Multi-Objective Reconfiguration of Radial Distribution Systems Using Reliability Indices

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Abstract—This paper deals with the distribution network reconfiguration problem in a multi-objective scope, aiming to determine the optimal radial configuration by means of minimizing the active power losses and a set of commonly used reliability indices formulated with reference to the number of customers. The indices are developed in a way consistent with a mixed-integer linear programming (MILP) approach. A key contribution of the paper is the efficient implementation of the ε-constraint method using lexicographic optimization in order to solve the multi-objective optimization problem. After the Pareto efficient solution set is generated, the resulting configurations are evaluated using a backward/forward sweep load-flow algorithm to verify that the solutions obtained are both non-dominated and feasible. Since the ε-constraint method generates the Pareto front but does not incorporate decision maker (DM) preferences, a multi-attribute decision making procedure, namely, the technique for order preference by similarity to ideal solution (TOPSIS) method, is used in order to rank the obtained solutions according to the DM preferences, facilitating the final selection. The applicability of the proposed method is assessed on a classical test system and on a practical distribution system.

Index Terms—Distribution system reconfiguration, epsilon-constraint method, loss minimization, mixed-integer linear programming, multi-objective optimization, reliability.

NOMENCLATURE

The main notation used in the text is listed below, sorted alphabetically. Other symbols and abbreviations are defined where they appear.

A. Sets and Indices

$\mathcal{B}$: Index (set) of branches.
$i$: Index (set) of customers.
$\mathcal{P}$: Index (set) of approximation points used for the SOS2 approximation.
$\Omega^t_b$: Set of receiving ("to") nodes of branch $b$.
$\Omega^r_b$: Set of sending ("from") nodes of branch $b$.
$\Omega^s$: Set of substation nodes.

B. Variables

$c_b$: Number of clients supplied by branch $b$.
$C_{i,j}^+$: Average similarity index.
$C_{i,j}^+$: Similarity index.
$f_{yi}$: Number of clients supplied by substation at bus $i$.
$P_b$: Active power flow through branch $b$ [kW].
$P_i^f$: Active power flow fed by substation at bus $i$ [kW].
$Q_b$: Reactive power flow through branch $b$ [kvar].
$Q_i^f$: Reactive power flow fed by substation at bus $i$ [kvar].
$x_b$: Binary variable (1 if branch $b$ is active, else 0).
$z^A_{b,p}$: $p$ continuous variables used to approximate the square of the active power flow through branch $b$.
$z^R_{b,p}$: $p$ continuous variables used to approximate the square of the reactive power flow through branch $b$.

C. Parameters

$M$: Total number of customers.
$N$: Number of buses.
$N_f$: Number of substation buses.
$N_i$: Number of customers connected at bus $i$.
$N^W$: Number of weight combinations.
$P_{iD}$: Active power demand of load at bus $i$ [kW].
$Q_{iD}$: Reactive power demand of load at bus $i$ [kvar].
$R_b$: Resistance of branch $b$ [Ω].
$S_b$: Apparent power flow limit of branch $b$ [kVA].
$U_b$: Average repair time of branch $b$ [h].
$V$: Nominal voltage of the system [kV].
$w^{loss}$: Weight of the losses.

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The distribution system (DS) is the part of the power system infrastructure that serves as a link between the high-voltage highly meshed transmission system and the end-users of electric energy. Typically, DS is designed as weakly meshed system.

However, it is operated in radial configuration in order to exploit the advantages this topology offers: easier coordination of protective measures, lower fault currents, easier voltage and power flow control and, definitely, lower cost [1]. This is the reason why in the majority of optimization problems regarding the DS, the preservation of radial topology constitutes an important constraint [2]. The reconfiguration of DS may be generally defined as the procedure of changing the status of the switches that activate or deactivate specific branches, in order to obtain a specific configuration of a DS. Two main types of switches exist in a DS: those that are normally closed (sectionalizing switches) and those that are normally open (tie switches). Reconfiguration is performed for various reasons, both in normal and emergency operation conditions. In general, DS reconfiguration constitutes a single or multi-objective nonlinear combinatorial problem. Historically, reconfiguration was initiated with the purpose of obtaining a radial topology leading to the lowest power losses [3].

However, being reported that the DS is the main contributor to the unavailability of energy supply to the end-users [4], significant attention should be paid to its reliability enhancement. The objective of improving the reliability of the DS stands for the reduction in the frequency and the duration of power interruptions that affect the customers [5]. This is generally achieved through network automation, efficiently designed protection schemes, reclosing and switching, fault prediction techniques, efficiently organized and fast repair teams and the improvement of the reliability of single components [6].

DS reconfiguration with respect to the enhancement of reliability emerges as a promising operational strategy to be considered together with the aforementioned techniques, both in planning and operational phases.

B. Literature Overview

Over the past three decades significant research has been conducted on the topic of optimal DS reconfiguration and a great number of different techniques have been presented in the technical literature. A classification of the DS problems can be found in [7].

The most commonly met objective function is the minimization of the active power losses often combined with other DS problems. Jabr et al. [8] presented an exact minimum loss network reconfiguration procedure based on mixed-integer convex programming. In [9] a harmony search algorithm was proposed and was further extended in [10] to consider also the effect of optimal placement of distributed generation (DG) on active power losses. In [11] optimal power flow together with Benders’ decomposition was employed in order to achieve a radial configuration with minimum power losses. Zhang et al. [12] presented a genetic algorithm based methodology to jointly optimize capacitor placement and least-losses reconfiguration. Simulated annealing algorithm was used in the reconfiguration of large-scale distribution systems in [13]. Finally, the minimum-current circular-updating-mechanism was presented in [14].

Other objective functions include the minimization of the node voltage deviation from its nominal value [15] and the minimization of the switching operations required [16].

Multi-objective approaches try to optimize a combination of the above-mentioned objectives mainly using meta-heuristic based techniques due to the possibility of widely exploring the search space [17].

In the emerging smart grid scheme, DG and energy storage systems play a key role and pose new challenges in the operation of the DS [18]–[20]. The quantification of reliability in order to be considered an objective for reconfiguration has been studied in several papers.

Amanulla [5] presented a probabilistic reliability assessment model with an algorithm to identify minimal cut sets, while reconfiguration was based on a swarm optimization. In [21] aggregated objective functions were used in order to consider reliability indices together with network losses. A neighborhood search algorithm was employed for the network reconfiguration. Uncertainty and customer damage related to the unreliability of the system were investigated through a clonal selection algorithm approach in [6]. Finally, in [22] an improved shuffled frog leaping algorithm was presented to formulate a multi-objective problem comprising the minimization of the active power losses and several reliability indices minimization as objectives.

Meta-heuristics based solution algorithms have resulted to be effective in finding out pseudo-optimal solutions also for large networks. However, an exact (deterministic) technique has the following advantages:

1) Transparency. The introduction of new stakeholders in the DS may increase the interest in the procedures used by DS operators (DSOs) relevant to the operation of the DS. This is a plausible concern since the experience from the restructured production and transmission systems suggests that many independent system operators (ISOs) base their scheduling procedures on mathematical programming models.

2) Determinacy. The solution of an exact method does not depend on any random seeds and hence it is totally reproducible. Conversely, for problems solved by using methods based on meta-heuristics the results are affected by the variability of the seed for random number extractions, which can be set up to an initial fixed value by using
a certain programming language. However, the results are not totally reproducible on any platform. This reduces the acceptability of random number-dependent solvers by DSOs.

C. Contribution and Organization of the Paper

The studies mentioned in the previous subsection did not implicitly incorporate the reliability indices into the optimization problem using an exact optimization method such as mixed-integer linear programming (MILP) in a multi-objective framework. The contributions presented in this paper are two-fold:
1) Instead of using heuristic methods as in most studies in the literature, in this paper an exact technique, namely, the augmented $\varepsilon$-constraint method (AUGMECON), is applied for the first time to the multi-objective reconfiguration problem in order to generate solution points belonging to the Pareto optimal set.
2) A number of reliability indices, such as $SAIFI$ (System Average Interruption Frequency Index), $SAIDI$ (System Average Interruption Duration Index) and $AENS$ (Average Energy Not Supplied), are considered within the MILP formulation. More specifically, a new technique is proposed in order to allow for the inclusion of the reliability indices into the MILP formulation.

The remainder of the paper is organized as follows: in Section II the mathematical model of the investigated problem is developed. Then, in Section III the solution procedure is described. Results following the application of the proposed methodology on a test system and on a practical system are presented and discussed in Section IV. Finally, the conclusions are drawn in Section V.

II. MODEL

A. Modeling Assumptions

The mathematical model used in this paper is formulated from the point of view of the DSO. In the practical conduct of DS, a reference network configuration is typically considered in normal operation. The identification of a well-established reference network configuration is relevant also for reliability purposes. In fact, the conventional reliability indicators that must be communicated to the Regulatory Authority for the determination of possible incentives or penalties referring to the continuity of supply are calculated on the reference network. When a fault occurs, the restoration process is executed, at the end of which the normal conditions are re-established. In practice, network faults are relatively rare events, and the duration of the restoration process is in general very short with respect to the time horizon used for long-term reliability studies [23]. As such, the distribution network will operate in the reference configuration for most of the time. Choosing a less effective reference configuration would lead to lower overall reliability of the system at the end of the operation period relevant to reliability (e.g., one year). These concepts are at the basis of the formulation of the reliability indices used in the standards and taken into account in this paper as well. On this basis, the following modeling assumptions are introduced:
1) The network reliability is assessed for a yearly period.
2) The active power demand at each point represents the annual average value.
3) The calculations are based on the reference network configuration.
4) Only faults at the network branches are considered by assigning failure rates to all branches. All other components of the DS are considered totally reliable, since reconfiguration would not induce any change to their behavior during faults.
5) The clients affected by a fault in a specific branch are the ones located downstream with respect to the faulted branch.
6) DG is not considered in this study. In fact, the DSO has no control over the DG units and is more likely to adopt conventional reliability indices described in the Standards, expressing the performance of a utility or DSO in providing continuity of supply. Currently, these indices do not consider the effects of DG. The impact of DG on the reliability assessment procedure require new metrics to be developed [24] and is left as a future development.

The proposed model is not used for planning purposes, for which the typical objective function is expressed in economic terms. In the proposed approach, the DSO has to choose the best network configuration for both reducing the losses and to be used as an effective reference network in terms of reliability.

B. Mathematical Formulation

1) Objective Functions: the optimal radial configuration of the DS is evaluated by means of simultaneously minimizing a set of objective functions. The reliability indices are adapted from the IEEE standard [25].

a) Active power losses: it has been reported that under heavy loading condition both underground cables, as well as overhead lines, tend to fail more often [22]. In [22] it is also suggested that any strategy that has as a result the limitation of the current has a positive effect on reliability. Moreover, its minimization renders a more economical operation of the power system since active power losses are limited as well.

To calculate the losses, the current flow through all the branches of the system should be known by solving the power flow equations.

The power flow problem is generally nonlinear. When dealing with the reconfiguration problem, it becomes mixed-integer nonlinear. Considering directly the power flow equations would render the total problem mixed-integer nonlinear. Due to the computational burden associated with this type of mathematical programming problems, several attempts to provide accurate linear approximations of the power flow equations have been proposed in the literature [26], [27].

In this paper, simplifications are adopted in order to avoid a complete power flow representation in AUGMECON. It has been proven in [27] that under normal operating conditions the voltage angles are very small (e.g., $[-5^\circ, 2^\circ]$) and that the voltage amplitudes are close to the nominal voltage level of the system, typically within the range of [0.9 p.u., 1 p.u.]. Moreover, these assumptions are natural under a smart-grid concept in which sufficient monitoring and control enforces further these quality constraints [28].
The total power losses of the system are given by [29]

\[ L_{tot} = \sum_{b \in B} \left( R_b \cdot \frac{P_b^2 + Q_b^2}{V_b^2} \right) \]  

(1)

The expression of the losses is linearized using the concept of Special Order Sets of Type 2 (SOS2) for the squares of active and reactive power flow through branches. This is described in detail in Section II-B2b.

b) System Average Interruption Frequency Index (SAIFI): SAIFI is a commonly used reliability indicator by electric power utilities. SAIFI is the average number of interruptions per year, calculated as the weighted mean of the number of interruptions assuming the number of customers as weights:

\[ SAIFI = \frac{\text{total number of customer interruptions}}{\text{total number of customers served}} \]  

(2)

\[ SAIFI = \frac{\sum_i \lambda_i N_i}{M} \]  

(3)

where \( \lambda_i \) is the rate of the failure \( i \) that affects \( N_i \) customers and \( M \) is the total number of customers served.

This study focuses on the impact of branch failures to the customers served, and thus this index is reformulated as

\[ SAIFI = \frac{\sum_b \lambda_b \cdot |c f_b|}{M} \]  

(4)

The analytical way to calculate the number of customers that are interrupted by the failure of branch \( b \) is indicated in Section II-B2c. It is known that longer lines tend to have a failure rate proportional to their length. Longer lines correspond to higher impedances. Data concerning the individual line failure rates are scarce.

To obtain the required parameters by the model the following procedure is adopted [22]: a failure rate is specified or assumed for both the line with the highest impedance (maximum) and the line with the least impedance (minimum). Then, the failure rate of the rest of the lines is determined through linear interpolation.

c) System Average Interruption Duration Index (SAIDI): SAIDI is also a common indicator of reliability. It states the average outage duration per year, calculated as the weighted mean of the duration of the interruptions assuming the number of customers as weights:

\[ SAIDI = \frac{\text{sum of customer interruption duration}}{\text{total number of customers served}} \]  

(5)

\[ SAIDI = \frac{\sum_i \lambda_i U_i N_i}{M} \]  

(6)

where \( U_i \) is the duration of the interruption due to failure \( i \). In this study it is also reformulated as

\[ SAIDI = \frac{\sum_b \lambda_b \cdot U_b \cdot |c f_b|}{M} \]  

(7)

SAIDI is not generally proportional to SAIFI. Some branches may not be easily accessible, so it is possible that a very rare failure takes a long time to be fixed, while a frequent one could be fixed within minutes.

d) Average Energy Not Supplied (AENS): although SAIFI and SAIDI focus on the interruptions experienced by the customers, they do not provide any direct information related to the effect of an interruption on the energy that is not supplied to the end-users during a failure. Some feeders may not electrify a large number of individual consumers but may transfer large amounts of energy (e.g., to a few industrial customers).

An index that provides that information is AENS, being defined as follows:

\[ AENS = \frac{\text{energy not served during interruptions}}{\text{total number of customers served}} \]  

(8)

\[ AENS = \frac{\sum_i \lambda_i U_i P_d}{M} \]  

(9)

where \( P_d \) is the active load not supplied during failure \( i \). To be able to calculate this index within the proposed approach, it has to be transformed as

\[ AENS = \frac{\sum_b \lambda_b \cdot U_b \cdot P_b}{M} \]  

(10)

The objective functions (4), (7), and (10) involve the absolute value function, which is nonlinear. The most common way to linearize these expressions is by using positive auxiliary variables. For example, suppose the absolute value of variable \( y \) is required. Defining the positive variables \( y^+ \) and \( y^- \), the constraints (11) and (12) yield the absolute value of the variable:

\[ y = y^+ - y^- \]  

(11)

\[ y = y^+ + y^- \]  

(12)

To point out that the previous objective functions are not generally co-optimized, the following example is provided. The illustrative 3-node system of Fig. 1 has 3 possible radial configurations. Supposing that the resistance of each branch is 0.01 \( \Omega \), the nominal network voltage is 20 kV and the branches have different failure rates, the results regarding all the possible radial configurations of the system are presented in Table I. It is clear that the objective functions are conflicting.
2) Constraints:

a) Radiality condition: to guarantee the radial topology of a DS, two conditions must be satisfied: 1) no loops should be formed (tree topology), and 2) every bus of the system should be connected to a substation point. Equation (13) guarantees that the topology of a DS with \( N \) nodes (\( N_f \) of which are substation nodes) consists of tree sub-graphs. Connectivity is guaranteed by the power balance equations:

\[
\sum_{b \in B} x_b = N - N_f.
\] (13)

In order to account for transfer nodes (i.e., nodes without production or consumption), a simple way to avoid complicate constraints is to consider a very small value of consumption (e.g., \( 10^{-5} \) p.u.) [2].

b) Active and reactive flow through branches: the active and reactive power balance is enforced at each node through the following constraints:

\[
\sum_{b \in B: i \in \Omega^a_b} P_b + \sum_{b \in B: i \in \Omega^r_b} P^f_i = P^d_i \forall i \in I
\] (14)

\[
\sum_{b \in B: i \in \Omega^a_b} Q_b + \sum_{b \in B: i \in \Omega^r_b} Q^f_i = Q^d_i \forall i \in I
\] (15)

Equations (14) and (15) enforce the active and reactive power balance at each node, respectively. Furthermore, the apparent power that flows through an active branch should be less than its rated apparent power limit, as stated by (16).

Constraints (17)–(19) stand for the linear expression of the squares of the active power flow through SOS2:

\[
\sum_{p \in P} z^{A}_{b,p} = 1 \forall b \in B
\] (17)

\[
P_b = \sum_{p \in P} X^{A}_{p} \cdot z^{A}_{b,p} \forall b \in B
\] (18)

\[
P_b^2 = \sum_{p \in P} Y^{A}_{p} \cdot z^{A}_{b,p} \forall b \in B.
\] (19)

Similarly, (20)–(22) state the linear expression of the reactive power flow:

\[
\sum_{p \in P} z^{R}_{b,p} = 1 \forall b \in B
\] (20)

\[
Q_b = \sum_{p \in P} X^{R}_{p} \cdot z^{R}_{b,p} \forall b \in B
\] (21)

\[
Q_b^2 = \sum_{p \in P} Y^{R}_{p} \cdot z^{R}_{b,p} \forall b \in B.
\] (22)

It should be noted that variables \( z^{A}_{b,p} \) and \( z^{R}_{b,p} \) are positive and continuous. By the definition of SOS2, it is also stipulated that no more than two adjacent values of \( z \) can be greater than zero. This can be enforced by trivial constraints (e.g. found in [30]), though modern solvers include this type of variables into their supported variable types. The accuracy of this approximation depends on the sampling of the nonlinear function, i.e., the number of samples and the intervals that are used.

b) Determination of the number of customers affected by a branch failure: the key idea to allow for the analytical consideration of the reliability indices is to establish a fictitious “flow of customers” through each branch. Naturally, a line failure leads to the interruption of the customers that “flow” through it. This is enforced by (25)–(28):

\[
\sum_{b \in B: i \in \Omega^a_b} c_{fb} - \sum_{b \in B: i \in \Omega^r_b} c_{fb} + f_{gi} = N_i \forall i
\] (25)

\[
0 \leq f_{gi} \leq TN \forall i \in \Omega^a
\] (26)

\[
f_{gi} = 0 \forall i \notin \Omega^a
\] (27)

\[-TN \cdot x_b \leq c_{fb} \leq TN \cdot x_b \forall b \in B.
\] (28)

To obtain the absolute values of the “customers that flow” through a branch, constraints (29)-(30) are used:

\[
c_{fb} = c_{fb}^+ - c_{fb}^- \forall b \in B
\] (29)

\[|c_{fb}^+ + c_{fb}^-| \forall b \in B.
\] (30)

The auxiliary variables \( c_{fb}^+ \) and \( c_{fb}^- \) are positive.

C. Optimization Problem

The optimization problem that needs to be solved is

minimize any combination of (1), (4), (7), (10)

subject to (13)–(30).

It is a multi-objective mathematical programming (MMP) problem with linear objective functions and constraints. Its solution is not straightforward and a special technique should be adopted. This is the subject of the following section.

III. SOLUTION METHODOLOGY

The proposed solution methodology is articulated into different stages. The flowchart of the proposed approach is presented in Fig. 2. The details on the various stages are illustrated in the next subsections.

A. Multi-Objective Optimization Method

A mathematical programming problem that has more than one objective function to be optimized is called MMP problem.

Unlike the single-objective mathematical programming, there is not in general a single solution that simultaneously optimizes all the objective functions. In such cases, the set of
Fig. 2. Flowchart of the proposed approach.

efficient (or Pareto optimal, non-dominated) solutions appears. Such solutions cannot be improved with enhancing one objective function without deteriorating at least one of the others.

Several techniques have been proposed in order to solve MMP problems. Since the solution comprises not a single optimal but several efficient alternatives, the intervention of a decision maker (DM) is required in order to make the final choice about the solution to be implemented.

Exact methods may be classified into three categories regarding the point at which the DM intervenes to express preferences over the objectives [32]:

1) A priori methods: the DM expresses preferences (i.e., weights) over the objectives before the solution processes.
2) A posteriori or Generation methods: the DM expresses preferences after the Pareto set is discovered.
3) Interactive methods: the DM expresses preferences during the solution procedure guiding the method to converge to the most preferable solution.

The drawback of a priori and interactive methods is that the DM does not have a picture of the Pareto front when called to express preferences while the generation methods address this issue, allowing for the DM to intervene only after the solution procedure, having all information at hand.

There are several classical generation methods, among which the most famous methods are the weighted sum method and the (simple) $\varepsilon$-constraint method [33]. The weighted sum method has a simple application but is outperformed by the (simple) $\varepsilon$-constraint method for the following reasons:

1) The $\varepsilon$-constraint method can return efficient solutions for both convex and non-convex Pareto optimal sets, while the weighted sum method is useable for convex Pareto sets only [34].
2) The $\varepsilon$-constraint method does not require scaling of the objective functions that can affect the results, while any method summing up different objectives needs to provide scaling factors, even though the variables are normalized.
3) Unlike the (simple) $\varepsilon$-constraint method, the weighted sum method suffers from the fact that there may be different combinations of weights that result into the same efficient solution. In practical terms, many more iterations would be needed in order to discover a given number of unique Pareto optimal solutions.
4) Insufficient selections of the weights may lead the weighted sum method to poorly map the Pareto front. However, despite its advantages over the weighted sum method, the (simple) $\varepsilon$-constraint method has several pitfalls. For instance, it may return weakly efficient solutions if the ranges of the objective functions over the Pareto optimal set are not appropriately identified, the efficiency of the returned solutions is not guaranteed, and the computational time increases when dealing with more than two objective functions.

The aforementioned weaknesses of the classical $\varepsilon$-constraint method are addressed through an improved version, namely, the AUGMECON, belonging to the generation methods as well, the basic functionalities of which are presented hereafter.

AUGMECON is a variant of the $\varepsilon$-constraint method retaining its advantages. Moreover, it addresses three major concerns that are linked with the application of the (simple) $\varepsilon$-constraint method: 1) the ranges of the objective functions are calculated using lexicographic optimization, 2) the efficiency of the returned solutions is proven and, 3) it enables the use of several acceleration techniques in order to achieve computational tractability.

Detailed presentations of the method, as well as a further improved version, are included in [34] and [35], respectively. Without loss of generality a MMP problem with $p$ objective functions to be maximized (to account for minimization of an objective function, the respective objective function is multiplied by $-1$) is considered (31):

$$\max \{f_1(\vec{x}), \ldots, f_i(\vec{x}), \ldots, f_p(\vec{x})\}$$

s.t. $\vec{x} \in S$

where $\vec{x}$ is the vector of the decision variables and $S$ is the feasible region of the problem.

The $\varepsilon$-constraint method is applied by selecting one of the $p$ objective functions as the objective function of the new problem, while the other objective functions are treated as constraints.

The transformed problem is the following (32):

---

\[ \max \{f_1(\vec{x}), \ldots, f_i(\vec{x}), \ldots, f_p(\vec{x})\} \]

\[ \text{s.t. } \vec{x} \in S \]

where $\vec{x}$ is the vector of the decision variables and $S$ is the feasible region of the problem.

The $\varepsilon$-constraint method is applied by selecting one of the $p$ objective functions as the objective function of the new problem, while the other objective functions are treated as constraints.

The transformed problem is the following (32):
max \( f_1(\bar{x}) \)

\[ s.t. \]

\( f_2(\bar{x}) \geq e_2 \)

\[ \vdots \]

\( f_i(\bar{x}) \geq e_i \)

\[ \vdots \]

\( f_p(\bar{x}) \geq e_p \quad \bar{x} \in S. \] (32)

By the parametrical variation of the right hand of the above constraints, efficient solutions are discovered. To apply this method the range of the \( p - 1 \) objective functions that are considered constraints should be known. This range is normally calculated through the pay-off table. The pay-off table is an \( p \times p \) array that contains the values of the individual optimization of each objective value.

The first difference between the classical \( \epsilon \)-constraint method and the AUGMECON is the calculation of the range of the objective functions through the pay-off table. In the first case, the best (ideal) value of the objective function is attained by the diagonal element, in other words, the individual optimization optimal value. The worst value is often approximated by the worst value of the corresponding column.

In the case of AUGMECON the pay-off table is calculated using lexicographic optimization, which is, optimizing consecutively the objective functions adding as an equality constraint the optimal value of the previous objective functions in order to keep the optimal solutions.

This technique guarantees that the obtained solutions of the individual optimization are Pareto optimal solutions (see the Appendix).

Afterwards, the range of the \( p - 1 \) objective functions is divided into \( q_i - 1 \) intervals using \( q_i \) grid points. Then, these \( q_i \) grid points are used to vary the right hand of the \( i \)th objective function.

The number of the sub-problems that has to be solved is \( \prod_{i=2}^{p} q_i \). The accuracy of the efficient set approximation depends on the number of grid points used, always at the expense of computational burden, so this trade-off should be carefully considered.

Nevertheless, the actual number of problems that have to be solved is in practice significantly less because of acceleration techniques that are implemented:

1) Early exit from nested loops: when the problem becomes infeasible there is no need to examine further more bounded cases since they will be de facto infeasible [34].

2) Use of a bypass coefficient: The objective functions are optimized sequentially by slightly altering the objective function as described in [35]. A surplus variable is defined (as an integer variable) and is evaluated after varying the right-hand side (RHS) of the objective functions. According to its value, it may be redundant to solve the next sub-problem since it will not return a new Pareto optimal solution but it will provide the same solution instead. As stated in [35] the use of the bypass coefficient is more significant when the grid density is increased (more grid points).

Besides, AUGMECON has drawn specific research attention recently and as a result several other acceleration techniques are available [36].

The implementation of these techniques and the fact that Algebraic Modeling Languages (e.g., GAMS) and commercial solvers (e.g., CPLEX) implement a pre-solve stage that directly identifies infeasible optimization problems leads to substantially reduced computational times in comparison with the direct application of the method.

The above concepts qualify AUGMECON as a significant and widely acceptable exact solution technique.

B. Validation Stage

The multi-objective problem solved by AUGMECON does not consider the complete load flow constraints in order to reduce the computational burden. This poses two challenges that should be carefully addressed. Firstly, several solutions may not be feasible in practice in terms of violating DS constraints, such as voltage amplitude and angle. Also, several solutions that are non-dominated from the point of view of the AUGMECON optimization method may turn out to be dominated after the complete load flow calculations.

Since all the configurations have radial topology, the backward-forward sweep method [37] (with implemented all the physical constraints of the network) is applied.

From the load flow results, objectives involving physical quantities of the network (e.g., losses, AENS and SAIDI) are computed in exact way (i.e., without any approximation), so that the set of the solution obtained by AUGMECON can be analyzed, by eliminating all the solutions resulting either infeasible or dominated after the load-flow computation.

After that, a further stage is applied, based on the comparison of the feasible and non-dominated solutions with either the reference Pareto front, which can be the complete Pareto front (calculated from exhaustive search on all the distribution network configurations [38]) for relatively small networks, or the best-known Pareto front (from application of other optimization methods) for large networks.

This stage is requested to assess whether or not the solutions considered as non-dominated really belong to the reference Pareto front.

C. Multi-Attribute Decision Making Method

As stated before, the solution of the MMP comprises a set of efficient solutions. Therefore, a DM should intervene and decide one single solution to be implemented, according to his/her preferences. The DM may decide without a systematic method, but by experience instead. However, when dealing with a very large set of relatively optimal solutions, a method to rank and present a narrower subset will be very useful, facilitating the selection. This falls under the umbrella of multi-attribute decision making (MADM) problems, for which several methods have been proposed in the literature. In this study, the technique for order preference by similarity to ideal solution (TOPSIS) [38] has been implemented.
Let the solution of the aforementioned \( p \)-objective MMP comprise \( n \) Pareto optimal alternative solutions. The TOPSIS method evaluates the following decision matrix:

\[
D = \begin{bmatrix}
    x_{11} & \cdots & x_{1j} & \cdots & x_{1p} \\
    \vdots & \ddots & \vdots & \ddots & \vdots \\
    x_{ij} & \cdots & x_{ij} & \cdots & x_{ij} \\
    \vdots & \ddots & \vdots & \ddots & \vdots \\
    x_{mj} & \cdots & x_{mj} & \cdots & x_{mp}
\end{bmatrix} \quad (33)
\]

Each line of (33) represents an alternative solution, while each column is associated with an objective (minimization or maximization). In the general case, each objective is expressed in different units.

Thus, the next step of the TOPSIS method is to transform the decision matrix into a non-dimensional attribute matrix in order to enable a comparison among the attributes. The normalization process is performed through the division of each element by the norm of the vector (column) of each criterion.

An element \( r_{ij} \) of the normalized matrix is given by (34):

\[
r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}. \quad (34)
\]

A set of weights \( w = \{w_1, \ldots, w_j, \ldots, w_p\} \), \( \sum_{j=1}^{n} w_j = 1 \) that express the relative importance of each objective (criterion) is provided by the DM at this point. The weighted normalized matrix with elements \( v_{ij} \) is created by multiplying each column of the matrix with elements \( r_{ij} \) by the weight \( w_j \).

Next, the ideal \((A^+)\) and the negative-ideal \((A^-)\) solution vectors must be specified:

\[
A^+ = \{\max_{i} \{v_{ij}\} \mid j \in J\}, \quad \{\min_{i} \{v_{ij}\} \mid j \in J'\}
\]

\* \( \forall i = 1, \ldots, m \) \* \( \forall j \in J \) \* \( \forall j' \in J' \) \* \( (35) \)

\[
A^- = \{\min_{i} \{v_{ij}\} \mid j \in J\}, \quad \{\max_{i} \{v_{ij}\} \mid j \in J'\}
\]

\* \( \forall i = 1, \ldots, m \) \* \( \forall j \in J \) \* \( \forall j' \in J' \) \* \( (36) \)

In (35) and (36), \( J \) is the set of objectives (criteria) to be maximized and \( J' \) is the set of objectives to be minimized. These artificial alternatives indicate the most preferable (ideal) solution and the least preferable (negative-ideal) solutions. Then, the separation measure of each alternative from the ideal \((S_i^+)\) and the negative-ideal \((S_i^-)\) solution is measured by the \( n \)-dimensional Euclidean distance:

\[
S_i^+ = \sqrt{\sum_{j=1}^{p} (v_{ij} - v_j^+)^2} \quad \forall i = 1, \ldots, m \quad (37)
\]

\[
S_i^- = \sqrt{\sum_{j=1}^{p} (v_{ij} - v_j^-)^2} \quad \forall i = 1, \ldots, m \quad (38)
\]

The final step in the application of the TOPSIS method is the calculation of the relative closeness to the ideal solution. According to the descending order of \( C_i^+ \), the ranking of the alternatives is performed with respect to the similarity index that is calculated by using (39):

\[
C_i^+ = \frac{S_i^-}{S_i^+ + S_i^-}, \quad 0 < C_i^+ < 1, \quad \forall i = 1, \ldots, m. \quad (39)
\]

The ideal solution corresponds to the value equal to unity.

**D. Sensitivity Analysis**

The sensitivity analysis is usually applied in design problems for getting information about the variation of a specific parameter (e.g., the rating of the equipment) by varying one of the inputs: the less is the variation of the parameter, the more results are robust.

The study presented here does not consider design problems; however, it is possible to examine the results of the multi-attribute decision making method according to the variation of the relevant parameter, i.e., the weights of the considered objectives.

In fact, the DM chooses the weights of the objectives, according to a given goal; in order to carry out a more complete analysis, the variation of the solution ranking when the weights vary is studied in this section. The results of this study can be applied in case of variation of the DSO priority.

As shown in Section III-C, the ranking of TOPSIS is made according to the value of the similarity index \( C_i^+ \). Then, it is possible to introduce the number of weight combinations \( N^W \), representing all the weight combinations available for the problem under analysis.

Furthermore, the average similarity index \( \bar{C}_i^+ \) is introduced as well, representing the average value of the similarity index referring to the solution \( i \) and obtained for all the \( N^W \) weight combinations, i.e., by indicating with \( C_{i,j}^+ \) the value of similarity index \( C_i^+ \) at the weight combination \( j \):

\[
\bar{C}_i^+ = \frac{1}{N^W} \sum_{j=1}^{N^W} C_{i,j}^+. \quad (40)
\]

The value of the average similarity index \( \bar{C}_i^+ \) represents a good indicator about the performance of the solution \( i \) by considering different weight combinations.

The application of the above concepts in the case study applications is reported in the next section.

**IV. TESTS AND RESULTS**

**A. Computer Implementation Details**

The AUGMECON method has been coded in GAMS, and the commercial solver CPLEX has performed the optimization for each sub-problem. The backward/forward sweep load flow and TOPSIS algorithms have been implemented in MATLAB. The calculation of the best-known Pareto front used in the validation phase is carried out with a genetic algorithm (GA) directly updating the pseudo-optimal Pareto front, typically used for DS optimal reconfiguration [17].

**B. 33-Node Distribution System Test Case**

1) **Network Data and Multi-Objective Optimization:** firstly, the application of the proposed approach to the 12.66-kV...
33-node system [39] is examined. The acceptable bus voltage range is set to [0.9 pu, 1.1 pu]. For the approximation of the squares of the active (reactive) power flow through the branches, the SOS2 techniques is applied by using 76 evenly spaced breakpoints in the range from 3750 kW (kvar) to 3750kW(kvar). The failure rate of the branch with the greatest impedance is considered to be 0.4 failures/year and of the branch with the least impedance 0.1 failures/year. For all the other branches these values are calculated using linear interpolation. The average repair time for each branch is considered equal to 2 hours. The total demand in active and reactive power are 3715 kW and 2000 kvar, respectively, while the total number of clients electrified by the DS is $M = 18200$. The number of clients connected to each bus is the same as in [22].

For the application of the AUGMECON method, 200 grid points are used and the set of non-dominated solutions is acquired. Then, the radial configurations that correspond to these solutions are validated in terms of feasibility and of being non-dominated after the complete load flow algorithm is applied. The updated set of non-dominated solutions and the corresponding radial configurations are presented in Table II and Table III, respectively.

2) Validation Stage With Pareto Front Assessment: since the 33-node network has 50 751 possible radial configurations [40], a complete analysis on these configurations has been carried out. The results are shown in Table IV, where non-dominated solutions from complete analysis are reported.

The results are also shown in Fig. 3, where the red circles represent the solutions found by AUGMECON and the “plus” markers represent the points of the complete Pareto front. From the comparison between Table II and Table IV, the excellent performance of AUGMECON in the creation of the complete Pareto front is clear. In fact, all the 14 solutions obtained by AUGMECON (after the validation stage) belong to the complete Pareto front, i.e., 14 over 16 solutions of the complete Pareto front are discovered. The solutions not discovered are solution #3 and solution #5.

3) Solution Ranking With TOPSIS: finally, the TOPSIS method is applied to the updated non-dominated set of solutions. For the application of TOPSIS the relative weights of the attributes are considered to be 0.3 for the active power losses, 0.35 for $SAIFI$ and 0.35 for $AENS$. Without loss of generality, these values have been assumed to represent a DM willing to give more importance to reliability aspects (70%) with respect to the system losses (30%).
TABLE V
PARETO OPTIMAL SOLUTIONS RANKED USING TOPSIS
FOR THE 33-NODE DISTRIBUTION SYSTEM

<table>
<thead>
<tr>
<th>solution #</th>
<th>similarity index</th>
<th>losses [kW]</th>
<th>SAIFI [failures/year]</th>
<th>AENS [kWh/customer/year]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.7855</td>
<td>146.2891</td>
<td>1.0042</td>
<td>0.3998</td>
</tr>
<tr>
<td>3</td>
<td>0.7850</td>
<td>141.9160</td>
<td>1.0173</td>
<td>0.4056</td>
</tr>
<tr>
<td>4</td>
<td>0.7843</td>
<td>142.4292</td>
<td>1.0162</td>
<td>0.4054</td>
</tr>
<tr>
<td>6</td>
<td>0.7842</td>
<td>146.5133</td>
<td>1.0031</td>
<td>0.3995</td>
</tr>
<tr>
<td>7</td>
<td>0.7812</td>
<td>146.6658</td>
<td>1.0021</td>
<td>0.3999</td>
</tr>
<tr>
<td>8</td>
<td>0.7503</td>
<td>148.6078</td>
<td>0.9982</td>
<td>0.3991</td>
</tr>
<tr>
<td>11</td>
<td>0.7227</td>
<td>150.9774</td>
<td>0.9910</td>
<td>0.3952</td>
</tr>
<tr>
<td>10</td>
<td>0.7183</td>
<td>150.2483</td>
<td>0.9991</td>
<td>0.3982</td>
</tr>
<tr>
<td>9</td>
<td>0.7166</td>
<td>150.2031</td>
<td>1.0003</td>
<td>0.3984</td>
</tr>
<tr>
<td>2</td>
<td>0.7106</td>
<td>139.9780</td>
<td>1.0327</td>
<td>0.4118</td>
</tr>
<tr>
<td>12</td>
<td>0.6979</td>
<td>152.5900</td>
<td>0.9871</td>
<td>0.3943</td>
</tr>
<tr>
<td>13</td>
<td>0.6429</td>
<td>156.0999</td>
<td>0.9847</td>
<td>0.3936</td>
</tr>
<tr>
<td>14</td>
<td>0.5715</td>
<td>161.5802</td>
<td>0.9841</td>
<td>0.3935</td>
</tr>
<tr>
<td>1</td>
<td>0.4285</td>
<td>139.5513</td>
<td>1.1048</td>
<td>0.4422</td>
</tr>
</tbody>
</table>

Fig. 4. Radial configuration for the top-ranked solution (33-node DS).

C. Taiwan Power Company (TPC) Distribution System Test Case

1) Network Data and Multi-Objective Optimization: to demonstrate the applicability of the proposed methodology on a practical DS, the DS of Taiwan Power Company (TPC) is examined [41]. The system has 11 feeders, 83 normally closed branches and 13 normally open branches. The data are provided considering a balanced three-phase system. The nominal voltage of the system is 11.4 kV and the acceptable voltage range is [0.9 pu, 1.1 pu].

For the approximation of the squares of the active (reactive) power flow through the branches, the SOS2 technique is applied by using 121 evenly spaced breakpoints in the range from −6000 kW (kvar) to 6000 kW (kvar). The failure rate of the branches is determined as for the 33-node system and the average repair time is considered 2 hours. The total active power and reactive power demand are 28350 kW and 20700 kvar, respectively, while the total number of clients electrified by the DS is $M = 17596$. The number of customers connected to each bus is decided assuming that each single client demands an apparent power equal to 2 kVA.

For the application of the AUGMECON method, 200 grid points are used to obtain the set of non-dominated solutions. The radial configurations that correspond to the non-dominated solutions have been validated in terms of feasibility and of being non-dominated after running the complete load flow algorithm. The updated set of non-dominated solutions and the corresponding radial configurations are presented in Table VIII and Table IX, respectively.

2) Validation Stage With Pareto Front Assessment: the total number of radial configurations for TPC network is 7,000, and hence the complete analysis of all of them is not feasible. The multi-objective GA [17] has been applied by running it 200 times with different seeds for random number extraction, for the purpose of widely exploring the solutions space. In this case, the number of consumers located at the various nodes and the variety of the consumer size results in mainly non-conflicting $SAIFI$ and $AENS$ objectives (from the results reported in Table IV). For this reason, only the losses and $SAIFI$ have been considered for further analysis. The results are summarized in Table VIII: AUGMECON discovers four non-dominated solutions, all belonging to the best-known Pareto front for the TPC network, using a single run.

3) Solution Ranking With TOPSIS: for the application of the TOPSIS method, the relative weights (0.3 for losses and 0.7 for $SAIFI$) have been chosen to be consistent with the decision-making used in the case of the 33-node system.

The relevant results of the ranking are presented in Table X. The radial topology that corresponds to the top-ranked non-dominated solution is portrayed in Fig. 5.

4) Results of the Sensitivity Analysis: in this case, with two objectives, by imposing the losses weight $w^{losses}$, the $SAIFI$ weight $w^{SAIFI}$ is obtained as $w^{SAIFI} = 1 - w^{losses}$. As for the 33-node network, from the results shown in Table XI the ranking obtained with $w^{losses} = 0$ and $w^{losses} = 1$ are opposite (i.e., reliability and losses are conflicting objectives), and there is a gradual transition of the top-ranked
solutions. From Table XII, the highest value of the average similarity index corresponds to the solution #2.
TABLE XI

<table>
<thead>
<tr>
<th>rank</th>
<th>sol. #</th>
<th>( C^*_1 )</th>
<th>sol. #</th>
<th>( C^*_2 )</th>
<th>sol. #</th>
<th>( C^*_3 )</th>
<th>sol. #</th>
<th>( C^*_4 )</th>
<th>sol. #</th>
<th>( C^*_5 )</th>
<th>sol. #</th>
<th>( C^*_6 )</th>
<th>sol. #</th>
<th>( C^*_7 )</th>
<th>sol. #</th>
<th>( C^*_8 )</th>
<th>sol. #</th>
<th>( C^*_9 )</th>
<th>sol. #</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
<td>0.987</td>
<td>3</td>
<td>0.972</td>
<td>3</td>
<td>0.954</td>
<td>3</td>
<td>0.930</td>
<td>3</td>
<td>0.899</td>
<td>2</td>
<td>0.858</td>
<td>2</td>
<td>0.855</td>
<td>2</td>
<td>0.847</td>
<td>2</td>
<td>0.825</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.995</td>
<td>4</td>
<td>0.969</td>
<td>4</td>
<td>0.949</td>
<td>4</td>
<td>0.922</td>
<td>4</td>
<td>0.883</td>
<td>3</td>
<td>0.856</td>
<td>3</td>
<td>0.793</td>
<td>3</td>
<td>0.692</td>
<td>1</td>
<td>0.533</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.861</td>
<td>2</td>
<td>0.860</td>
<td>2</td>
<td>0.861</td>
<td>2</td>
<td>0.860</td>
<td>2</td>
<td>0.860</td>
<td>4</td>
<td>0.840</td>
<td>4</td>
<td>0.772</td>
<td>4</td>
<td>0.664</td>
<td>3</td>
<td>0.501</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.014</td>
<td>1</td>
<td>0.031</td>
<td>1</td>
<td>0.052</td>
<td>1</td>
<td>0.078</td>
<td>1</td>
<td>0.113</td>
<td>1</td>
<td>0.160</td>
<td>1</td>
<td>0.228</td>
<td>1</td>
<td>0.336</td>
<td>4</td>
<td>0.467</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE XII

<table>
<thead>
<tr>
<th>sol. #</th>
<th>( C^*_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.230</td>
</tr>
<tr>
<td>2</td>
<td>0.850</td>
</tr>
<tr>
<td>3</td>
<td>0.791</td>
</tr>
<tr>
<td>4</td>
<td>0.769</td>
</tr>
</tbody>
</table>

TABLE XIII

<table>
<thead>
<tr>
<th>network</th>
<th>33 nodes</th>
<th>TPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimality gap (%)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>#Equations/sub-problem</td>
<td>656</td>
<td>1727</td>
</tr>
<tr>
<td>#Variables/sub-problem</td>
<td>6172</td>
<td>24959</td>
</tr>
<tr>
<td>#Discrete Variables/sub-problem</td>
<td>37</td>
<td>96</td>
</tr>
<tr>
<td>#Grid Points</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Total Time (s)</td>
<td>192</td>
<td>512</td>
</tr>
</tbody>
</table>

TABLE XIV

<table>
<thead>
<tr>
<th>network</th>
<th>33 nodes</th>
<th>TPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean value [s]</td>
<td>281</td>
<td>2138</td>
</tr>
<tr>
<td>standard deviation [s]</td>
<td>51</td>
<td>531</td>
</tr>
</tbody>
</table>

TABLE XV

<table>
<thead>
<tr>
<th>step [kW]</th>
<th>number of breakpoints</th>
<th>absolute average error [%]</th>
<th>time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>751</td>
<td>(reference case)</td>
<td>623</td>
</tr>
<tr>
<td>15</td>
<td>501</td>
<td>0.004</td>
<td>345</td>
</tr>
<tr>
<td>25</td>
<td>301</td>
<td>0.020</td>
<td>186</td>
</tr>
<tr>
<td>50</td>
<td>151</td>
<td>0.092</td>
<td>109</td>
</tr>
<tr>
<td>100</td>
<td>76</td>
<td>0.452</td>
<td>64</td>
</tr>
<tr>
<td>150</td>
<td>51</td>
<td>0.949</td>
<td>38</td>
</tr>
</tbody>
</table>

with the computational burden of the GA, it is clear that AUGMECON is convenient in terms of computation time.

The number of breakpoints used in order to approximate the squares of the active and reactive power using the SOS2 technique directly affects the computational performance of the method. Like any other approximation technique, perfunctory selection of the number of the breakpoints may affect the result. Nevertheless, to overcome such problems several selection strategies have been proposed [42] in order to optimally select the number and the coordinates of the breakpoints.

In the presented study, the number of breakpoints that are used has an impact only on the values of the total losses. The \( SAIFI \) and \( SAIDI \) results are clearly not affected since they do not involve the values of active and reactive power in their calculation.

\( AENS \) depends in a linear way on the active power flow through the branches. Since the SOS2 approximation applies only to the nonlinear components, the value of \( AENS \) is not affected by the SOS2 approximation.

To examine the potential effects of the SOS2 approximation, several indicative tests have been performed on the 33-node system using the active power losses and \( SAIFI \) minimization objectives. One thousand grid points have been used in order to construct the Pareto optimal set. The active (reactive) power flows have been evaluated in the range from \(-3750 \text{ kW} \) to \(3750 \text{ kW} \) (kvar). The results are presented in Table XV, with the absolute average error of the approximate active power losses calculated by taking the case with 5-kW step as the reference case.

It is noticed that for relatively insignificant errors the computation time is significantly reduced. Nevertheless, the selection of the appropriate number of breakpoints depends on the specific problem. In case significant errors appear it would be advisable to develop a break-point selection technique. The number of the Pareto optimal solutions discovered and the respective radial network topologies are the same for all the cases. It should be stated that several linear AC optimal power flow models [30] explicitly utilize SOS2 variables to approximate quadratic functions because the SOS2 technique does not introduce any extra binary variables to the optimization problem and the approximation errors are generally considered as minor.

V. CONCLUSIONS

In this study, the DS radial reconfiguration problem was formulated as a MMP problem using a MILP approach. The objectives considered were the minimization of the active power losses and the minimization of commonly used reliability indices (\( SAIFI \), \( SAIDI \), and \( AENS \)), which were explicitly treated within the MILP formulation. To solve the multi-objective problem, firstly, an exact technique was employed, namely, the AUGMECON in order to generate an initial set of non-dominated solutions. At that stage, a reduced network representation was enforced disregarding power flow constraints. To validate the results, the resulting configurations were evaluated in terms of a backward/forward sweep load flow algorithm. In that way, an updated set of non-dominated solutions was generated comprising only feasible (e.g., in terms of bus voltage and branch current limits) solutions and actually non-dominated
The AUGMECON addresses this problem by transforming the objective function constraints to equalities using positive continuous slack variables:

\[
\begin{align*}
\max & \quad f_1(\bar{x}) + \epsilon \cdot (s_2 + \cdots + s_p) \\
\text{s.t.} & \quad f_2(\bar{x}) - s_2 = e_2, \\
& \quad \cdots \\
& \quad f_p(\bar{x}) - s_p = e_p, \\
& \quad \bar{x} \in \mathcal{S}, \quad \epsilon \in [10^{-6}, 10^{-3}].
\end{align*}
\] (A2)

To prove that the formulation (A2) produces only Pareto optimal solutions, let us assume that (A1) has alternative optima and one of them ($\bar{x}'$) dominates the optimal solution $\bar{x}$ obtained from (A2). This means that the vector $(e_1, e_2 + s_2, \ldots, e_p + s_p)$ is dominated by $(e_1, e_2 + s_2', \ldots, e_p + s_p')$, in turn equivalent to

\[
\begin{align*}
e_2 + s_2 & \leq e_2 + s_2', \\
& \quad \cdots \\
e_p + s_p & \leq e_p + s_p',
\end{align*}
\] (A3)

with at least one strict inequality. It is to be stated that $x_1$ is the same for (A1) and (A2). The sum of the relations presented above turns into: $\sum_{i=2}^{p} s_i < \sum_{i=2}^{p} s_i'$. This contradicts the initial assumption that the sum of $s_i$ is maximized by the optimal solution of (A2). Hence, it is concluded that there is no solution that dominates $\bar{x}$.

APPENDIX

This appendix provides the proof that the AUGMECON method produces only non-dominated solutions. The interested reader may refer to [34] for the rigorous mathematical treatment of the method.

Proof: Given a general MMP problem in which $f_1(\bar{x}), \ldots, f_p(\bar{x})$ are the $p$ objective functions to be (without loss of generality) maximized, $S$ the feasible space and $\bar{x}$ the vector of decision variables, the classical $\epsilon$-constraint method is described by (A1) where the RHS is varied in order to obtain the efficient solutions of the problem:

\[
\begin{align*}
\max & \quad f_1(\bar{x}) \\
\text{s.t.} & \quad f_2(\bar{x}) \geq e_2, \\
& \quad \cdots \\
& \quad f_p(\bar{x}) > e_p, \\
& \quad \bar{x} \in \mathcal{S}.
\end{align*}
\] (A1)

A solution of the problem (A1) is Pareto optimal if and only if the constraints of the $p - 1$ objective functions are binding. In case of alternative optima, an optimal solution of the problem is not in fact Pareto optimal.

REFERENCES


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