Gain and Noise Figure of Ytterbium Doped Lead Fluoroborate Optical Fiber Amplifiers

Osama Mahran
Faculty of Science-University of Alexandria-Egypt

Abstract: The calculation of the gain and noise figure of ytterbium doped with a lead fluoroborate glass (PbO-PbF\(_2\)-B\(_2\)O\(_3\)) is presented. This based on the emission and absorption cross sections of ytterbium in this glass, the lead fluoroborate glass doped of ytterbium is chosen because exhibited a very good mechanical resistance under high-brightness diode laser pumping. The one with the best spectroscopic properties, doped with 1.153x10\(^{20}\) ions/cm\(^3\) of Yb\(^{3+}\), has fluorescence lifetime of 0.81 ms, emission cross-section of 1.07x10\(^{-20}\) cm\(^2\) at 1022 nm, fluorescence effective linewidth of 60 nm, high absorption cross-section of 2.56x10\(^{-20}\) cm\(^2\) at 968 nm. The effect of pump power, saturation parameter and the z position along the fiber length on the gain and noise figure is studied. Samples with a lead fluoroborate glass at concentration 1.153x10\(^{24}\) ion/cm\(^3\) of ytterbium is calculated for both gain and noise figure. Through this calculation, the saturated single pass gain in a three-level fiber amplifier is proposed. This calculation is valid in any conditions of pumping and signal injection and takes into account the effect of both the saturation of absorption and saturation of the signal and ASE.

Key words: Ytterbium doped- lead fluoroborate glass-noise figure- optical amplifiers.

INTRODUCTION

Quasi-three-level systems such as erbium and ytterbium-doped glass fibers take part in commonly used technologies in many applications such as telecommunication devices and high average-power systems. Because of the importance of these devices, much theoretical research has been undertaken to understand and optimize them. Two main areas have been prospected in order to characterize the behavior of the fiber amplifiers. On one hand, fully numerical models allow us to predict accurately the gain and the amplified spontaneous emission (ASE) spectra, but these approaches usually take a lot of computing time and are not very suitable for a better understanding of the fiber device properties (Digonnet, 1990).

The increasing importance of glasses doped with rare-earth ions as possible lasing materials has created considerable interest in the study of their optical properties. Materials doped with trivalent ytterbium ions exhibit highly efficient emission using InGaAs laser diodes as pump source. As there are only two manifolds in the Yb\(^{3+}\) energy level scheme, the \(^2\)F\(_{7/2}\) ground state and \(^2\)F\(_{5/2}\) excitation state, it is commonly believed that concentration quenching and multiphonon relaxation should not affect the excitation wavelength. The lack of intermediate levels and the large separation between the excited state and the ground state manifolds reduces non-radiative-decay. The Yb\(^{3+}\) ions are of interest for the next generation of high field lasers and also as a sensitiser of energy transfer for infrared to visible up-conversion and infrared lasers (Nees et al., 1998; Diening et al., 1998). It is known that knowledge of the spectroscopic properties of Yb\(^{3+}\) ions is of fundamental importance for laser action. These properties include emission cross-section, peak wavelengths, fluorescence lifetime and fluorescence quenching processes.

Laser glasses are usually evaluated by means of emission cross-section and fluorescence lifetime. These properties are calculated using intensity parameters based on the Judd-Ofelt theory (Judd, 1962; Ofeil, 1962). Up to now, there are only a few papers involving the effect of composition on the emission cross-section of Yb\(^{3+}\) in simple systems as borate, phosphate, silicate and telluride glasses (Zou and Toratani, 1995; Jiang et al., 1999). Noise figure (NF) measurements have been established for the characterization of the commonly used erbium doped fiber amplifiers in the telecommunication business (Desurvire, 1994; Haus, 2000). Actually, there are several different electrical and optical methods of measuring the NF which should, from the theoretical point of view; all give the same results (Baney et al., 2000).
In this work the calculation of the gain and noise figure of ytterbium doped with a lead fluoroborate glass (PbO-PbF₂-B₂O₃) is done. This based on the emission and absorption cross sections of ytterbium in this glass. The effect of pump power, saturation parameter and the z position along the fiber length on the gain and noise figure is studied. Samples with a lead fluoroborate glass at concentration 1.153x10^{24} ion/cm³ of ytterbium is calculated for both gain and noise figure.

II- Model and Equations:

1- The gain equation:

The behaviour of rare-earth-doped fiber devices can be described in terms of rate equations for the population inversion density \( N_2(z) \), the pump field \( P(z) \), the signal field \( P_s(z) \) and the ASE \( P_f(z) \), where \( z \) is the location along the fiber (Jarabo and Rebolledo, 1995).

\[
\frac{dp(z)}{dz} = -\eta_p \sigma_{abs} P(z) [N_{tot} - (1 + \delta)N_2(z)] - \eta_p \sigma_{esa} P(z) N_2(z) \tag{1}
\]

\[
\frac{dp_s(z,v)}{dz} = \eta_s \sigma_s(v) P_s^*(z,v) [(1 + \alpha(v))N_2(z) - \alpha(v)N_{tot}] \tag{2}
\]

\[
\frac{dp_f(z,v)}{dz} = -\eta_s \sigma_s(v) P_s^*(z,v) [(1 + \alpha(v))N_2(z) - \alpha(v)N_{tot}] \tag{3}
\]

\[
\frac{dp_f^*(z,v)}{dz} = \eta_s \sigma_s(v) [P_s(z,v) ((1 + \alpha(v))N_2(z) - \alpha(v)N_{tot})] + h\nu \delta \nu N_2(z) \tag{4}
\]

\[
\frac{dp_f^*(z,v)}{dz} = -\eta_s \sigma_s(v) [P_s(z,v) ((1 + \alpha(v))N_2(z) - \alpha(v)N_{tot})] + h\nu \delta \nu N_2(z) \tag{5}
\]

Here, the overlapping factors \( \eta_p \) and \( \eta_s \) are respectively the proportion of the pump and signal propagated within the fiber core, \( P_s(z,v) \), \( P_s^*(z,v) \), \( P_f(z,v) \) and \( P_f^*(z,v) \) are respectively the power spectral densities of the injected co-propagation and counter-propagating signals and ASE. \( N_{tot} \) is the dopant concentration, \( N_2(z) \) is the metastable level density population. \( \sigma_{abs}, \sigma_{esa}, \sigma_s(v) \) are the pump absorption, pump ESA and stimulated emission cross section respectively. \( \delta = \frac{\sigma_e(v)}{\sigma_{abs}} \) is the ratio between the stimulated emission absorption cross section at the pump wavelength, and \( \alpha(v) = \frac{\sigma_s(v)}{\sigma_e(v)} \) is the ratio between the stimulated emission absorption cross section at the signal wavelength. \( \alpha(v) = 1 \) for an ideal three-level system, and 0 for a pure four-level scheme (Cioc, 2002). In the ASE power expression, the factor \( h\nu \delta \nu \) is the noise power corresponding to one photon per mode in bandwidth \( \delta \nu \); if the two polarizations can be propagated in the fiber, the noise power becomes \( 2h\nu \delta \nu \).

The pump and stimulated emission rates are given by and

\[
W_s(z,v) = \frac{\eta_s \sigma_{abs}}{h\nu a} P(z)
\]

\[
W_s(z,v) = \frac{\eta_s \sigma_e(v)}{h\nu a} \left[ P_s^*(z,v) + P_s^*(z,v) + P_f^*(z,v) + P_f^*(z,v) \right] \tag{6}
\]

where \( a \) is the core area of the fiber.

We can describe the density of metastable level \( N_2(z) \) in equilibrium as:

\[
N_2(z) = N_{tot} \times \frac{\left[ \int_{v=--}^{v=-\infty} \alpha(v)W_s(z,v)dv \right]^{1/\tau_f}}{W_s(z)[1 + \delta] + \int_{v=--}^{v=-\infty} [(1 + \alpha(v))W_s(z,v)dv + 1/\tau_f]} \tag{7}
\]
An approximate expression for the metastable level \( N_2(z) \) is developed (Cioc, 2002), assuming that \( N_2(z) \) is independent of the stimulated emission rate \( W_s(z) \). We therefore introduce a new variable \( S \) with no dimension and no dependence on \( z \) or \( \nu \). If we also make the assumption that the three-level coefficient \( \alpha(\nu) \) is slowly varying with \( W_s(z) \), then we can replace in the expression of \( N_2 \) the quantities \( a \) and \( b \) by \( S \) and \( \alpha_m \) respectively, where \( \alpha_m \) is the mean value of \( \alpha(\nu) \) over the signal emission, and \( S \) is the new parameter describing the saturation of the medium.

Replacing \( \alpha(\nu) \) by a mean value \( \alpha_m \) is valid only if \( \alpha(\nu) \) is not varying significantly with \( \nu \) over the signal bandwidth or if we know exactly the recovering integral between \( \alpha(\nu) \) and the signal.

If we write the expression of \( N_2(z) \) with the \( S \) parameter, and we write the equality between this new expression and (7), we can express, after integration along \( z \) and by neglecting the coefficients \( \alpha \) and \( \delta \), an approximation of the dimensionless parameter \( S \):

\[
S = \frac{\int_{z=0}^{z=L} \int_{\nu=-\infty}^{\nu=\infty} W_s(z,\nu) \tau_f \, d\nu \, dz}{\int_{z=0}^{z=L} \int_{\nu=-\infty}^{\nu=\infty} W_s(z,\nu) \, d\nu \, dz}
\]

\( S \) is affected by the pump repartition over \( z \): this means that the value of \( S \) reaches a maximum value when the pump is completely absorbed. Under these conditions, we can introduce some normalized parameters in order to solve the differential equations. We now define the following parameters in order to give simplified forms of the expressions:

\[
\begin{align*}
\gamma_a(\nu) &= \eta_c \sigma_a(\nu) N_{in} \\
\gamma_p &= \eta_c \sigma_{abs} N_{tot} \\
\beta &= \frac{(1+S) - \alpha_m \delta}{1+\delta} \\
\beta' &= \eta_c \sigma_{sat} N_{sat} \\
\eta &= \frac{1+(1+\alpha_m) S}{1+\delta} \\
\varepsilon &= \alpha_m S
\end{align*}
\]

With the help of these parameters, we can write a reduced form of the initial set of equations (1)–(5) governing the evolution of pump, signal and ASE. To achieve the normalization of the set of equations, we shall use the expression of the pump \( P_p(z) \) defined as

\[
P_p(z) = P(z) \quad \text{with} \quad P_{sat} = \frac{hv}{\sigma_{abs} \eta_p \tau_f}
\]

\( P_p(z) \) is a dimensionless parameter and we can express the set of differential equations as functions of \( P_p(z) \), \( \beta \), \( \eta \), \( \varepsilon \) and \( \gamma_a(\nu) \). Because the parameter \( S \) does not depend on \( z \), the set of differential equations (1)–(5) describing the amplification can be integrated analytically without any restrictions on the values of pump, signal and ASE powers; we only suppose that there is no excited absorption (\( \beta' = 0 \)).

The integration of the differential equations (2)–(5) for the signal and ASE has been already performed by Jarabo and Rebolledo (Jarabo and Rebolledo, 1995). Using the reduced parameters, the signal and ASE powers, the gain \( G(z, \nu) \) can be expressed as (Cioc, 2002)

\[
G(z, \nu) = \exp \left[ \gamma_a(\nu) \frac{1 - \alpha(\nu) \delta}{1 + \delta} \right] \left[ \frac{P_p(z)}{P_p(0)} \right]^{\frac{\gamma_a(\nu)(1+\alpha(\nu))}{\gamma_p(1+\delta)}}
\]

Where

\[
P_p(z) = \eta C(z)
\]
With
\[
C(z) = W\left( \frac{P_x(0)}{\eta} \right) \exp\left[ -\frac{[z\beta - P_x(0)]}{\eta} \right] \tag{13}
\]

\(W(x)\) is the Lambert function (Corless et al., 4) which obeys the following equation \(W(x)\cdot\exp[W(x)] = x\).

2- The noise figure:

The noise figure (NF) is defined as the degradation of the signal-to-noise ratio (SNR) caused by any attenuation or amplification of the signal, expressed in dB:
\[
NF = \frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{in}}} = \frac{1}{G^2} \frac{\langle \Delta^2 P_{\text{out}} \rangle}{\langle \Delta^2 P_{\text{in}} \rangle} \tag{14}
\]

with \(P_{\text{out}} = P^s(z,\nu)\) and \(P_{\text{in}} = P^s(0,\nu)\)

\(\langle \Delta^2 P_{\text{out}} \rangle\) and \(\langle \Delta^2 P_{\text{in}} \rangle\) are the power variances of the optical output and input signals, respectively. Hence the NF can simply be determined by measuring the power variances before and after the amplifier. As this measurement depends on the relative intensity noise of the input signal \(\langle \Delta^2 P_{\text{in}} \rangle / \langle P_{\text{in}} \rangle^2\) (WeBels and Fallnich, 2003), there is a common agreement that the pre-amplified signal has to be shot-noise limited.

According to the quantum theory for ideal quantum limited attenuation or amplification of an optical signal, the input variance of the optical signal is changed to (WeBels and Fallnich, 2003).
\[
\langle \Delta^2 P_{\text{out}} \rangle = G^2 \langle \Delta^2 P_{\text{in}} \rangle + \left| G - 1 \right| \langle \Delta^2 P_{\text{out}} \rangle_{\text{shot}} \tag{15}
\]

where \(\langle \Delta^2 P_{\text{out}} \rangle_{\text{shot}}\) is the shot noise associated with the optical signal. Assuming a shot noise limited input signal the NF for quantum limited amplification or attenuation results in (WeBels and Fallnich, 2003).
\[
NF_{\text{ideal}} = 1 + \frac{\left| G - 1 \right|}{G} \tag{16}
\]

RESULTS AND DISCUSSION

The values of peak emission \(\sigma_{\text{em}}\) and absorption cross sections \(\sigma_{\text{abs}}\) with fluorescence lifetime \(\tau_f\) of ytterbium doped with lead fluoroborate glass (PbO-PbF\(_2\)-B\(_2\)O\(_3\)) at concentration 1.153x10\(^{20}\) ion/cm\(^3\) is shown in table 1[15].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentration ((N_{\text{tot}}))</td>
<td>1.153x10(^{20}) ion/cm(^3)</td>
</tr>
<tr>
<td>(\sigma_{\text{em}}) ((\lambda=1022\ \text{nm}))</td>
<td>1.07x10(^{-20}) cm(^2)</td>
</tr>
<tr>
<td>(\sigma_{\text{abs}})</td>
<td>2.56x10(^{-20}) cm(^2)</td>
</tr>
<tr>
<td>(\tau_f)</td>
<td>0.81 ms</td>
</tr>
<tr>
<td>(\Delta\lambda) (emission)</td>
<td>60.7 nm</td>
</tr>
<tr>
<td>Pump wavelength ((\lambda_p))</td>
<td>968 nm</td>
</tr>
<tr>
<td>at (\sigma_{\text{em}}) (968 nm) = (\sigma_{\text{abs}}) (968 nm), (\delta)</td>
<td></td>
</tr>
<tr>
<td>Core radius ((a))</td>
<td>2.1 (\mu)</td>
</tr>
<tr>
<td>Overlap factor of the pump (\eta_p)</td>
<td>0.85</td>
</tr>
<tr>
<td>Overlap factor of the signal (\eta_s)</td>
<td>0.85</td>
</tr>
<tr>
<td>Signal wavelength ((\lambda_s))</td>
<td>1022</td>
</tr>
<tr>
<td>(\Delta\lambda) (absorption)</td>
<td>8 nm</td>
</tr>
<tr>
<td>(\alpha_{\text{m}})</td>
<td>0, 0.05</td>
</tr>
</tbody>
</table>

The normalized parameters according to the model and equation (9) are calculate using the data in table 1 and are given in table 2.
Table 2: The normalized parameters according to equation (9)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{esa}$</td>
<td>0 (no ESA)</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>0 (no excited absorption)</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>1.0486535</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>2.54813</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.54813 for $S = 0$</td>
</tr>
<tr>
<td>$S$</td>
<td>0-10</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0, 1</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0</td>
</tr>
<tr>
<td>$P_{sat}$</td>
<td>1.5 mW</td>
</tr>
</tbody>
</table>

Fig. 1: The output pump power versus the saturation $s$ parameter for different values of the input power for Yb$^{3+}$ doped with lead fluoroborate glass as fiber amplifier.

Fig. 2: The output pump power versus z position along the fiber for different values of the input power for Yb$^{3+}$ doped with lead fluoroborate glass as fiber amplifier.

Fig. 1 and Fig. 2 represent the output pump power as a function of the saturation parameter $S$ and the $z$ position along the fiber for $P_p(0) = 50, 100, 150, 200$ and $250$ mW respectively with $z = 17$ (Fig.1) and $S = 0$ (Fig.2). The other parameters are listed in table 1 and table 2. The results show that for a strong pump power the absorption is saturated and the pump power becomes linear with $z$. For a low pumping rate, there
is no saturation of the absorption and the pump power become exponential with $z$. With our approximation consisting of an averaged value of the saturation $S$ over $z$, when the saturation of the signal and ASE transition is increased, the reduction of the pumping rate also appears. This behaviour is not realistic but only an approximation which allows us to calculate signal and ASE input and output power.

In Fig. 3 we explain the strong dependence of the output pump power on the input pump power at the values of saturation parameter $S = 0, 0.3, 0.6, 0.9$ and $1.2$, where the figure shows at minimum input pump power no output pump power and after defined values of input pump power, the output pump power increase linearly with the input pump power.

![Graph showing output pump power versus input pump power for different values of saturation parameter.](image1)

**Fig. 3:** The output pump power versus input pump power for different values of saturation $s$ parameter for Yb$^{3+}$ doped with lead fluoroborate glass as fiber amplifier.

![Graph showing gain in dB versus the saturation parameter.](image2)

**Fig. 4:** Gain in dB versus the saturation $s$ parameter for different values of the input power for Yb$^{3+}$ doped with lead fluoroborate glass as fiber amplifier.
In Fig. 4 we describe the evolution of the gain in dB from (11) as a function of the saturation parameter $S$ for $P_p(0) = 50, 100, 150, 200$ and $250$ mW. The parameters of the fiber are the same as in table 1 and table 2. We observe that for a low pump power, the gain decreases exponentially with $S$ because of the saturation of the stored energy. For a high pump power, the behaviour of the gain is not exponential because, for a low value of $S$, the medium is totally inverted and the gain is equal to $\eta_s c_q \frac{v_1}{\sigma} N_{tot}$. In reality, for high pump power, $S$ is always greater than zero because of the existing ASE.

![Graph](image1.png)

**Fig. 5:** Gain in dB versus the input pump power for different values of saturation $S$ parameter for $\text{Yb}^{3+}$ doped with lead fluoroborate glass as fiber amplifier.

![Graph](image2.png)

**Fig. 6:** Gain in dB versus $z$ position along the fiber for different values of the input power for $\text{Yb}^{3+}$ doped with lead fluoroborate glass as fiber amplifier.

In Fig. 5 we describe the effect of input pump power on the gain in dB at the values of saturation parameter $S = 0, 0.3, 0.6, 0.9$ and 1.2, the figure shows that the gain increase with the input pump power and reaches to saturation with more increase in the input pump power. Also Fig. 6 shows the $z$ position variation of the gain along the fiber length for $P_p(0) = 50, 100, 150, 200$ and $250$ mW, which the gain becomes linear for more increase in the input power.
Fig. 7: Noise Figure NF in dB the gain for 50 mW of the input power for Yb$^{3+}$ doped with lead fluoroborate glass as fiber amplifier.

Fig. 8: Noise Figure NF in dB the gain for 250 mW of the input power for Yb$^{3+}$ doped with lead fluoroborate glass as fiber amplifier.

Fig. 7 and Fig. 8 represent the Noise Figure NF in dB against the gain at two values of input pump power which 50 and 250 mW for Yb$^{3+}$ doped with lead fluoroborate glass as fiber amplifier respectively. The NF is 1.88 dB below the gain value 10 and increase slowly as the gain increase at 50 mw of input pump power as in Fig.7, but its value 1.98 below the gain value 66.5 and increase slowly as the gain increase at 250 mw of input pump power as in Fig.8.

Fig. 9: Noise Figure NF in dB versus the input pump power for $S$ saturation parameter = 10 for Yb$^{3+}$ doped with lead fluoroborate glass as fiber amplifier.

The behaviour of NF in dB with input pump power at saturation parameter $S = 10$ mW for Yb$^{3+}$ doped with lead fluoroborate glass as fiber amplifier is shown in Fig.9, at lower values of the input pump power the NF is increase and becomes saturated at 190 mW pump power.

**Conclusion:**

A numerical calculation describing the behaviour of lead fluoroborate glass doped of ytterbium amplifier has been proposed, the calculation is valid in any conditions of pumping and signal injection and takes into account the effects of both the saturation of absorption and saturation of the signal and ASE.

The lead fluoroborate glass doped of ytterbium is chosen because exhibited a very good mechanical resistance under high-brightness diode laser pumping. The one with the best spectroscopic properties, doped
with $1.153 \times 10^{20}$ ions/cm$^3$ of Yb$^{3+}$, has fluorescence lifetime of 0.81 ms, emission cross-section of $1.07 \times 10^{-20}$ cm$^2$ at 1022 nm, fluorescence effective linewidth of 60 nm, high absorption cross-section of $2.56 \times 10^{-20}$ cm$^2$ at 968 nm.

The lead fluoroborate glass doped of ytterbium amplifier exhibited a higher gain and lower NF, and its noise figure is lower comparable to values reported for Er doped fiber amplifiers [16, 17]. According to this calculations the gain depend on the S-parameters introduced in this model of calculation and the input pump power and the input pump power. Also the output pump power depend on both the z-direction through the amplifier and the input pump power.

As the NF decreases with increasing seed power it is expected that towards the multi-watt regime the Yb doped lead fluoroborate glass amplifier will perform even better than other types of glasses.

REFERENCES


Corless, R.M., Gonnet G.H., D.E.G. Hare, D.J. Jeffrey and D.E. Knuth On the Lambert function W MapleV release 4


