Numerical study of the subgrid fluid turbulence effects on the statistics of heavy colliding particles

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The main purpose of this article is to investigate the effects of the subgrid fluid turbulence on the motion of nonsettling colliding particles suspended in steady homogeneous isotropic turbulent flow. An additional goal is to characterize the statistical properties of the subgrid fluid turbulence “viewed” by inertial particles to support the development of large eddy simulation (LES) approach for particle-laden turbulent flows. Two types of numerical experiments have been carried out: first, the discrete particle trajectories were computed using the fluid velocity field given by direct numerical simulation (DNS) in order to characterize the small-scale fluid velocity fluctuations “seen” by the particles. In a second stage, the particle trajectory simulations were performed using several filtered velocity fields computed from the DNS data to evaluate the effect of the subgrid fluid turbulence on the particle statistics (turbulent dispersion, kinetic energy, accumulation efficiency, particle-particle relative velocity and collision frequency). The first part of this study shows that particle inertia has a limited effect on the subgrid fluid turbulent statistics: subgrid kinetic energy and Lagrangian integral time scale measured along the particle trajectories. In addition, the subgrid fluid Lagrangian integral time scale is found to be nearly equal to the subgrid Eulerian integral time scale and proportional to the ratio of the filter width to the whole fluid turbulent velocity. The second part shows that the particle turbulent dispersion and kinetic energy are affected by the filtering only when a significant percentage of the turbulent kinetic energy was removed from the velocity field seen by the particles. In contrast, accumulation and collision phenomena are found to be significantly influenced by the subgrid fluid velocity fluctuations when the particle response time is of the same order or smaller than the subgrid Lagrangian integral time scale measured along particle paths. Finally, these results are used to characterize, in terms of particle response time to subgrid time scale ratio, the validity range of LES for the computation of gas-solid turbulent flow.


I. INTRODUCTION

The dynamical behavior of particles suspended in turbulent flows continues to receive attention because of its relevance to a wide range of applications. Examples are, in the field of engineering: rocket booster, liquid sprays injection, and fluidized beds; and for environmental aspects: pollutant dispersion, sand or sediment transport, and rain droplet growth. Each of these areas needs a strong understanding of the underlying phenomena that drive the interactions between the particulate phase and the turbulence.

In the past few decades, the numerical simulation has become a powerful tool to investigate complex phenomena taking place in gas-solid turbulent flow. Two approaches are used for unsteady numerical prediction of turbulent flows: direct numerical simulation (DNS) and large eddy simulation (LES). From theoretical point of view, DNS is the most accurate solution because all fluid turbulent scales are fully solved. The drawback of this approach is the computational cost which restricts its application to a range of small Reynolds number values that are far from those found in industrial applications. To avoid this restriction, the LES approach has been developed in which the large eddy field is directly solved and the subgrid scales need to be modeled. Actually, the effect of the subgrid fluid turbulence on the motion of solid particles is an open issue receiving much attention. Wang and Squires1 and Armenio et al.2 investigated the effect of the subgrid model on the particle motion suspended in channel flow. More recently, Yamamoto et al.3 validated LES of channel laden using an approach based on a filtering of DNS velocity field. They show that the limiting cutoff wave number, for which the subgrid scales do not modify the particle dispersion, is depending on the Stokes number. These studies have shown that the one-point statistics are not strongly affected by the subgrid scales which is favorable for LES accuracy. In contrast, local/spatial phenomena such as particle segregation, particle-particle relative motion and particle collisions are much more driven by the small scales of the turbulence and remain open modeling challenges for LES approach.

In particle-laden flows, depending on particle response time, the turbulence may cause the particles to accumulate in low-vorticity regions.4,5 It is fully understood that preferential concentration results from the competition between the drag and centrifugal forces, but the fluid eddies responsible
for such an effect are not clearly identified. In the literature we found two approaches: on one hand,\textsuperscript{5,7} particle segregation is driven by eddies with maximum of vorticity, thus the Kolmogorov eddies; on the other hand, experimental data\textsuperscript{5} and numerical simulation\textsuperscript{9} show that particles with a response time larger than the Kolmogorov time scale are concentrated by turbulent macroscales.

In a general way, effect of the turbulence on the particle relative motion is separated by two mechanisms.\textsuperscript{7,10-12} The first contribution, called the shear mechanism, corresponds to the relative motion induced by the local gradient of fluid velocity. The second contribution, called the acceleration mechanism, is caused by the difference in the relative fluctuating velocity between the particles and the fluid. Following Saffman and Turner,\textsuperscript{13} collision rate of particles with fluctuating velocity between the particles and the fluid. Following Saffman and Turner,\textsuperscript{13} collision rate of particles with small response time ($\tau_p < \tau_K$, where $\tau_p$ is the particle response time and $\tau_K$ is the Kolmogorov time scale) is due to only the shear mechanism and is determined by their interaction with energy-dissipating turbulent eddies. In contrast, the collision rate of coarse particles ($\tau_p \ll \tau_K$), where $\tau_p$ is the Lagrangian fluid integral time scale) is analogous to the chaotic motion of molecules in a gas. In this frame and considering only the particle interaction with energy-containing turbulent eddies, Abrahamson\textsuperscript{14} derived a theoretical expression of the collision frequency. Between both limiting cases, e.g., $\tau_p \approx \tau_K \approx \tau_L$, the relative velocity between neighboring particles is decreased by interaction with turbulent fluid eddies inducing correlations between neighboring particle velocities.\textsuperscript{7,10,11,15}

In the present study, we have carried out DNS of turbulent homogeneous isotropic flows coupled with discrete particle simulation. The mass loading is assumed sufficiently small to allow two-way coupling not to be taken into account (no turbulence modulation by the particles). But, as we are interested in the effect of the subgrid fluid turbulence on the collision, we artificially increase the volumetric fraction of the dispersed phase in order to increase the efficiency of the collision detection algorithm. Nevertheless, the solid volume fraction is chosen smaller than 5% to insure that the particle mean free path remains much larger that the particle diameter and the dispersed phase still obeys the kinetic regime rather than the collisional one. The transfer of particle properties, like momentum and kinetic energy, due to collisions is negligible compared to the transport of particle properties by the fluctuating motion.\textsuperscript{16} Also, the probable positions of two colliding particles is not influenced by the finite size of the particles (no close-packing effect).

The paper is organized as follows. Governing equations required for numerical simulations are presented in Sec. II. The analysis of subgrid turbulence is performed in Sec. III where special attention is taken in order to determine the particle inertia effect on the subgrid turbulence statistic measured along particle trajectories. Section IV is dedicated to the analysis of simulations where the particle trajectories are computed from the filtered turbulent fluid velocity field. In this case, the discussion deals with the effect of the subgrid turbulence on: particle turbulent dispersion, preferential concentration, relative velocity between neighboring particles and collision rate. Finally, in Sec. V we have interpreted our results in the frame of the LES approach, to characterize the effect of the subgrid turbulence and to derive accuracy criteria.

II. NUMERICAL SIMULATION OVERVIEW

The prohibitive cost of computing particle-laden flows with particle-particle collisions is a strong limitation of numerical simulation. Sundaram and Collins\textsuperscript{7} performed DNS coupled with Lagrangian tracking of many thousand discrete particles. They investigated the motion of particles with a response time too large to reach the regime of Saffman and Turner.\textsuperscript{13} In contrast, Zhou et al.\textsuperscript{12} and Wang et al.\textsuperscript{7} performed a numerical simulation of particles with very small response time but using a frozen turbulence. In this approach a frozen fluid velocity field is used for the computation of the particle trajectories. As pointed out by Zhou et al.\textsuperscript{12} such an approach overestimates the preferential concentration which may modify the collision rate. Lavièville et al.\textsuperscript{18} coupled LES with discrete particle tracking presuming that particle motion is not affected by subgrid scales. More recently, Bec et al.\textsuperscript{19} investigate clustering and collision using numerical simulation in random smooth flows.

A. Direct numerical simulation

The fluid flow in this study is governed by the incompressible Navier-Stokes equations,

\[
\frac{\partial u_{fi}}{\partial t} + u_{fi} \frac{\partial u_{fi}}{\partial x_j} = -\frac{1}{\rho_f} \frac{\partial P_f}{\partial x_i} + v_f \frac{\partial^2 u_{fi}}{\partial x_j \partial x_j},
\]

where $u_{fi}$ is the $i$th fluid velocity component, $\rho_f$ the density, $P_f$ the pressure, and $v_f$ the kinematic viscosity. The Navier-Stokes equations are discretized using a finite-volume method on staggered mesh (with $N$ grid points) with a second-order centered scheme.\textsuperscript{20} The solution is time-advanced using a second order Runge-Kutta scheme and the Poisson equation is solved with a spectral method. The computational domain is a cubic box of length $L_x=0.128$ m with periodic boundary conditions.

Turbulent steady flow is obtained with a stochastic spectral forcing proposed by Eswaran and Pope.\textsuperscript{21} The forcing is accomplished in Fourier space using a stochastic force added to the given range of wave numbers: $[2\kappa_0, 6\kappa_0]$ (where $\kappa_0$ is the first solved wave number). This parameter leads to energy-containing eddies ten times smaller than the computational length box. The stochastic force is parameterized in order to ensure that the Eulerian time macroscale $\tau_E$, defined as $\tau_E=L_u/u_f$, is nearly identical to the Eulerian integral time scale $\tau_E$ computed from the time correlation function measured on fixed points.\textsuperscript{22}

Simulations were performed with fluid kinematic viscosity $\nu_f=1.47 \times 10^{-5}$ m$^2$ s$^{-1}$ and density $\rho_f=1.17$ kg m$^{-3}$. The main fluid properties for the homogeneous isotropic turbu-
TABLE I. Turbulent fluid flow statistics computed in DNS with $N=128^3$ mesh points.

<table>
<thead>
<tr>
<th>Reynolds number</th>
<th>$Re_L$</th>
<th>61</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_\lambda$</td>
<td></td>
<td>34.1</td>
</tr>
<tr>
<td>Fluid kinetic energy ($m^2 s^{-2}$)</td>
<td>$q_f^2$</td>
<td>$6.56 \times 10^{-3}$</td>
</tr>
<tr>
<td>r.m.s. fluid velocity ($m/s$)</td>
<td>$u'_f$</td>
<td>$6.61 \times 10^{-2}$</td>
</tr>
<tr>
<td>Dissipation rate ($m^2 s^{-3}$)</td>
<td>$\varepsilon_f$</td>
<td>$16.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>Dissipation time scale ($\tau_e / \tau_k$)</td>
<td>$\tau_e / \tau_k$</td>
<td>13.96</td>
</tr>
<tr>
<td>Integral length scale</td>
<td>$L_f / L_0$</td>
<td>0.11</td>
</tr>
<tr>
<td>Eulerian time macroscale ($L_e / u'_f$)</td>
<td>$\tau_e / \tau_k$</td>
<td>7.15</td>
</tr>
<tr>
<td>Eulerian integral time scale</td>
<td>$\tau_e / \tau_k$</td>
<td>6.94</td>
</tr>
<tr>
<td>Lagrangian integral time scale</td>
<td>$\tau_L / \tau_k$</td>
<td>5.63</td>
</tr>
<tr>
<td>Taylor microscale</td>
<td>$\lambda_g / L_f$</td>
<td>0.56</td>
</tr>
<tr>
<td>Kolmogorov time scale (s)</td>
<td>$\tau_k$</td>
<td>$28.8 \times 10^{-3}$</td>
</tr>
<tr>
<td>Kolmogorov length scale</td>
<td>$\eta_k / L_f$</td>
<td>0.048</td>
</tr>
<tr>
<td>$k_{max} \eta_k$</td>
<td>2.01</td>
<td></td>
</tr>
</tbody>
</table>

The Taylor microscale, $\lambda_g$, is defined as

$$\lambda_g^2 = 2u'_f \int_0^{+\infty} \frac{E(\kappa)}{\kappa} d\kappa.$$  

(7)

where $f(r)$ is the fluid velocity longitudinal correlation function. In homogeneous isotropic turbulent flow, the integral length scale may be also computed from the three-dimensional turbulent spectrum $E(\kappa)$,

$$L_f = \frac{\pi}{(4/3)q_f^2} \int_0^{+\infty} \frac{E(\kappa)}{\kappa} d\kappa.$$  

(6)

The integral length scale is defined as

$$L_f = \int_0^{+\infty} f(r) dr.$$  

(5)

The Kolmogorov time and length scales are calculated with

$$\tau_k = \left( \frac{\varepsilon}{u'_f} \right)^{1/2}, \quad \eta_k = \left( \frac{\varepsilon}{\varepsilon_f} \right)^{1/4}.$$  

(9)

B. Discrete particle simulation

1. Particle motion

This study is restricted to dispersed phase composed of $N_p$ solid, spherical, identical particles ($N_p=200,000$). Studies on the motion of single particle suspended in turbulent flow field have shown that many forces act on the particle. In gas-solid turbulent flows, the assumption of a large ratio between particle and fluid densities ($\rho_p/\rho_f \approx 1$) is commonly used to reduce forces to the only drag force. In the present study, the collision detection algorithm requires the particle diameter to be of the same order as the Kolmogorov length scale (see Sec. II B 2). Therefore, in order to simulate particle with small response time (to observe the collision regime of Saffman and Turner) the particle density is in the range of $\rho_p/\rho_f \in [4,130]$ (see Table II). Nevertheless, the single particle motion governing equation is reduced to

$$\frac{dx_{p,i}}{dt} = u_{p,i},$$  

(10)

$$\frac{du_{p,i}}{dt} = - \frac{u_{p,i} - u_{f@p,i}}{\tau_p},$$  

(11)

where $x_{p,i}$ and $u_{p,i}$ are the $i$th component of the particle position and velocity. The particle response time, $\tau_p$, is defined as

$$\tau_p = \frac{\rho_p 4 \varepsilon_f}{\rho_f^3 C_d} \frac{1}{|u_f@p - u_{p,i}|},$$  

(12)

where $u_f@p,i$ is the fluid velocity at the particle position (locally undisturbed by the particle) also called the fluid velocity seen by the particle. As the two-way coupling is not considered, the undisturbed fluid velocity is given by the DNS and evaluated at the particle position by a cubic spline interpolation scheme. The drag coefficient, $C_d$, is given by the correlation of Schiller and Nauman,

$$C_d = \frac{24}{Re_p} \left[ 1 + 0.154 Re_p^{0.827} \right]$$  

(13)

with the particle Reynolds number given by...
\[
\text{Re}_p = \frac{d_p |\mathbf{u}_p - \mathbf{u}_{\nu\text{p}}|}{\nu_f}.
\]
(14)

The particle diameter, \(d_p\), is taken such as \(d_p/\eta_p = 0.92\). For the analysis, we introduce \(r_f^{jp}\) the mean particle response time defined as

\[
r_f^{jp} = \left[ \frac{1}{\langle T_p \rangle_p} \right]^{-1},
\]
(15)

where \(\langle \cdot \rangle_p\) is the particle average operator. Table II gives Stokes numbers and main particle statistics for the simulation without filtering.

The numerical method used for the calculation of particle positions (10) and velocities (11) is a second-order Runge-Kutta scheme with the same time step as the DNS but imposes by collision detection algorithm (see Sec. II B 2).

2. Interparticle collision

Depending on the particle number, the collision detection may be very expensive because it requires computing the distance between all neighboring particles \(O(N_p^2)\). To avoid this problem, several sophisticated algorithms have been developed.27 In our numerical simulations we use an algorithm proposed originally by Hopkins and Lough28 for rapid granular flows. A detection grid is superimposed on the computational domain. The grid mesh, \(\Delta_{\text{grid}}\), is determined by two criteria: first, one cell contains only one particle and second, two particles separated by two cells cannot overlap. These assumptions lead to \(d_p/2 < \Delta_{\text{grid}} < d_p/\sqrt{3}\) and the collision detection now involves \(O(124N_p)\) operations.

The equations governing the colliding velocities, in the frame of the hard sphere collision model, are

\[
\mathbf{u}_A' = \mathbf{u}_A + \frac{1}{2}(1 + e_r)w_r \cdot \mathbf{k},
\]
(16)

\[
\mathbf{u}_B' = \mathbf{u}_B - \frac{1}{2}(1 + e_r)w_r \cdot \mathbf{k},
\]
(17)

where the prime denotes the velocity after the collision and \(w_r\) is the radial relative velocity defined as \(w_r = \mathbf{w} \cdot \mathbf{k}\), where \(\mathbf{k}\) stands for the unit vector linking the center of \(A\) particle to the one of \(B\) particle and \(\mathbf{w}\) the relative velocity: \(\mathbf{w} = \mathbf{u}_B - \mathbf{u}_A\). In the present study the numerical simulations were carried out for elastic particles therefore the particle restitution coefficient, \(e_r\), is equal to 1.

This algorithm has been validated by comparison of numerical simulation for homogeneous isotropic dry granular flow with theoretical result from standard kinetic theory of granular media. Comparisons have been made for probability density functions of relative velocity and collision angle between colliding particles because both statistics are very sensitive to the time step. For large time step, collisions with small angles may not be detected by the algorithm29 because such angles lead to very small overlaps. Therefore, to reduce this effect, we impose a time step so that the mean particle displacement during a time step is less than 15% of the particle diameter. In our numerical simulation, this condition is more restrictive than the CFL for the computation of the Navier-Stokes equations. Typically, the time step is of the order of \(\Delta t = 4.0 \times 10^{-4} \text{ s}\).

III. SUBGRID FLUID TURBULENCE PROPERTIES

This section is dedicated to the characterization of the turbulence at small-scale measured along the solid particle paths. The particle trajectories are computed with the whole scales of the turbulent spectrum but for the statistics we apply a spectral cutoff filter on the fluid velocity field seen by the particles. The filtered fluid velocity field, \(\tilde{u}_{ij}(x,t)\), is computed as

\[
\tilde{u}_{ij}(x,t) = \text{FT}^{-1}\left[ \begin{array}{c}
\tilde{u}_{ij}^{*}(\kappa,t) \\
0
\end{array} \right]_{\kappa}.
\]
(18)

where FT is the Fourier transform, \(\kappa\) is the cutoff wave number, and \(u_{ij}^{*}\) is the fluid velocity field in Fourier space \(u_{ij}(\kappa,t) = \text{FT}[u_{ij}(x,t)]\). The subgrid velocity field, \(\delta u_{ij}\), is obtained with

\[
\delta u_{ij}(x,t) = u_{ij}(x,t) - \tilde{u}_{ij}(x,t).
\]
(19)

Simulations have been performed for several cutoff wave numbers summarized in Table III. Figure 1 shows the cutoff wave number positions on the energetic and dissipative spectra. In our DNS all turbulent scales, even energy-dissipating scales, are well solved allowing accurate measurements of the subgrid fluid turbulence statistics.

![FIG. 1. Energetic (—) and dissipative (⋯) spectra extracted from DNS. The dashed lines correspond to \(\kappa, \eta_K\) (see Table III).](image)
A. One point statistics

The filtered and subgrid kinetic energies are defined by

$$\overline{\overline{q_i^2}} = \int_0^{\kappa} E(\kappa) d\kappa, \quad (20)$$

$$\delta q_i^2 = \int_{\kappa_c}^{+\infty} E(\kappa) d\kappa. \quad (21)$$

The variations of the filtered kinetic energy for different values of the dimensionless product $\kappa_L f$ is shown by Fig. 2. The filtered kinetic energy is computed directly by integration of the measured three-dimensional DNS spectrum using (20), on one hand, and by averaging on fixed points of the filtered fluid velocity transformed back in the physical space, on the other hand. As expected, for low values of this product, e.g., when the cutoff approaches the production region, we observe that the filtered kinetic energy is decreased. For the typical energy spectrum shape of our simulations, the filtered kinetic energy is about 95% the full turbulent kinetic energy for the intermediate value $\kappa_c L_f = 10$. According to Fig. 2 and Table III, the cutoff wave numbers of the considered cases were chosen sufficiently far from the production range to cause a fluid kinetic energy reduction by filtering of less than 15%.

In the case of fully developed flow at very large Reynolds numbers ($L_f \eta_k \gg 1$), Kolmogorov's statistical description of the turbulence implies the existence of an inertial range with the universal form of the energy spectrum,\textsuperscript{23,30}

$$E(\kappa) = C_0 \nu f^{2/3} \kappa^{-5/3}, \quad (22)$$

where $C_0$ is the Kolmogorov's constant ($C_0 = 1.5$).

Assuming that the cutoff wave number is in the inertial subrange, $\kappa_c \eta_k \ll 1$ and $\kappa_c L_f \approx 1$, (21) and (22) lead the following expression for the subgrid kinetic energy:

$$\delta q_i^2 = \frac{3}{2} C_0 \nu f^{2/3} \kappa_c^{-2/3}. \quad (23)$$

When the condition $\kappa_c \eta_k \ll 1$ is not perfectly respected, the subgrid kinetic energy is depending on the shape of the spectrum in the dissipation range. In such a case, a better evaluation of the subgrid energy can be done by taking into account the fall of energy at large wave number in the spectrum,\textsuperscript{31}

$$E(\kappa) = C_0 \nu f^{2/3} \kappa^{-5/3} f(\kappa \eta_k) \quad (24)$$

with, e.g.,\textsuperscript{23}

$$f(x) = \exp \left[ -\frac{3}{2} C_0 x^{4/3} \right]. \quad (25)$$

Then the subgrid kinetic energy can be written as

$$\delta q_i^2 = \frac{3}{2} C_0 \nu f^{2/3} \kappa_c^{-2/3} g(\kappa, \eta_k) \quad (26)$$

where the function $g(x)$, the integral of the function $f(y)$ between $x$ and infinity, is monotonic decreasing with the following asymptotic behavior:

$$g(x) = 1 \quad \text{for} \quad x \ll 1, \quad (27)$$

$$g(x) = 0 \quad \text{for} \quad x \gg 1. \quad (28)$$

According to Fig. 3, the subgrid fluid kinetic energy computed from numerical simulation on fixed points decreases faster than predicted by (23). This trend is predicted by (26) which takes into account the fall of energy in the dissipation range.

The subgrid fluid dissipation by viscous effects of the subgrid kinetic energy is defined as

$$\delta e_i = \int_{\kappa_c}^{+\infty} 2 \nu \kappa^2 E(\kappa) d\kappa. \quad (29)$$

Evolution of $\delta e_i$ normalized by the full fluid dissipation is shown in Fig. 4. As expected, the subgrid dissipation tends toward the full dissipation rate when the cutoff-filtering is
applied far from the dissipation range, e.g., \( \kappa_c \eta_k \ll 1 \). In Fig. 4, we see that the theoretical spectrum (24) modified for large wave numbers (dashed lines) gives the same trend but overestimates the subgrid fluid dissipation.

Using the subgrid kinetic energy, \( \delta q_f^2 \), and the subgrid fluid dissipation, \( \delta e_f \), we define the subgrid dissipation time scale

\[
\delta \tau_f = \frac{\delta q_f^2}{\delta e_f}.
\]

According to Fig. 5, the subgrid dissipation time scale decreases when the cutoff wave number is increased. The theoretical spectrum (24) and (25) predict this decrease as resulting from the fall of energy at large wave numbers.

B. Spatial and time correlations

To characterize the spatial correlations we introduce \( \delta L_f \), the subgrid integral length scale. This length scale, expected to be proportional to \( 1/\kappa_c \), is computed with the following extension of (6):

\[
\delta L_f = \frac{\pi}{(4/3)} \overline{\delta q_f \cdot \delta e_f} \frac{E(\kappa)}{\kappa} d\kappa.
\]

The theoretical spectrum shape (22) leads to the following expression:

\[
\delta L_f = \frac{3 \pi}{10} \frac{1}{\kappa_c}.
\]

Figure 6 shows that the subgrid integral length scale ratio is larger than 1 and increases with \( \kappa_c \). Similar to the subgrid kinetic energy, this effect is due to the influence of the peculiar spectrum shape in the dissipation range. This effect is taken into account in the theoretical evaluation of the integral length scale by using the modified spectrum shape given by (24). The subgrid integral length scale computed from Eqs. (24) and (25) gives

\[
\delta L_f = \frac{3 \pi}{10} \frac{h(\kappa_c \eta_k)}{g(\kappa_c \eta_k)}.
\]

where the function \( h(x) \) is the integral of the function \( f(y)/y \) between \( x \) and infinity and the ratio \( h(x)/g(x) \) is a monotonic function increasing with \( x \). Figure 6 shows that the modeled subgrid integral length scale using the theoretical shape of the spectrum given by (24) and (25) is in agreement with our numerical simulations.

To conclude the discussion on the subgrid spatial correlation, it must be noticed that according to Figs. 3 and 6, the measured to theoretical expression for the subgrid kinetic energy ratio is found to cover a very broad range \([0.004, 0.4]\) whereas the analytical to measured subgrid integral length scale ratio is found in a much narrow range \([1.9, 2.3]\), confirming that the shape of the spectrum in the dissipation...
range has much more influence on the subgrid kinetic energy
than on the subgrid length scale. The subgrid length scale is
mainly dominated by the filter cutoff width.

Temporal correlations are investigated through the sub-
grid Eulerian correlation function measured at fixed points

$$\delta R_{E}(\tau) = \frac{\langle \delta u_x^f(\tau_0, x) \delta u_x^f(\tau_0 + \tau, x) \rangle}{2\delta q_f^2}$$  (34)

and the subgrid Eulerian integral time scale is defined by

$$\delta \tau_E = \int_0^{+\infty} \delta R_E(\tau) d\tau.$$  (35)

Figure 7 shows the subgrid Eulerian correlation function measured from the DNS. We observe that the filtering does not significantly modify the correlation function shape and we note that the subgrid Eulerian correlation function does not exhibit significant negative loops. In Fig. 7 we compare the DNS results with two available models from the literature: the exponential function:31

$$\delta R_{E}(\tau) = \exp \left( -\frac{\tau}{\delta \tau_E} \right),$$  (36)

and the bi-exponential correlation function:32

$$\delta R_{E}(\tau) = \frac{1}{2\sqrt{1-z^2}} \left[ \frac{1 + \sqrt{1-2z^2}}{1 - \sqrt{1-2z^2}} \exp \left( -\frac{2}{1 + \sqrt{1-2z^2}} \frac{\tau}{\delta \tau_E} \right) - \frac{1 - \sqrt{1-2z^2}}{1 - \sqrt{1-2z^2}} \exp \left( -\frac{2}{1 - \sqrt{1-2z^2}} \frac{\tau}{\delta \tau_E} \right) \right],$$  (37)

where $z$ is a typical ratio of two time scales.11 Here we have chosen $z$ to fit our numerical simulations and we found $z=0.85$. Figure 7 shows that qualitatively (36) gives good results. The bi-exponential model (37) improves quantitatively the prediction of the subgrid Eulerian correlation function because it takes into account the incomplete separation of turbulent scales due to the low Reynolds number value of the subgrid turbulence.

As shown in Fig. 8, the subgrid Eulerian integral time scale decreases when the cutoff wave number is increased. Depending on $\kappa, \eta_K$ the subgrid integral time scale may be smaller than the Kolmogorov time scale. To model the subgrid Eulerian integral time scale, we assume that the subgrid fluid velocity fluctuations are transported by energy-containing eddies with a characteristic velocity $u_f^* = \sqrt{2q_f^2}/3$. Based on this hypothesis we propose the following relation:

$$\delta \tau_E = \frac{\delta L_f}{\sqrt{2q_f^2}/3}.$$  (38)

To validate our assumption, we have plotted in Fig. 8 the evolution of $\delta \tau_E$ computed by integration of the measured subgrid Eulerian correlation (35) and the predictions of (38) computed with $\delta L_f$ obtained with (31) and the one given by the three-dimensional DNS spectrum. We observe that both results are in very good agreement indicating that the subgrid fluid fluctuations are transported by the energy-containing eddies. Using (38) and (32), we derive the following expression:

$$\delta \tau_E = \frac{3\pi}{10} \kappa K_{\eta_K} \left( \frac{\sqrt{3}}{\delta q_f} \right)^{-1}.$$  (39)

In Fig. 8 we have added the predictions given by (39) and the ones given by (38) computed with $\delta L_f$ modeled by (33). Expression (39) underestimates $\delta \tau_E$ whereas the modified spectrum is in agreement with the DNS results. This improvement was expected because the subgrid integral length scale is more accurately modeled by (33) than what can be derived using the spectrum modified at large wave numbers (see Fig. 6).

The subgrid Lagrangian correlation function measured along fluid element trajectories is defined as
The subgrid Lagrangian integral time scale seen by the particles is defined as

$$\delta t_L(\tau) = \int_0^{+\infty} \delta R_f(\tau) d\tau.$$  

Evolution of $t_f^{\tau}$ with respect to the subgrid Stokes number is plotted in Fig. 13. For very light particles, $t_f^{\tau} \to 0$, the dynamical behavior of solid particles is equivalent to the motion of fluid elements. Hence, the fluid integral time scale seen by such solid particles tends toward the fluid Lagrangian integral time scale $t_L$. For very large response time, $t_f^{\tau} \to +\infty$, the particles are not correlated with the fluid and then see the fluid Eulerian integral time scale, $t_E$. Wang and Stock\textsuperscript{33} proposed the following semiempirical model:

C. Subgrid fluid turbulence viewed by the particles

As shown in Fig. 10 the subgrid fluid kinetic energy measured along solid particle paths is weakly dependent on the particle inertia. The maximum difference is about $5\%$.

The dependence of the time-correlation with the particle inertia is investigated through the Lagrangian correlation function of the subgrid fluid velocity along the particle path,

$$\delta R_{f@\tau} = \left\langle \delta u_f^0(\tau_0, x) \delta u_f^0(\tau_0 + \tau, x + u_f(\tau)) \right\rangle_{\tau_0}.$$  

Figure 11 shows that the particle inertia has no effect on the shape of $\delta R_{f@\tau}$. As pointed out for the subgrid Eulerian correlation function in the previous section, the bi-exponential function improves the prediction of $\delta R_{f@\tau}$. Figure 12 shows that the filtering has a limited effect on the shape of $\delta R_{f@\tau}$ that is well predicted by the two-exponential model (37).

FIG. 9. Subgrid Lagrangian integral time scale measured on fluid elements normalized by the Kolmogorov time scale. The solid line stands for the subgrid dissipation time scale (30) and the dashed line stands for the subgrid Eulerian integral time scale (38), both computed using the theoretical spectrum (24) with correction (25).

FIG. 10. Subgrid fluid kinetic energy measured along particle trajectories normalized by the one measured on fixed points with respect to the subgrid Stokes number $t_f / t_L$. The dashed line correspond to $q_f / t_f$ and $t_f / t_L$. For symbol legend see Table II.

FIG. 11. Particle inertia effect on the subgrid Lagrangian correlation function measured along solid particle paths for $\kappa_\eta=0.51$. The dashed and solid lines correspond to exponential (36) and bi-exponential (37) presumed functions, respectively. Symbol legend is given in Table II.

FIG. 12. Subgrid fluid kinetic energy measured along particle trajectories normalized by the one measured on fixed points with respect to the subgrid Stokes number $t_f / t_L$. The dashed line correspond to $q_f / t_f$ and $t_f / t_L$. For symbol legend see Table II.
interaction of particle with the subgrid turbulence is much more complicated and does not allow an easy physical interpretation.

IV. EFFECT OF THE FILTERING ON PARTICLE MOTION

This section is dedicated to the effect of the filtering on the particle dispersion, particle segregation, particle relative velocity motion and the collision rate. In these numerical simulations the particle trajectories are computed using a filtered fluid velocity field. Therefore the particle motion governing equations become

\[
\frac{dx_{p,i}}{dt} = \bar{u}_{p,i},
\]

\[
\frac{d\bar{u}_{p,i}}{dt} = - \frac{\bar{u}_{p,i} - \bar{u}_{fp,i}}{t_p}.
\]

A. Particle dispersion

Dispersion coefficient, \( D_p \), of solid particles suspended in homogeneous isotropic turbulent flow can be related to the particle kinetic energy, \( q^2_p \), and particle Lagrangian integral time scale, \( \tau_p \), with the relation\(^{34,35} \)

\[
D_p = \frac{2}{3} q^2_p \tau_p. 
\]

The particle Lagrangian integral time scale is defined as

\[
\tau_p = \int_0^{+\infty} R_p(\tau) d\tau,
\]

where \( R_p(\tau) \) is the particle Lagrangian velocity correlation function given by

\[
R_p(\tau) = \frac{\langle u'_{p,i}(\tau_0,x)u'_{p,i}(\tau_0 + \tau,x + u_p\tau) \rangle_p}{2q^2_p}.
\]

The effect of the particle inertia on the dispersion coefficient in the case without filtering is shown in Fig. 15 where the particle dispersion coefficient is normalized by the fluid turbulent diffusivity,

\[
D_f = \frac{2}{3} q^2_f \tau_L. 
\]

As expected, the particle dispersion differs from the fluid turbulent diffusivity coefficient because of particle-particle collisions and preferential concentration. On one hand, it has been shown that the preferential concentration may lead to increase the particle dispersion.\(^{5,36} \) On the other hand, the particle-particle collisions lead to decreased \( D_p \).\(^{18} \) This phenomenon results from the decrease of the particle Lagrangian integral time scale, \( \tau_p \), resulting from the diminution of the particle mean free path by collisions. According to Fig. 16 the particle dispersion coefficient, measured in DNS using (47) and (48), is slightly modified by the filtering. The difference is less than 2% for \( \kappa L_f \approx 10 \). This result is in accor-
dance with the study of Armenio et al.,\textsuperscript{2} which showed that the particle dispersion is not affected by subgrid LES model.

The variations of $D_p$ result from the separate changes in the particle Lagrangian integral time scale and the particle kinetic energy as shown in Figs. 17 and 18. Dependence of $\tilde{q}_p^2$ with the particle inertia is shown by Fig. 18 where turbulent agitation of heavy particles is less affected by the filtering than the one of lighter particles. This means that the particle dispersion is caused by the particle interaction with energy-containing fluid eddies. In the frame of Tchen's theory\textsuperscript{31} the particle kinetic energy can be represented in terms of turbulent fluid agitation,

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig14.png}
\caption{Subgrid Lagrangian integral time scale measured along particle trajectories with respect to the particle response time. The symbols are $\delta t_{fp}/\delta t_L$ and the dashed lines on the right-hand side correspond to $\delta t_L/\delta t_L$. Gray symbols are available data with $\tau_p/\tau_K=9.16 (\triangledown); 13.5 (\triangle); 17.9 (\bullet); 26.6 (\triangleright); 35.2 (\bigstar); \text{and} 43.9 (\blacksquare)$.
\end{figure}
\[ q^2_p = f_{fp}^2, \]

(51)

where \( f_{fp} \) is the filtered fluid-particle correlation function depending on filtered particle response time and filtered fluid velocity correlation function \( \tilde{R}_{fp}(\tau) \) defined by

\[ \tilde{R}_{fp}(\tau) = \frac{\langle \tilde{u}_{fp}(\tau, x) \rangle}{2 \tilde{q}_{fp}} \]

(52)

and the corresponding integral time scale is

\[ \tilde{\tau}_{fp} = \int_0^{\infty} \tilde{R}_{fp}(\tau) d\tau. \]

(53)

Modification of turbulent fluid agitation by filtering is plotted in Fig. 19 showing that \( q^2_p \) is not depending on the particle inertia. Further, Figs. 18 and 19 point out that for large response time (\( \square, \diamond, \) and \( \triangleright \)), the modification of the fluid kinetic energy by the filtering is larger than the decreasing of the particle kinetic energy.

Evolution of the fluid Lagrangian integral time scale with respect to the cutoff wave number is given in Fig. 20. Our numerical simulations show that the filtering leads to increased \( \tilde{\tau}_{fp} \). The modification of the fluid Lagrangian integral time scale does not exceed 5% whereas the cutoff wave number is \( \kappa_c L_f < 10 \).

Assuming that \( \tilde{R}_{fp}(\tau) \) is an exponential function of characteristic time scale \( \tilde{\tau}_{fp} \), the filtered fluid-particle correlation function becomes

\[ f_{fp} = \frac{1}{1 + \tilde{\eta}} \]

(54)

with \( \tilde{\eta} = \frac{\tilde{\tau}_{fp}}{\tilde{\tau}_{fp}} \),

where \( \tilde{\tau}_{fp} \) is the mean filtered particle response time. In our numerical simulations the maximum difference between \( \tilde{\tau}_{fp} \) with the one computed without filtering did not exceed 1.5%.

As shown in Fig. 21, the filtered particle kinetic energy for particles with large response time (squares and diamonds) is well predicted by (54). As pointed out by Zaichik et al., \( 11 \) for particles with response time smaller than the Taylor time scale, \( \tau_T \), the exponential shape of the fluid velocity correlation function is not suitable. To improve the predictions,
Zaichik et al.\textsuperscript{11} proposed to use the bi-exponential correlation function\textsuperscript{37} of Sawford\textsuperscript{32} with the parameter $z$ defined as the ratio between the Taylor and the integral Lagrangian time scale: $z = \tau_T/\tau_L = 66.7 \times 10^{-2}$. The Taylor time scale is given by

$$\tau_T = \left( \frac{2R e_\lambda}{a_0 \sqrt{15}} \right)^{1/2} \tau_K,$$

where $a_0$ is associated with the acceleration variance in isotropic turbulence. According to numerical simulations by Yeung and Pope\textsuperscript{36} and Vedula and Yeung\textsuperscript{37} for low to moderate Reynolds numbers and the experiments of Voth et al.\textsuperscript{38} for reasonably large Reynolds number, the dependence of $a_0$ on $R e_\lambda$ is approximated by Zaichik et al.\textsuperscript{11} as

$$a_0 = a_{01} + a_{02} R e_\lambda, \quad a_{01} = 11, \quad a_{02} = 205, \quad a_{0c} = 7.$$

Using the bi-exponential function the filtered fluid-particle correlation function becomes

$$\tilde{f}_{fp} = \frac{2\tilde{f} + z^2}{2\tilde{f} + 2\tilde{f} + z^2}. \quad (57)$$

Figure 21 is showing that (57) improves the prediction for particles with a small response time. Hence, we found that the filtering does not modify the link between the particle and fluid kinetic energy, modeled with (51). In other words, even with the filtering, the particles satisfy the Tchen’s theory, meaning that the transfer of fluid energy to the particles is always well described by the fluid-particle correlation function.

\section*{B. Particle segregation}

The particle segregation is analyzed through the global particle accumulation, $\Sigma_p$, defined as

$$\Sigma_p = \frac{1}{\lambda} (\sigma - \sigma_{Pois}) \quad \text{with} \quad \lambda = \frac{N_p}{N_{cell}}, \quad (58)$$

where $\sigma$ is the standard deviation of $f(C)$, the particle-concentration probability density function measured in the DNS, and $\sigma_{Pois}$ the standard deviation of the Poisson distribution. The function $f(C)$ is computed on a grid containing $N_{cell}$ cells, of volume $\Omega_{cell}$, and covering the computational box. The parameter $\lambda$ is the mean number of particles in one cell for a random uniform particle distribution. The drawback of such a method is the dependence of $\Sigma_p$ on the cell size.\textsuperscript{8} To avoid this problem, Février et al.\textsuperscript{9} proposed to compute $f(C)$ for several values of $\Omega_{cell}$ and to keep only the largest value of $\Sigma_p$. Finally, we note that the Poisson distribution corresponds to a random uniform distribution of non-interacting particles. Here, as the collisions are taken into account we may have a modification of the Poisson distribution. Nevertheless we assume that the dispersed phase is diluted enough in order for the nonoverlapping criterion, due to the collision treatment, to not have a significant effect on the measurement of $\Sigma_p$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig19.png}
\caption{Effect of the filtering on the fluid kinetic energy measured along the particle trajectories. For legend see Table II.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig20.png}
\caption{Effect of the filtering on the fluid Lagrangian integral time scale seen by the particles. For legend see Table II.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig21.png}
\caption{Evolution of filtered particle kinetic energy with respect to the filtered Stokes number $\tilde{f}_{fp}$. Lines correspond to predictions in the frame of Tchen’s theory. The dashed and solid lines correspond to the predictions using exponential (54) and bi-exponential (57) presumed correlation functions, respectively. For symbol legend see Table II.}
\end{figure}
Particle accumulation, in the simulation without filtering, with respect to Stokes number is plotted in Fig. 22. We observe that for both limiting cases, $\tau_{fp} \to 0$ and $\tau_{fp} \to +\infty$, the particles tend to be uniformly distributed. Both asymptotic behaviors were expected because the particles with a small response time are distributed like fluid elements and the coarse particles have a chaotic motion. Between these two limiting cases, the particles interact with specific fluid eddies leading to particle segregation in low-vorticity regions.5

The effect of the filtering on the particle accumulation is shown in Figs. 22 and 23. These figures point out that depending on the ratio $\tau_{fp}^F/\tau_{fp}^f$, the filtering lead to three different behaviors.

- $\tau_{fp}^F/\tau_{fp}^f < 0.5$: The effect of the filtering is to decrease the particle accumulation meaning that the energy-dissipating fluid eddies drive this phenomenon.6,7

- $\tau_{fp}^F/\tau_{fp}^f \in [0.5, 5]$: According to Figs. 22 and 23 the particle accumulation is found to increase with the filtering, suggesting that the energy-dissipating eddies give a random contribution to the particle motion leading to decrease the particle segregation.8,9 For such particles, the particle segregation is performed through the particle interaction with the energy-containing fluid eddies.

- $\tau_{fp}^F/\tau_{fp}^f > 5$: In this case the filtering has no effect on the particle distribution meaning that the preferential concentration of such inertial particles is dominated by large eddy-containing eddies which are slightly affected by the filtering.

These different regimes show that the particle accumulation results from two mechanisms that may act at all scales of the turbulent spectrum.

The small Reynolds number of the fluid turbulence numerical simulation might have impaired the results and further studies with larger Reynolds number values are needed to confirm the above conclusions. However the three different regimes characterizing the effect of the filtering on particle accumulation will surely occur even if the width of the intermediate range of the particle to subgrid time scale ratio might be dependent of the Reynolds number. In addition, some recent DNS results40 showed that the particle segregation effect saturates at large Reynolds numbers.

### C. Relative velocity of neighboring particles

Effect of turbulent microscales on the collision mechanism is investigated through the relative mean radial velocity between neighboring particles. This statistic is computed using the collision detection algorithm which list all particles separated by $|r| < 2d_p$ (where $|r|$ is the distance between the particle centers). As shown by Fig. 24, the mean radial relative velocity measured for light particles tends toward zero for $|r| = d_p$. This effect is due to the interpolation scheme accuracy which should be improved for such particles. Nevertheless, to determine an approximated value of $\langle |w_r| \rangle_p$ at $|r| = d_p$ we use linear regression (see Fig. 24).

As expected, Fig. 25 shows that the relative velocity variance measured between lightest particles tend to the limiting value derived by Saffman and Turner.13
To establish this relation Saffman and Turner\textsuperscript{13} assume that the relative motion of such particles is induced by the local gradient of the turbulence.

Effect of the filtering on the radial relative velocity variance measured on the neighboring particles is shown in Fig. 26. We observe two behaviors depending on the subgrid Stokes number. First, relative motion of heavy particles (\(\square\) and \(\vartriangle\)) is not affected by higher filters. The variance of the relative radial velocity is modified only when a given percentage of the particle turbulent agitation is removed by the filtering, namely for \(\kappa_f L_f < 10\). In contrast, the relative motion of particles with a small response time is more modified by the filtering even for higher filters. Therefore, for such particles, the energy-dissipating turbulent eddies drive the relative motion as assumed by Saffman and Turner.\textsuperscript{13}

To point out both effects we have plotted, in Fig. 27, the ratio \(\langle w^2 \rangle_p / \sigma^2_p\) normalized by its value in the numerical simulation without filtering. We found that as \(\tau_{fp}^F / \tau_{fp@p}^F\) is less than 5, the relative motion between particles is not significantly modified by the filtering.

### D. Collision rate

The collision time scale is written as\textsuperscript{7,11,12}

\[
\tau_{coll}^F = \left[ \frac{1}{2} n_p \bar{d}_p^2 g_0 \langle |w_r| \rangle \right]^{-1} \quad \text{with} \quad n_p = \frac{N_p}{L_p},
\]

where \(n_p\) is the mean particle number in the domain, \(g_0\) the radial distribution function, depending on preferential concentration, and \(\langle |w_r| \rangle\) the mean relative radial velocity. As expected, Fig. 28 shows that the collision time scale is increasing when \(\tau_{fp}^F / \delta \tau_{fp@p}^F < 0.5\). This effect results from the decreasing of the particle accumulation and mean radial relative velocity between neighboring particles observed in Figs. 23 and 27. For \(\tau_{fp}^F / \delta \tau_{fp@p}^F \in [0.5, 5]\), the modification of the collision rate results from the competition between two opposite behaviors. Indeed, as shown in Fig. 23, the particle
segregation is increased and the mean relative radial velocity is decreased in Fig. 27. In our DNS both opposite behaviors lead to a rise in the collision time scale, meaning that the modification of the particle relative motion by the filtering is more important than the modification of the particle accumulation. Finally, for the case $\overline{\tau}_{ip}/\delta \tau_{ip} > 5$ the numerical results show that the modification of the collision rate by the filtering is smaller than 5%.

V. CONSEQUENCES FOR LES OF GAS-SOLID TURBULENT FLOWS

An interesting consequence of this study is to derive criteria for the feasibility and accuracy of LES for particle-laden turbulent flow.

As shown by Refs. 1–3, we found that the particle dispersion is weakly dependent to the subgrid fluid velocity field. Indeed, as expected, the particle dispersion is caused by the large energy-containing turbulent eddies which are fully resolved in LES approach. We found that for $\kappa L_f > 10$ the particle dispersion coefficient modification by the filtering are lower than 5%. We observed the same trends for the particle kinetic energy, the fluid kinetic energy seen by the particles, the particle Lagrangian integral time scale and the fluid Lagrangian integral time scale seen by the particle.

For phenomena driven by the particle interaction with the local fluid velocity, such as the preferential concentration or the particle-particle collisions, a second criteria is necessary. Our analysis shows that the relevant criterion is the Stokes number defined as the ratio between the particle response time and the subgrid fluid integral time scale: $\tau_{ip}^{F}/\delta \tau_{ip}^{F}$.

For $\tau_{ip}^{F}/\delta \tau_{ip}^{F} > 5$ we have shown that it is not necessary to take into account the subgrid fluid fluctuations in the particle trajectory computation. The preferential concentration and the collision rate are not affected by the subgrid fluid velocity hence LES is able to represent these local phenomena. For $\tau_{ip}^{F}/\delta \tau_{ip}^{F} < 5$, in agreement with previous studies, we found a dependence of the particle statistics on the subgrid turbulence. Hence, for such particles, we should account for the subgrid fluid velocity fluctuations in the particle trajectory computation.

Further, for $0.5 < \tau_{ip}^{F}/\delta \tau_{ip}^{F} < 5$, according to our numerical simulations the subgrid turbulence acts only as a random force and should be accurately taken into account using stochastic approaches such as “Langevin” model equation or “eddy-lifetime” model. These approaches need the modeling of the subgrid kinetic energy and the subgrid Lagrangian integral time scale both measured along solid particle paths. Our analysis shows that, in first approximation, it is possible to neglect the statistical bias induced by particle conditioning in the modeling of the subgrid fluid turbulent kinetic energy and integral time scales. Nevertheless more investigations are necessary to evaluate the effect of crossing trajectory on the subgrid Lagrangian integral time scale when a large drift between the fluid and the particle takes place. Finally for $\tau_{ip}^{F}/\delta \tau_{ip}^{F} < 0.5$, according to our results, the particle segregation and the collisions are driven by the interaction with the subgrid fluid velocity gradient that are unknown in LES. At this time no Lagrangian model of the subgrid fluid turbulence along particle trajectories accounting for such mechanisms are available. But recently11 proposed an Eulerian modeling of the subgrid fluid fluctuation based on deconvolution of the filtered velocity field.

VI. CONCLUSION

Direct numerical simulations of homogeneous isotropic turbulent flow coupled with discrete particle simulation taking into account the particle-particle collisions have been performed. Particle Stokes numbers have been chosen in order to ensure preferential concentration effect and a collision rate close to the regime of Saffman and Turner.13

The subgrid turbulence statistics have been investigated using a spectral filter applied to the fluid velocity field given by the DNS. Fixed points and fluid elements were used to compute the Eulerian, $\delta \tau_p$, and Lagrangian, $\delta \tau_L$, integral time scale associated to the subgrid fluid velocity field. A model for the subgrid Eulerian integral time scale based on the assumption that the subgrid fluid velocity fluctuation is transported by the energy-containing eddies has been proposed and validated by comparison with the DNS results.

We investigated the dependence of the subgrid turbulence statistics measured along solid particle trajectories with respect to the particle inertia. We showed that the particle inertia has a limited effect on the measured subgrid kinetic energy. In contrast, the subgrid Lagrangian integral time scale measured along solid particle paths is much more affected by the particle inertia. However we found that for the asymptotic case of very large response time, the subgrid Lagrangian integral time scale seen by the particles tends toward the subgrid Eulerian time scale (fixed points limit case) and for very small response time the subgrid integral time scale seen by the particle tends toward the subgrid Lagrangian time scale (fluid elements limit case). Between both limiting behaviors the interaction of particle with the turbulence is much more complicated mainly because of the preferential concentration effect.

Subgrid turbulence effect on the particle dispersion, preferential concentration, relative velocity and collision rate have been investigated by performing particle trajectory computations using a filtered fluid velocity field given by the DNS. As pointed out by previous study7 the particle dispersion and kinetic energy are found to be dominated by energy-containing turbulent eddies and slightly dependent on the filtering. The detailed analysis of the subgrid turbulence influence on the preferential concentration effect leads to characterize three kinds of behavior depending on a subgrid Stokes number, defined as the particle response time to the subgrid integral time scale ratio. For large value of the subgrid Stokes number the particle segregation is not modified by the filtering. In contrast, for intermediate subgrid Stokes number, typically in the range of $[0.5, 5]$, the subgrid turbulent fluid eddies is limiting the mechanism of preferential concentration. For this case the particle segregation is caused by the particle interaction with the energy-containing fluid eddies and the subgrid fluid turbulence acts like a stochastic force. For small subgrid Stokes numbers, our numerical
simulations showed that the preferential concentration is induced by the subgrid eddies. Last points investigated deal with the particle-particle relative motion and collisions. For such phenomena, the role of the turbulence microscales is much more complex. As expected, the relative velocity of particle with small response time tends toward the limit derived by Saffman and Turner. For larger response times, the relative motion is caused by the particle turbulent agitation, but the turbulence velocity field may induce a correlation between neighboring particle velocities that leads to collision rate decrease.

Finally, we have interpreted our results in the frame of the LES approach, to characterize the effect of the subgrid turbulence and to derive accuracy criteria. The non-dependence of the preferential concentration and relative velocity with the subgrid fluid turbulent scales, when the subgrid Stokes number is larger than 5, demonstrate that the LES approach is accurate for simulating gas-solid turbulent flow without any modeling of the subgrid fluid velocity in the particle trajectory equation. In contrast, for lower values of the subgrid Stokes number the subgrid turbulence should be account for. But for intermediate subgrid Stokes number ([0.5, 5]), as the contribution of the subgrid fluid velocity fluctuation is identified as a random force, the modeling could be carried out in the frame of the classical stochastic Lagrangian approach, using Langevin or eddy-lifetime model. Finally, for a subgrid Stokes number lower than 0.5, the modeling seems very tricky because of the strong and complex interaction of the particles with the subgrid turbulence velocity field.

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