td>
<td>827.3</td>
<td>827.3</td>
<td>0</td>
<td>11.22</td>
</tr>
<tr>
<td>c103</td>
<td>100</td>
<td>826.3</td>
<td>826.3</td>
<td>0</td>
<td>12.44</td>
</tr>
<tr>
<td>c104</td>
<td>100</td>
<td>822.9</td>
<td>822.9</td>
<td>0</td>
<td>11.17</td>
</tr>
<tr>
<td>c105</td>
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<td>827.3</td>
<td>827.3</td>
<td>0</td>
<td>12.41</td>
</tr>
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<td>827.3</td>
<td>827.3</td>
<td>0</td>
<td>13.41</td>
</tr>
<tr>
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<td>827.3</td>
<td>0</td>
<td>13.56</td>
</tr>
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<td>827.3</td>
<td>0</td>
<td>13.01</td>
</tr>
<tr>
<td>c109</td>
<td>100</td>
<td>827.3</td>
<td>827.3</td>
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</tr>
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<td>1,637.7</td>
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<td>19.93</td>
</tr>
<tr>
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<td>1,466.6</td>
<td>1,466.6</td>
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<td>18.64</td>
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<tr>
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<td>1,208.7</td>
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<td>16.65</td>
</tr>
<tr>
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<td>971.5</td>
<td>0</td>
<td>15.58</td>
</tr>
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<td>r105</td>
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<td>1,355.3</td>
<td>0</td>
<td>17.17</td>
</tr>
<tr>
<td>r106</td>
<td>100</td>
<td>1,234.6</td>
<td>1,234.6</td>
<td>0</td>
<td>18.49</td>
</tr>
<tr>
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<td>1,064.6</td>
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<td>19.04</td>
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<tr>
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<td>1,068</td>
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<td>1,048.7</td>
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<td>18.29</td>
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<tr>
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<td>1,619.8</td>
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<td>17.89</td>
</tr>
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<td>1,258</td>
<td>0</td>
<td>17.23</td>
</tr>
<tr>
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<td>100</td>
<td>1,513.7</td>
<td>1,513.7</td>
<td>0</td>
<td>18.92</td>
</tr>
<tr>
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<td>100</td>
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<td>1,207.8</td>
<td>0</td>
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</tr>
<tr>
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<td>1,114.2</td>
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</table>

Average RD = 0

5.1.2  Twenty-five derived datasets of IRARPTW from Solomon (1987)

The IRARPTW datasets are derived from the VRPTW datasets of Solomon (1987). We superimpose the IRARPTW features on this set of problems. The best reported or optimal solutions of VRPTW are taken as LBs for IRARPTW datasets. Table 2 shows the results of the derived IRARPTW datasets from VRPTW datasets of Solomon (1987). These datasets are numbered as Z1 to Z25. The RD for the solution obtained by CLING for IRARPTW datasets when compared to lower bound is 9.8%. On an average, CLING performs equally better in terms of computational time when compared to computational time of BKS heuristic for IRARPTW datasets.
Table 2: Comparison of CLING with 25 derived datasets of IRARPTW from Solomon (1987) with the lower bound of Solomon (1987)

<table>
<thead>
<tr>
<th>Dataset</th>
<th>N</th>
<th>BKS</th>
<th>CLING</th>
<th>RD</th>
<th>CPU by CLING (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c101</td>
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<td>827.3</td>
<td>901.23</td>
<td>8.94</td>
<td>44.13</td>
</tr>
<tr>
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<td>100</td>
<td>827.3</td>
<td>901.12</td>
<td>8.92</td>
<td>42.12</td>
</tr>
<tr>
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<td>100</td>
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<td>903.04</td>
<td>9.29</td>
<td>45.57</td>
</tr>
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<td>892.63</td>
<td>8.47</td>
<td>47.77</td>
</tr>
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<td>100</td>
<td>827.3</td>
<td>903.96</td>
<td>9.27</td>
<td>43.13</td>
</tr>
<tr>
<td>c106</td>
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<td>827.3</td>
<td>903.96</td>
<td>9.27</td>
<td>52.23</td>
</tr>
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<td>903.96</td>
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<td>46.23</td>
</tr>
<tr>
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<td>900.21</td>
<td>8.81</td>
<td>48.18</td>
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<td>903.96</td>
<td>9.27</td>
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</tr>
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<td>78.62</td>
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<td>1,114.2</td>
<td>1,248.17</td>
<td>12.02</td>
<td>77.93</td>
</tr>
</tbody>
</table>

Average RD = 9.80

6 Conclusions

This paper has addressed two variants VRPTW and IRARPTW. For this non-deterministic polynomial (NP)-hard problem, we have developed unified heuristic CLING as solution methodology. The CLING heuristic is tested for standard benchmark datasets of VRPTW and derived datasets of IRARPTW. The sizes of the datasets are 100 nodes. When compared with the best results reported so far for VRPTW datasets, CLING has performed equally competitive both in terms of solution quality and computational time. The heuristic CLING was tested using publicly available sets of benchmark problem for VRPTW and derived datasets of IRARPTW from VRPTW datasets. When compared with the lower bound of VRPTW datasets for the derived
IRARPTW datasets, CLING has performed with the deviation of 9.8% in terms of solution cost and equally competitive in terms of solution computational time. The overall findings seem to justify the employment of CLING in general, as suitable techniques for solving the VRPTW and IRARPTW. The critical contribution of the research is the development of unified CLING to solve both VRPTW and IRARPTW. The limitations of the research are of two-fold:

1. only medium size datasets are addressed in the research
2. there is no comparative heuristic is developed for this research apart from CLING.

Further research may consider constraints on precedence and time windows of nodes in both VRPTW and IRARPTW. The future scope also includes development of analyst’s tool-kit for finding quick and effective solutions and can be embedded into Decision support systems (DSS). The incorporation of ‘What If’ rules in a DSS along with the software for the heuristic is a potential way forward.

References


CLING: heuristic to solve integrated resource allocation and routing problem


R.A. Malairajan et al.


