Abstract—This paper investigates the effect of channel estimation errors in presence of spatial correlation for the fading MIMO multiple access channels. Specifically, the capacity is derived under the notion of reliable communication based on the average of the error probability over all channel estimation errors. Although not necessarily optimal with channel estimation errors, we restrict the input distributions to be Gaussian allowing the notion of achievable rate. For different amount of transmitter estimation errors, we formulate the optimal transmission that permits to maximize the achievable sum-rate. Numerical results show that the performance of covariance feedback is quite sensitive to channel accuracy while the gap between the open-loop and the closed-loop scenario increases with the uncertainty. Simulations also assess the performance degradation of transmit/receive correlation and reveal that their impact behaves independently of the estimation errors.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems are widely recognized to increase substantially the spectral efficiency of wireless channels. Specifically, for MIMO Multiple Access Channel (MAC), the capacity region [1] defines the information rates that enable users to communicate reliably and simultaneously. From this region, a particular interest has been drawn on the maximization of the sum-capacity. It consists in computing the optimal input covariance matrices according to the Channel State Information (CSI) available. The capacity benefits of multiuser MIMO are highly dependent on the accuracy of the CSI available. Making use of perfect CSI at the transmitter and the receiver, the iterative waterfilling [2] turns out to be an efficient algorithm to maximize the sum-capacity. This upper limit of communication involves a perfect CSIT, which is a challenging requirement in wireless MIMO communications.

A more conceivable scenario is first to rely on a limited feedback link. In presence of transmit correlation, statistical channel information may incur some capacity improvement. For MISO channels, the optimal solution was shown in [3] to consist of independent Gaussian inputs along the eigenvectors of the transmit correlation matrix. Generalization of this work to the MIMO MAC case can be found in [4]. For single-user MIMO, an additional receive correlation has been proved to be only involved in the power allocation [5]. Besides, the estimation process at the receiver side would surely fail to give an error free channel estimate, leading to an overall performance degradation. The effect of channel uncertainty has been addressed for SISO and MIMO channels in [6] and [7]. It has been shown that the gap between the lower and upper bound on mutual information is small for Gaussian inputs. Extensions of these works have also been proposed for MIMO MAC in [8][9]. The combined effect of partial feedback and channel uncertainty on capacity was addressed in [10]. It was shown that covariance feedback achieves a great part of the instantaneous capacity in the high SNR regime. However, these capacity bounds are only function of the channel estimation variance and do not consider, in the channel model, the estimation method. A more general framework for reliable communication has been recently introduced, see for instance [11], where a composite channel model results from the average of the error probability over all channel estimation errors. Thus, the maximal achievable sum-rate is defined as the maximum over all input distribution of the mutual information of this composite channel.

In this paper, we focus on this latter notion of reliable communication. From this model of uncertainty, we investigate the impact of channel estimation error in a MIMO MAC scenario, in presence of spatial correlation at both side of the link. This allows us to study the impact of channel correlation on the achievable sum-rate with (and without) estimation errors. We first derive a lower bound on mutual information. Then we determine the optimal transmission strategy of each user for different CSI accuracy at transmitter. The remainder of this paper is organized as follows. Section II describes the communication model and the estimation process in details. The achievable rate region is derived in Section III. In Section IV, optimal strategies are characterized under various CSIT. Section V provides simulation results. Conclusions are drawn in Section VI.

Notation

- The superscripts $^\top$ and $^\dagger$ indicate transpose and Hermitian transpose.
- Lower-case and capital bold letters are used to denote vectors and matrices, respectively.
- Let $H$ be a matrix, $h_{ij}$ denotes the $j$th column, and $h_{i,j}$ or $(H)_{i,j}$ the $(i,j)$th entry of $H$.
- Calligraphic letters denote a set of matrices, e.g., the set of $K$ matrices $A_k$, $k \in [1,K]$, is $\mathcal{A} = \{ A_k \}_{k=1}^K$.
- $\det(\cdot)$, $\text{tr}(\cdot)$, $\text{diag}(\cdot)$ stand for the determinant, the trace and the diagonal operator of square matrices, respectively.
II. COMMUNICATION MODEL

A. Channel model

We consider a $K$-user MIMO-MAC communication system where the $K$ transmitters are equipped with $N_1 \cdots N_K$ antennas and the receiver with $N_R$ antennas. Transmission occurs over a Rayleigh flat fading channel. The received signal $y \in \mathbb{C}^{N_R}$ at time $t$ is

$$y(t) = \sum_{k=1}^{K} H_k(t)x_k(t) + n(t)$$

where $x_k(t) \in \mathbb{C}^{N_k}$ and $H_k \in \mathbb{C}^{N_R \times N_k}$ denote, respectively, the vector of the transmitted signal and the channel matrix of user $k \in [1, K]$. Entries of $H_k(t)$ are zero-mean circularly symmetric complex Gaussian random values with variance $\sigma_n^2$. The additive noise vector $n(t) \in \mathbb{C}^{N_R}$ is independent and identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian with covariance matrix $\Sigma_n = \sigma_n^2 I_{N_R}$. We suppose both $\{H_k(t)\}_{t=1}^T$ and $n(t)$ are ergodic and stationary random processes. In the sequel of this paper, we can then omit the time index $t$. We further assume that the elements of $H_k$ are correlated. The channel matrix of user $k$ can thus be factorized as

$$H_k = R_{N_R}^{1/2} H_{k,w} R_{N_k}^{1/2}$$

where $H_{k,w}$ is the spatially white channel matrix. Separate transmit and receive correlation matrices, denoted respectively by $R_{N_k}$ and $R_{N_R}$, capture reasonably the effect of spatial correlation. The pdf of $H_k$ is $\psi_{H_k} = \mathcal{CN}(0, R_{N_k} \otimes \sum_{H_k})$, where $\sum_{H_k} = \sigma_h^2 R_{N_R}$. Each transmitter is subject to a power constraint $P_k$ such that $\text{tr}(E[x_k^H x_k]) \leq P_k$. The total transmit power from all users is $\sum_{k=1}^K P_k = P$. We define $x$ as the set of the transmitted signal, $x = \{x_k\}_{k=1}^K$. If perfect Channel State Information is available at the Receiver (CSIR), it follows that the MIMO multiple access channel is distributed as $W(y|x, H) = \mathcal{CN}(\sum_{k=1}^K H_k x_k, \sum_0)$. The mutual information is given by

$$I(y;x) = \log_2 \left| I_{N_R} + \sum_{k=1}^{K} \sum_0^{-1} H_k \Theta_{x_k} H_k^H \right|$$

where $\Theta_{x_k} = E[x_k^H x_k^H]$ is the covariance matrix of the input signal of user $k$. However, tracking perfectly the channel gains at the receiver seems implausible. Therefore, we consider that only noisy estimates $\hat{H}$ are available.

B. Estimation model

Each channel matrix estimate $\hat{H}_k$ may be obtained by the use of a training sequence sent from user $k$ to the receiver, before transmitting the data. $L_k$ vectors $X_{k,T} = (x_{k,T}, \cdots , x_{k,T}, L_k)$ constitute the training sequence such that $L_k$ is much smaller than the coherence time. We assume that the receiver can capture the statistics of the channel accurately, notably the transmit correlation. Indeed, correlation varies more slowly than the fading gains. Therefore, we choose a training sequence $X_{k,T}$ that decorrelates the transmit correlation, i.e., $X_{k,T} = R_{N}^{-1/2} x_{k,T}$. The average energy of the training symbols may be expressed as $P_{k,T} = \frac{1}{L_k} \text{tr}(X_{k,T} X_{k,T}^H)$. The corresponding received signal $Y_T = \sum_{k=1}^{K} R_{N_R}^{1/2} H_{k,w} X_{k,T} + N_T$ allows the receiver to perform ML estimation. Since to estimate $R_{N_R}^{1/2} H_{k,w}$ we need at least $N_R N_k$ measurements, and each symbol time yields $N_R$ samples, we must have $L_k \geq N_k$ provided that $X_{k,T}$ is full rank. Let us then consider that $\hat{H}_k = \left( R_{N_R}^{1/2} H_{k,w} + E_{k,w} \right) R_{N_k}^{1/2}$, where $E_{k,w}$ is a white estimation error matrix if we assume that $X_{k,T}$ is orthogonal. The estimation variance is $\sigma_{e,k}^2 = \text{SNR}_{k,T}^{-1}$ with $\text{SNR}_{k,T} = \frac{L_k P_{k,T}}{\sigma_n^2}$. As a result, the conditional pdf of $\hat{H}_k$ given $H_k$ is

$$\psi_{\hat{H}_k|H_k} = \mathcal{CN}(H_k, R_{N_k} \otimes \sum_{E_k})$$

with $\sum_{E_k} = \sigma_{e,k}^2 I_{N_R}$. The a posteriori pdf $\psi_{\hat{H}_k|Y_T}$ can be derived from (4) and $\psi_{\hat{H}_k|H_k}$ in a quite similar way as [12], and is expressed as $\psi_{\hat{H}_k|Y_T} = \mathcal{CN}(\sum_{\Delta_k} \hat{H}_k, R_{N_k} \otimes \sum_{\Delta_k} \sum_{E_k})$ where $\sum_{\Delta_k} = \sum_{H_k} + \sum_{E_k}^{-1}$. Then by averaging the channel $W(y|x, \hat{H})$ over all channel estimation errors and after some algebra, we obtain the composite channel

$$\tilde{W}(y|x, \hat{H}) = \mathcal{CN}(\sum_{k=1}^{K} \sum_{\Delta_k} \hat{H}_k x_k, \sum_0)$$

for which, in the next sections, we investigate the achievable rate.

III. MUTUAL INFORMATION WITH IMPERFECT CSIR

The performance limits of MIMO MAC is characterized by the capacity region. This region is the convex closure of the union of rate regions. For independent input distributions with a given set of covariance matrices $\{\Theta_{x_k}\}_{k=1}^{K}$ that satisfies the individual power constraints $\{P_k\}_{k=1}^{K}$, the rate region of a $K$-user channel [2] can be expressed as

$$A = \left\{ (R_1, \cdots , R_K) : \sum_{i \in S} R_i \leq I_S, \forall S \subseteq \{1, \cdots , K\} \right\}$$

Let $x_S$ and $x_S$ be the set of $\{x_i\}_{i \in S}$ and $\{x_j\}_{j \in S}$, respectively. The joint mutual information of users in $S$ is defined as $I_S = I(y; x_S | x_S, \hat{H})$. For the Gaussian MAC, it has been proved that the convex hull operation is not needed [2]. However, estimation errors involve a channel noise correlated to $x$ ensuing that Gaussian inputs do not necessary maximize the mutual information. Restricting the input distributions to Gaussian allows a closed-form expression for the joint mutual information. This defines a lower bound of the rate region, resulting in the notion of achievable rate region.

To characterize this region, we need to derive the mutual information for the composite channel $\tilde{W}$. For any subset $S \subseteq \{1, \cdots , K\}$, expanding the mutual information into entropies, leads to

$$I_S = h(x_S | x_S, \hat{H}) - h(x_S | y, x_S, \hat{H}).$$

(7)
Since the input distributions are independent, the conditioning of the first entropy in (7) can be omitted. As all input distributions are supposed Gaussian, the entropy of $x_S$ equals $h(x_S) = \sum_{i \in S} \log_2 \left( \pi e / |\Theta_{x_i}| \right)$. On the other hand, the received signal $y$ is linearly dependent with the channels inputs. Since conditioning reduces the uncertainty, it follows that the second term can be upper bounded as
\[
h \left( x_S | y, x_S, \tilde{H} \right) \leq \sum_{i \in S} h \left( x_i | y, x_S, \tilde{H} \right). \tag{8}
\]

The entropy is also a function of the additive Gaussian noise component $\sum_{i=1}^{K} \Sigma_{\Delta_i}^{1/2} E_{k,w} R_{N_k} x_k + n$ resulting from the uncertainty of every users’ channel in addition to the AWGN noise. The covariance of the $k$th vector $\sum_{i=1}^{K} \Sigma_{\Delta_i}^{1/2} E_{k,w} R_{N_k} x_k$, when the realization of $x_k$ is not known, is given by $\Sigma_{\Delta_i} \Sigma_{E_{k,w}} \text{tr}(R_{N_k} \Theta_{x_k})$. Alternatively, the covariance turns out to be $\Sigma_{\Delta_i} \Sigma_{E_{k,w}} \text{tr}(R_{N_k} x_k x_k^\dagger)$. Hence, the covariance of the additive noise component $\Sigma_S$ is expressed as
\[
\Sigma_S = \Sigma_0 + \sum_{i \in S} \Sigma_{\Delta_i} \Sigma_{E_{i,w}} \text{tr}(R_{N_i} \Theta_{x_i}) + \sum_{j \in S} \sum_{i \in S} \Sigma_{\Delta_i} \Sigma_{E_{j,w}} \text{tr}(R_{N_j} x_j x_j^\dagger). \tag{9}
\]

Besides, as adding a constant does not change the differential entropy a novel upper bound of (8) after a reduction of conditioning is
\[
h \left( x_S | y, x_S, \tilde{H} \right) \leq \sum_{i \in S} h \left( x_i - F_i \tilde{y}_S | x_S \right), \tag{10}
\]
with $\tilde{y}_S$ the received signal after subtraction of conditional components,
\[
\tilde{y}_S = y - \sum_{j \in S} \Sigma_{\Delta_j} \tilde{H}_j x_j. \tag{11}
\]

The entropy in (10) is function of the covariance matrix $E[(x_i - F_i \tilde{y}_S)(x_i - F_i \tilde{y}_S)^\dagger]$. Let the constant $F_i$ be such that the MMSE estimate of $x_i$ is given by $F_i \tilde{y}_S$, whose expression is
\[
F_i = \Theta_{x_i} \tilde{H}_i \Sigma_{\Delta_i} \left( \sum_{j \in S} \tilde{H}_j \Theta_{x_j} \tilde{H}_j \Sigma_{\Delta_j} + \Sigma_S \right)^{-1}. \tag{12}
\]

Finally, combining expressions from (7) to (12) gives the joint mutual information of users contained in $S$ after observing users in $\bar{S}$. $I(y; x_S | x_S, \tilde{H})$ is then defined as
\[
I_S \geq \sum_{i \in S} \log_2 \left( \frac{1}{I + \Sigma_S^{-1} \Sigma_{\Delta_i} \tilde{H}_i \Theta_{x_i} \tilde{H}_i \Sigma_{\Delta_i}^{-1}} \right) \geq \log_2 \left( \frac{1}{1 + \sum_{i \in S} \Sigma_S^{-1} \Sigma_{\Delta_i} \tilde{H}_i \Theta_{x_i} \tilde{H}_i \Sigma_{\Delta_i}^{-1}} \right) \tag{13}
\]
where the last approximation comes from the Jensen’s inequality. Let us define $I_{\text{low}}(y; x_S | x_S, \tilde{H})$ as the lower bound on mutual information whose expression is given by (14).

IV. SUM-RATE OF VARIOUS CSIT

Once the lower bound on mutual information of the composite channel is defined, let us look more precisely at the sum-rate. Recall that the MIMO MAC sum-rate is defined as
\[
C = \max_{(p(x_k))_{k=1}^K} \sum_{k=1}^K \log_2 \left( 1 + \Sigma_k \tilde{H}_k \Theta_{x_k} \tilde{H}_k \Sigma_{\Delta_k} \right) \tag{15}
\]

where the inequality first comes from the assumption that inputs $\{p(x_k)\}_{k=1}^K$ are Gaussian distributed. We are now able to investigate the achievable sum-rate for various CSIT.

A. Covariance feedback

Covariance feedback, or statistical CSI, means that each user knows its own channel covariance matrix. In the case of transmit and receive correlation with channel uncertainty (where the elements of $H_k$ are all zero-mean complex Gaussians), optimal transmission does not depend on the receive correlation matrix. Each user has to transmit independent streams of data along the eigenvectors of its own channel correlation matrix. This condition holds whatever the decoding order is and for any channel correlation of the rest of the users. It solves the following optimization problem
\[
C = \max_{\forall k \in S, \Theta_{x_k}, \text{tr}(\Theta_{x_k}) = P_k} E \left[ \log_2 \left( 1 + \Sigma_k \tilde{H}_k \Theta_{x_k} \tilde{H}_k \Sigma_{\Delta_k} \right) \right]. \tag{16}
\]

B. Instantaneous feedback

Perfect knowledge of the (non-errorfree) channel estimates at transmitter enables each user to adapt its covariance matrix to the channel realizations and other user’s power distribution. The objective function to optimize is then given by
\[
C = E \left[ \max_{\forall x_k, \text{tr}(\Theta_{x_k}) = P_k} \log_2 \left( 1 + \Sigma_k \tilde{H}_k \Theta_{x_k} \tilde{H}_k \Sigma_{\Delta_k} \right) \right]. \tag{17}
\]

The prior problem is not concave in $\Theta_{x_k}$. Nevertheless, in absence of transmit correlation, i.e., fixing $R_{N_k} = I$, (18)
of SNR values \((K = 2, N_R = 4, N_T = 2, \alpha = 0.7)\).

becomes a concave maximization. It can be proved that the
sum-rate is achieved by performing the iterative waterfilling
algorithm for the composite channel \(\Sigma^{-1/2} \Sigma_{\Delta_{k}} \tilde{H}_k\). In con-
trast, with transmit correlation, the optimal solution is difficult
to obtain. An alternative proposal may be to use brute force
method, i.e., exhaustively look for the term \(\text{tr}(R_{\Theta_k} \Theta_{x_k})\), then
for each tested value perform the iterative waterfilling on the
composite channel \(\Sigma^{-1/2} \Sigma_{\Delta_{k}} \tilde{H}_k\), and finally recompute
\(\text{tr}(R_{\Theta_k} \Theta_{x_k})\). This method is never able to decorrelate \(R_{\Theta_k}\)
as the iterative waterfilling solution inherently tends to a scaled
identity matrix \(\frac{1}{N_k} I_{N_k}\). However, for some cases, it is more
beneficial to invert the transmit correlation than performing the
iterative waterfilling. Consequently, we choose a sub-optimal
approach where the input covariance matrix is of the form
\(\Theta_{x_k} = V_k \Lambda_k V_k^\dagger\) where \(V_k\) are the eigenvectors of \(R_{\Theta_k}\) and
\(\Lambda_k\) is a diagonal matrix, as in \([10]\). The sub-optimality clearly
comes from the eigenvectors. The optimization problem \((18)\)
can be rewritten as
\[
C = E \left[ \max_{\forall k, \Theta_{x_k}} \log_2 \left| I + \Sigma^{-1} \sum_{\Delta_{k}} \tilde{H}_k \right| \right] \tag{19}
\]
Again, no closed-form solution exits to solve the prior problem
and optimization methods are then helpful to find the optimal
powers \(\{\Lambda_k\}_{k=1}^K\).

V. SIMULATION RESULTS

This section presents numerical results in order to compare
the influence of estimation errors. Average performance are
obtained through Monte Carlo simulations. We consider an
uplink transmission with \(K\) users occurring on flat fading
Rayleigh channels. Entries of the correlation matrices are
modelled following
\[
(R_N)_{i,j} = \alpha^{|i-j|} \tag{20}
\]
with \(\alpha\), the correlation parameter, varying in \([0, 1]\). Subscripts
\(R\) and \(k \in [1, K]\) are used to denote the receive or transmit
correlation parameters, respectively. We further assume that
each user follows the same statistics. This means that all user
have equal fading distributions \(\sigma_{\Delta_{k}}^2 = 1\), the same number
of antennas \(N_k = N_T\), the same training length \(L_k = L\), the
same correlation parameter \(\alpha_{\Delta_{k}} = \alpha_T\) \(\forall k\). It thus seems logical
that the individual power constraints are similar and given by
\(P_k = P/K, \forall k\). We set the average energy of the training sym-
bits to be equal to the power constraint, i.e., \(P_{k,T} = P_k, \forall k\).

Fig. 2 shows the impact of correlation at the transmitters
and at the receiver. We have assumed when investigating the
transmit correlation effect that the receive correlation
parameter was null \(\alpha_R = 0\), and vice versa. We first observe
that both receive and transmit correlation has a significant
effect on the achievable sum-rate. We notice that the receive
correlation has a larger detrimental effect. This conclusion
holds for other configurations, for instance when the number
of antennas $N_T$ is increased. This can be explained from (14) as the term $\Sigma_{\Delta_k}$ is function of the receive correlation and multiply the estimated channel $\tilde{H}_k$ of user $k$. When $\alpha_R > 0.4$ with 2 users, an additional increase of the correlation induces a substantial loss of capacity. An increased number of users emphasizes the capacity loss due to $\alpha_R$ while it reduces the impact of the transmit correlation. Note that in this case we increase the training sequence length so as to have the same estimation variance. Regarding the influence of correlation in presence of channel estimation errors, it seems that the effect of correlation is quite independent of the channel estimation process since the gap between the dashed lines ($L = 2$ vs. perfect CSIR, i.e., $L = \infty$) stays nearly constant whatever the correlation coefficient. However, when $\alpha$ becomes close to 1, the dashed curves tend to join.

So far the covariance matrix was set to a scaled identity matrix, Fig. 3 now shows the effect of the feedback load. To this end, in addition to the open-loop case, we plot average achievable sum-rate with covariance feedback (17) and with instantaneous feedback (19). Note that solving (19) provides the same performance as the brute force method, suggesting that it is close to the optimal solution. It can be seen that with perfect CSIR (i.e., $\sigma^2_L = 0$) the covariance feedback case performs very close to the closed-loop curve. However, in presence of channels estimation errors, this behaviour is not so favourable. Indeed, for $L = 4$, benefits over the open-loop case are seen only for low SNR values. See for instance that at 4 bits/c.u., covariance feedback outperforms the open-loop by 0.7 dB whereas instantaneous feedback attains a gain of 5 dB. We further notice that the gap between the open-loop and the closed-loop case is increasing with the uncertainty.

To conclude, if channel estimation errors are reduced (e.g., $L = 10$), covariance feedback realizes a good trade-off between performance and feedback.

VI. CONCLUSIONS

In this paper, we have formulated the achievable rate region of a multiuser uplink channel in presence of channel uncertainty. Although not necessarily optimal with channel estimation errors, we restrict the input distributions to be Gaussian allowing a closed-form expression for the mutual information. This ensures a sum-rate lower bound, for which we have investigated the optimal power allocation of various CSIT. We have shown, for covariance feedback, that the optimal transmission does not depend on the receive correlation and each user has to transmit along the eigenvectors of its own channel correlation matrix. As for instantaneous feedback, we have derived the optimal formulation and proposed an efficient numerical solution. Numerical results indicate that the relative performance gain of covariance feedback is quite sensitive to channel estimation errors, since at 4 bits/c.u. it performs close to the closed-loop scenario (achieving almost 100% of the potential gain) with perfect CSIR whereas it achieves only 15% of the potential gain when $L = 4$. The effect of channel correlation is also studied in this work, and it can be seen that performance degradation of channel estimation errors and correlation impact independently on the achievable sum-rate. The negative impact of receive correlation predominates over the transmit correlation. The latter decreases when the number of users is raised.

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