Comparison of Analytical and Numerical Analysis Design Methods for Piled Embankments

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Abstract

A comparison of analytical methods for estimating the magnitude of arching in piled embankment applications is presented and compared with three-dimensional numerical analysis of the problem. The piled embankment application is a truly three-dimensional problem that cannot be simulated by two-dimensional or axi-symmetric analyses. The magnitude of arching calculated for the geometries analyzed varied considerably between methods. Considerable variation in the calculated tension in the reinforcement was also observed for the embankment geometries analyzed. Finally, a new technique is proposed for constructing piled embankments.

Introduction

It is becoming increasingly necessary to construct infrastructure on land that was previously considered unsuitable. Construction of road and rail embankments over soft clay and peat foundations can result in large, often differential, settlements occurring at the surface of the embankment. This leads to expensive ongoing long-term maintenance of the road or rail surface (Davitt & Killeen, 1996 and Jennings, 1994). Other, more innovative solutions such as piled embankments are required.

A piled embankment consists of piles, usually in a square grid, driven through the unsuitable foundation soil to a firm-bearing stratum. A geosynthetic layer is installed over the pile caps at the base of the embankment. Due to the higher stiffness of the piles in relation to the surrounding soft soil, the vertical stresses from the embankment are concentrated on the piles. Soil arching develops as a result of differential settlements between the stiff pile heads and the soft ground between them. The three-dimensional arches span the soft soil and the applied load is transferred onto the piles and then the firm-bearing stratum (Kempfert *et al*, 2004).

Soil arching is a natural phenomenon encountered in geotechnical engineering (Terzaghi, 1943). Arching develops whenever a localized area at the base of a soil mass yields relative to the rest of the soil mass. Shear forces are generated at the transition zone between the yielding and unyielding zones of the soil mass and result in a reduction of the stress on the yielding mass and an increase in the stress on the adjacent stationary parts (Russell & Pierpoint, 1997). A reduction in the vertical load on the soft soil is a result of the aching effect in the embankment fill and a membrane effect of the geosynthetic reinforcement. The precise mechanism by which the load is transferred to the geosynthetics remains poorly understood (Love & Milligan, 2003). While several methods currently exist for estimating the magnitude of arching (Kempfert *et al.*, 2004, Russell *et al.*, 2003, Jenner *et al.*, 1998, BS8006, 1995, Hewlett & Randolph, 1988 and Terzaghi, 1943,) none yet captures the key characteristics of these complex structures (Love & Milligan, 2003).

The design of piled embankments can be divided into three principal steps:

1. Estimation of the degree of arching taking place in the embankment fill,
2. Calculation of the tension in the geosynthetic reinforcement layer,
3. Calculation of the lateral thrust of the embankment fill.

The analysis presented in this paper is concerned with Points 1 and 2 only. The lateral thrust of the embankment fill while of critical importance is considered beyond the scope of this paper.

The analysis presented expands the comparison of design methods made by Russell & Pierpoint (1997) to include methods described by Kempfert *et al.* (2004) and Russell *et al.* (2003). The comparison of the design methods is based on the analysis of two actually constructed embankments that are representative of embankments constructed in the United Kingdom. No support from the subsoil was included in the analysis as extreme care should be exercised in calculating support from the subsoil, as often this support cannot be relied upon in the long term. Over-reliance on the foundation soil can lead to large settlements and in extreme cases large differential movements as the piles punch through at the surface (Azam *et al.*, 1990). Also, no factors of safety or material reduction factors were applied in the analysis to facilitate a direct comparison between methods.

Review of German recommendations on piled embankment design

Kempfert *et al*. (2004) present a new design method derived from 1:3 laboratory models of the piled embankment problem. The method firstly estimates the magnitude of load on the soft soil with no reinforcement included, before estimating the tension in the reinforcement required to carry that load. In the laboratory study it was observed that a higher tension was generated in the reinforcement spanning directly between adjacent piles. The method proposed by Kempfert *et al*. (2004) allows for support from the subgrade to be included.

The tension in the reinforcement is estimated based on the theory of elastically embedded membranes, resulting in a design chart that relates the interdependency of the applied force, magnitude of the subgrade reaction, stiffness of the reinforcement and the maximum design strain. The magnitude of subsoil support is calculated based on a modulus of subgrade reaction.

Review of Russell et al. (2003) recommendations on piled embankment design

Russell *et al.* (2003) presented a new design method for piled embankments. The estimation of the magnitude of arching is based on a re-evaluation of the stress distribution in the yielding soil mass between piles. The tension carried by the reinforcement assumes the unsupported deflected shape of the reinforcement is a parabola.

Assessment of arching in piled embankments

Numerous methods are available for estimating the magnitude of soil arching occurring in a piled embankment. The more popular two- and three-dimensional methods include: the BS 8006 method (BS 8006, 1995), the Hewlett and Randolph method (Hewlett & Randolph, 1988), the Guido method (Jenner *et al.*, 1998) and the Terzaghi method (Terzaghi, 1943). In order to compare the magnitude of arching occurring a dimensionless parameter, the Stress Reduction Ratio, S_{3D}, (Low *et al*. 1994) has been defined as the ratio of the average vertical stress carried by the reinforcement to the average vertical stress due to the embankment fill.

Recently, Love & Milligan (2003) in reviewing BS 8006 (1995) questioned the wisdom of using a modified two-dimensional solution for designing a truly three-dimensional problem. Kempton *et al.* (1998) demonstrated that a three-dimensional analysis was essential in the prediction of piled embankment performance. For these reasons the problem requires truly three-dimensional analysis techniques. Axi-symmetric analyses would produce an umbrella shape arching mechanism resting on a single pile cap, which is unrepresentative of the actual case.

Detailed numerical analysis of piled embankments was conducted using the $FLAC^{3D}$ computer code (Itasca, 1993). Three-dimensional numerical analysis was required, with the arching thought of as a dome, resting on four pile caps. In the numerical simulations the embankment fill was modeled as a linear elastic material, with a Mohr-Coulomb yield criteria. The reinforcement installed at the base of the fill, consisted of one-dimensional linear cable elements. Two layers were installed, spanning between adjacent pile caps, perpendicular to each other. Initial parametric studies were carried out to determine the finite difference grid size and convergence tolerances for the numerical analysis. A more detailed account of the numerical analysis is contained in Kempton *et al.* (1998). The numerical analysis omitted the foundation soil and constructed the embankment in a single layer.

The geometries of the embankments selected for the analysis are representative of current construction techniques in the United Kingdom. The primary differences between Embankments A and B, Table 1, is the clear spacing between the edges of the pile caps and the significant differences between the stiffness of the geosynthetic reinforcement materials used in the construction of the embankments. It should be noted that the stiffness of the reinforcement in Embankment B is approximately 15 times less than that in Embankment A, even though the clear spacing between piles in Embankment B is 0.5 m greater than Embankment A.

Table 1. Summary of embankment geometry and material properties used in the numerical analysis.

Property	Embankment A	Embankment B
Height (m)	5.8	4.3
Pile cap width (m)	1.0	0.5
Pile spacing (m)	2.5	2.5
Longitude reinforcement stiffness (kN/m)	5500	294
Transverse reinforcement stiffness (kN/m)	9500	738
Fill Poisson's ratio	0.2	0.2
Fill stiffness (MPa)	20	40
Average fill density (kN/m^3)	18.2	19.0
Fill angle of friction (^{0})	30	40
Fill dilation (^{0})	0	0
Fill cohesion (kPa)	0	10

The results of the numerical analysis (Russell & Pierpoint, 1997) show that large deformations occur at the base of the embankments, although the differential movement at the surface of the embankments was, in both cases, negligible. The numerical analysis also showed that, in both cases, the reinforcement tension was concentrated in the reinforcement elements spanning directly between adjacent pile caps, Figure 1, which is consistent with the experimental findings reported by Kempfert *et al.*(2004).

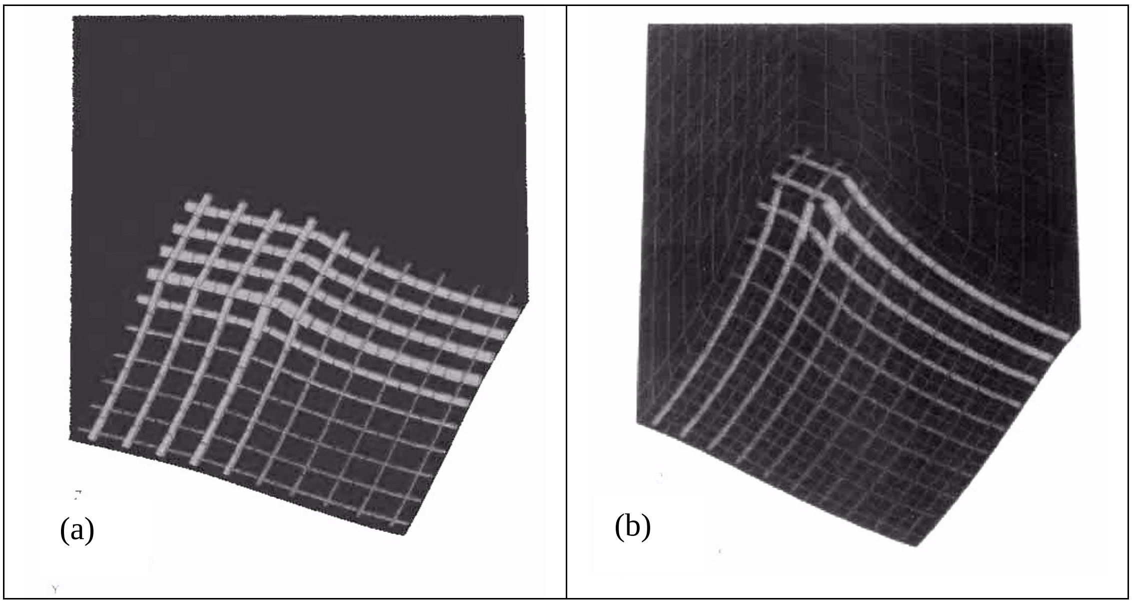


Figure 1. Distribution of reinforcement tension in (a) Embankment A and (b) Embankment B (after Russell & Pierpoint, 1997).

It should be noted that the low stiffness of the reinforcement used in Embankment B required that an artificial cohesion be included for the embankment fill to prevent instability in the FLAC^{3D} analysis. The consequence of the artificial cohesion was to reduce the Stress Reduction Ratio and also to reduce the reinforcement tension proportionally.

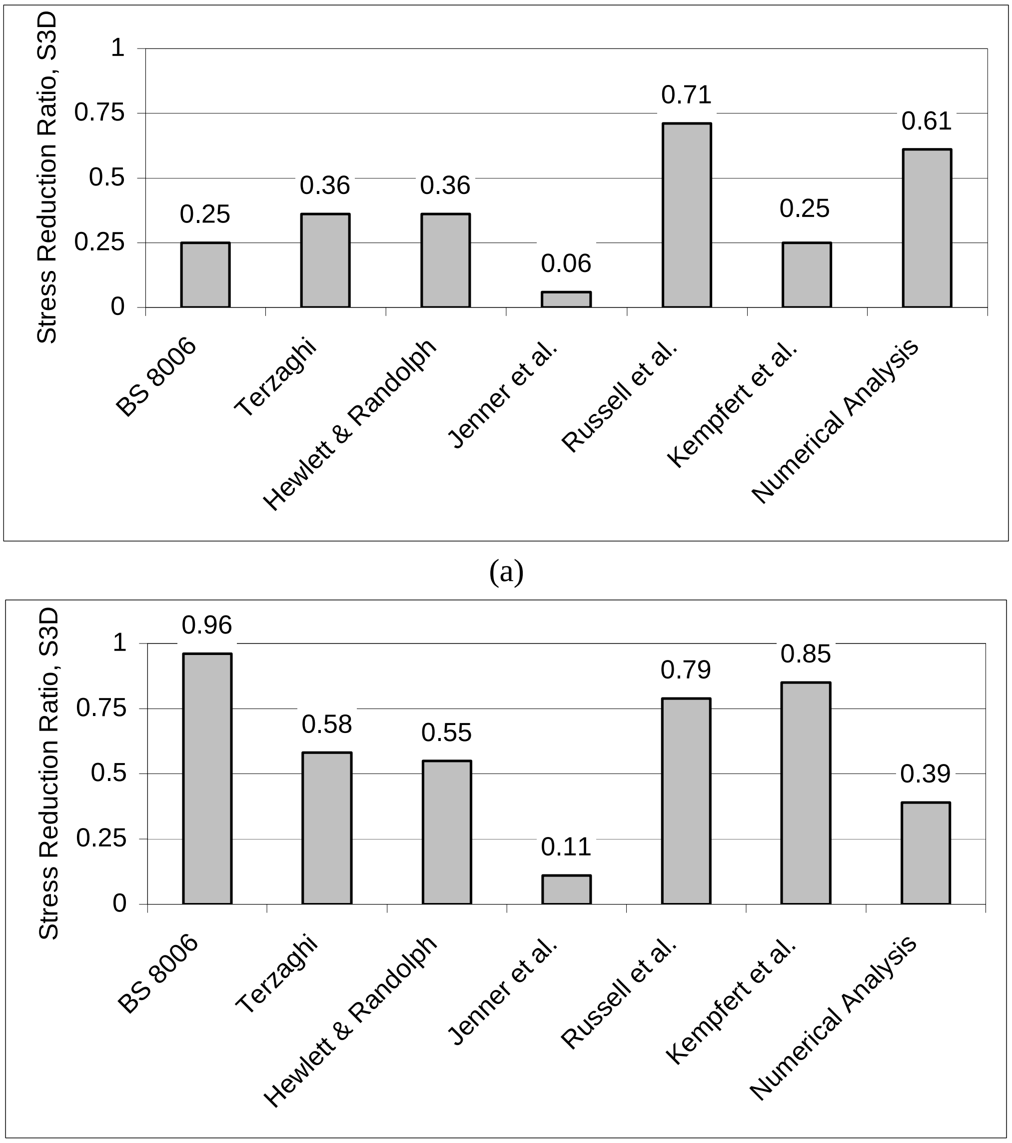
In Figure 1 the magnitude of the reinforcement tension was proportional to the thickness of the reinforcement elements displayed in light gray. Compatibility between the surface settlement, the reinforcement strain and the reinforcement tension must be considered since the magnitude of the reinforcement strain assumed in design has a significant effect on the design loads.

Magnitude of arching from different design methods

The Stress Reduction Ratio for Embankments A and B, determined from the various design methods and the numerical analysis, are presented in Figure 2. The design methods perform very differently. The BS 8006 and Kempfert *et al*. design methods appear inconsistent when compared with the numerical analysis; at Embankment A the Stress Reduction Ratio was under-predicted and at Embankment B was over-predicted. The design method following the Guido method appears to consistently under-predict the numerical analysis and all other analytical design methods. The Terzaghi and Hewlett and Randolph design methods predict similar values and appear reasonably consistent although they do under-predict the Stress Reduction Ratio at Embankment B relative to the numerical analysis. Good agreement between Russell *et al.* (2003) and the numerical analysis is found for Embankment A and between Russell *et al.* and Kempfert *et al.* (2004) for Embankment B, although, in the latter case, both do have Stress Reduction Ratios almost double that of the numerical analysis.

Calculation of the reinforcement tension

In calculating the required tension in the reinforcement it was assumed that the reinforcement layers receive no support from the foundation soil and that the arching mechanism results in a uniformly distributed load along the reinforcement. The resultant deflected shape of the reinforcement is therefore a parabola, with the maximum deflection mid span between adjacent pile caps (Russell & Pierpoint, 1997). Figure 3 presented the reinforcement tensions required for each method based on the Stress Reduction Ratios reported in Figure 2 and the actual measured values from the numerical analysis. In all methods a design strain of 5 % was used in calculating the reinforcement tension. The tension in the reinforcement based on Kempfert *et al.* (2004) was estimated using both their recommended method and assuming a parabolic shape for the deflected reinforcement. The design chart in Kempfert *et al.* (2004) recommends a reinforcement tension approximately 1.45-1.5 times that calculated assuming a parabolic deflected shape.

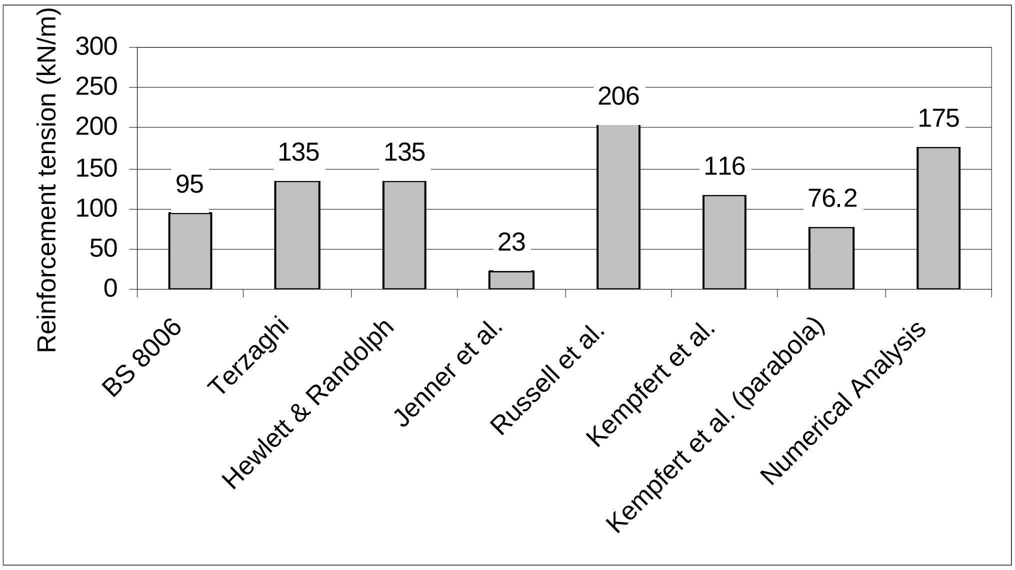


(b)

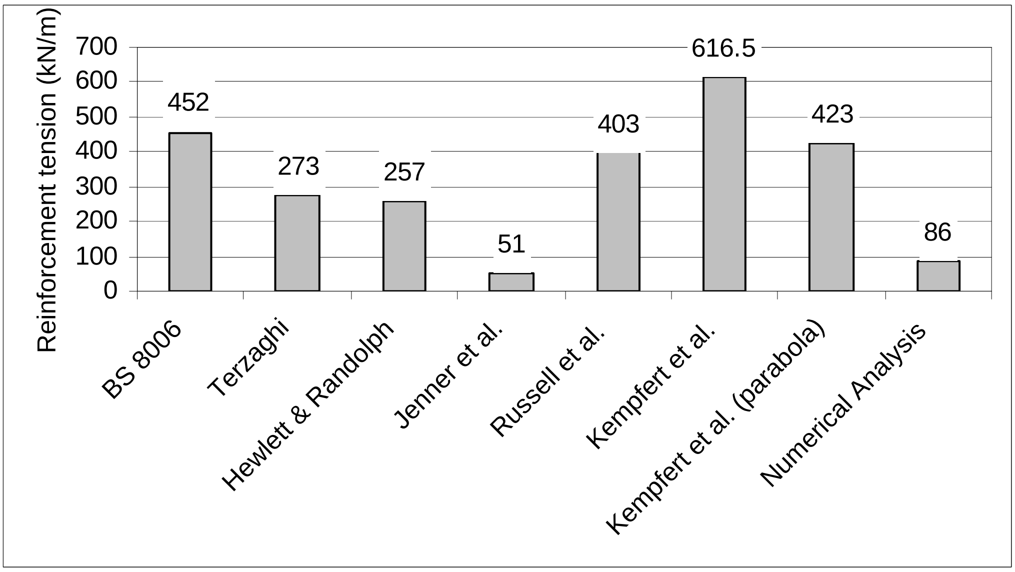
Figure 2. Stress Reduction Ratio for (a) Embankment and (b) Embankment B.

It should be noted that the method recommended by Kempfert *et al.* (2004) for calculating the reinforcement tension results in very high strength for Embankment B and below average strength in Embankment A. Otherwise reasonable agreement was found between the different methods, with the exception of the Guido method. The numerical analysis of Embankment B under-estimates, relative to the other methods, the tension in the reinforcement due to the low stiffness reinforcement used in constructing this embankment.

Considerable variation in the tension requirement (51 kN/m to 616.5 kN/m) was observed for Embankment B. The reinforcement tension from the numerical analysis was based on the actual stiffness of the reinforcement used in the construction of the embankments. The average strain in the reinforcement was estimated from the deformations determined at the base of the embankment. The strain in the transverse reinforcement in Embankment A was estimated as 6.8 %. If this strain were used in the design calculations then the reinforcement tensions reported in Figure 3 would be reduced by 14 %. For Embankment B, the strain in the transverse direction was estimated as 14.6 %. This higher strain value was a result of the lower stiffness reinforcement used in the construction of this embankment. It should be noted that the reinforcement strain calculated for this embankment was greater than that sustainable by most geosynthetics. The low stiffness reinforcement used in Embankment B has a profound effect on the calculated reinforcement tension from the numerical analysis.



(a)



(b)

Figure 3. Reinforcement tension for (a) Embankment A and (b) Embankment B.

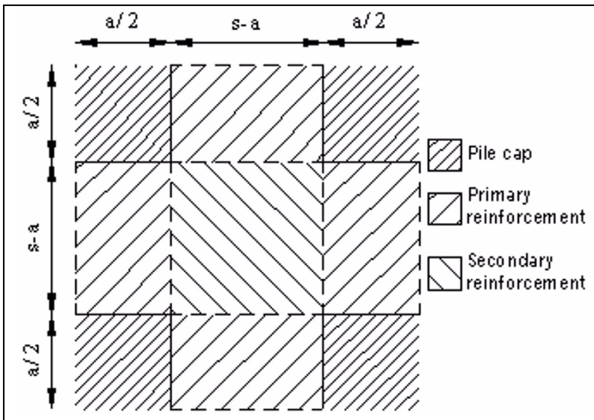
Comment on Guido Method

The Guido method for piled embankments (Jenner *et al.*, 1998) was developed from laboratory plate loading tests carried out by Guido *et al.* (1987) on samples of geogrid reinforced sand in a confined rigid box. There seems to be a conceptual difficulty with the Guido method, as the embankment gravity acts in the opposite direction to that used in the laboratory trials (Love & Milligan, 2003). This method also relies on support from the underlying soil, which may go some way to explain the results presented in this study.

Advances in the design of piled embankments

The numerical analysis of the piled embankment problem resulted in the development of a new method of construction (Russell *et al.*, 2003). Traditionally, piled embankments provided reinforcement layers with the design strength over the entire piled area. However, the numerical analysis reported in this paper has demonstrated that the reinforcement tension was concentrated in the area immediately between the pile caps. By providing high strength reinforcement elements in this area only, considerable economic savings can be achieved.

The new construction technique is illustrated in Figure 4. The reinforcement material provided is divided into primary reinforcement, which is a high strength reinforcement material that spans between adjacent pile caps, and secondary reinforcement, which is of lower strength but which covers the entire piled area. The design of both the primary and secondary reinforcements requires that suitable checks be performed in design to ensure strain compatibility. Currently both reinforcement types are designed for a short-term strain of 5 % and a long-term (end of design life) strain of not greater than 6 % (BS 8006, 1995).



(a)


(b)

Figure 4. (a) Plan view of base of the embankment, showing locations of primary and secondary reinforcement layers and (b) installation of primary reinforcement.

Conclusions

Piled embankments are often the only practical and economic method available for constructing embankments on low bearing capacity or highly compressible soils. The piled embankment application is a truly three-dimensional problem and should be modeled as such.

Numerical analysis and experimental investigations have found that the tension in the reinforcement was concentrated in the region directly between adjacent piles, with the highest tensions measured at the edge of the pile caps.

Difficulties were experienced in modeling piled embankments incorporating low stiffness reinforcement as the primary load carrying elements.

Several methods are currently available for estimating the magnitude of arching in the embankment fill between the pile caps. Considerable variations in the Stress Reduction Ratio and reinforcement tension were observed for the different design methods.

The Guido method continually underestimates the magnitude of arching and tension in the reinforcement and is inconsistent with other analytical methods and the numerical analysis.

Calculation of the reinforcement tension from the theory of elastically embedded membranes results in a tension 1.45 – 1.5 times that based on an assumed parabolic deflected shape of the reinforcement for the unsupported case.

Based on a comprehensive study of piled embankments using numerical analysis a new construction method is proposed. Primary reinforcement is installed between the pile caps to carry the concentrated loads that develop in these locations, while secondary reinforcement layers cover the entire piled area and transfers the remaining loads to the pile caps. This technique offers considerable economic and technical advantages over the traditional methods of construction.

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