A macroscopic turbulence model based on a two-scale analysis for incompressible flows in porous media

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Abstract. In this paper, turbulent flows in media laden with solid structures are considered. A complete set of macroscopic transport equations is derived by spatially averaging the Reynolds averaged governing equations. A two-scale analysis highlights energy transfers between macroscopic and sub-filter mean kinetic energies and turbulent kinetic energy. Additional terms representing solids / fluid interactions and turbulent contributions are modeled. Closure expressions are determined using physical considerations and spatial averaging of microscopic computations. Results of the present model are successfully compared to volume-averaged reference results coming from fine scale computations. Furthermore, this model is able to provide accurate boundary conditions for clear flow turbulent simulations.

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INTRODUCTION

The macroscopic modeling of turbulent flows passing through porous media concerns many practical applications such as nuclear reactors, chemical reactors or canopy flows. The primary interest for industrial purposes is not the microscopic details of the flow, but rather the description on a large scale of mean flow quantities and the ability to provide accurate boundary conditions for clear flows upstream and downstream the porous media. The objective of this work is to derive an extended and more practical version of the macroscopic turbulence model proposed by Pinson *et al.* [9]. To this aim, we apply two average operators to the Navier-Stokes equations to derive macroscopic balance equations. In a strict mathematical point of view, the order of application of statistical and spatial averages is immaterial in regard to the mean flow quantities equations [8]. Nevertheless, the procedure based on the statistical averaging of the spatially averaged equations will require small eddies to be modeled. Therefore, applying spatial filtering with a characteristic length scale larger than a pore before statistical average would only allow the treatment of large-scale turbulence. This is questionnable since the eddies larger than the scale of the porous structure are not likely to survive long enough to be detected [7]. Following [1, 7, 8, 10], we choose to apply first the statistical average in order to get a structured picture of the turbulent flow and to benefit from the large amount of knowledge available for Reynolds Averaged Navier-Stokes (RANS) turbulence modeling. RANS equations are then integrated over a representative elementary volume ΔV, which is assumed to be well adapted to the geometrical characteristics of the media under study [11, 12]. The spatial average is then defined by

$$\langle \xi \rangle_f(\mathbf{x}) = \frac{1}{\Delta V_f(\mathbf{x})} \int_{\Delta V_f(\mathbf{x})} \xi \, dV, \tag{1}$$

where ΔV_f is the volume of fluid embedded within ΔV. It can be assumed idempotent if variation length scales of the macroscopic quantities are large with respect to the filter size [11, 12]. For each average, any quantity ξ may be split into mean and fluctuating components as $\xi = \bar{\xi} + \xi' = \langle \xi \rangle_f + \delta\xi$, and one can write $\xi = \langle \bar{\xi} \rangle_f + \langle \xi' \rangle_f + \delta\bar{\xi} + \delta\xi'$.

A complete set of spatially averaged equations is first derived. Then, a two-scale analysis allows us to identify different types of kinetic energy transfer between scales and additional source terms in the macroscopic kinetic energy balance equations. Transfers between large scale and sub-filter scale are highlighted and two distinct transfer modes are identified: the classical turbulent cascade transfer, also existing in clear turbulence, and the supplementary transfer induced by the coupling between the drag of the solid structures and the mean macroscopic flow [5]. This description also leads us to define the sub-filter production, which represents energy transfer between the mean part and the turbulent part of the flow [9], and the wake dissipation due to the presence of the solid matrix. A macroscopic model based on three balance equations for the turbulent kinetic energy, the mean sub-filter kinetic energy and the wake dissipation is then proposed. Finally, results of the model are successfully compared to volume-averaged reference results.

NOMENCLATURE

D_h hydraulic diameter of the pores (m)
f_p friction coefficient
Re Reynolds number (UD_h/ν_f)
ν_f kinematic viscosity of the fluid ($m^2 \cdot s^{-1}$)
ρ density of the fluid ($kg \cdot m^{-3}$)
$\bar{\cdot}$ statistical average
$\cdot'$ fluctuation from the statistical average
$\langle\ \rangle_f$ fluid volume average
$\delta\cdot$ deviation from the fluid volume average

GOVERNING EQUATIONS

In this study, uncompressible, adiabatic, single phase flows in saturated, rigid porous media are considered. Fluid properties (density, viscosity) and the porosity of the medium are assumed constant. The Reynolds-averaged set of governing equations, namely the continuity equation, the Navier-Stokes equation and the turbulent kinetic energy equation, are then given by

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0, \tag{2}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu_f \frac{\partial \bar{u}_i}{\partial x_j}\right) - \frac{\partial \overline{u_i' u_j'}}{\partial x_j}, \tag{3}$$

$$\frac{\partial \bar{k}}{\partial t} + \frac{\partial \bar{u}_i \bar{k}}{\partial x_i} = -\frac{\partial}{\partial x_i}\overline{\left(\frac{P'}{\rho} + k'\right)u_i'} - \frac{\partial}{\partial x_i}\left(\nu_f \frac{\partial \bar{k}}{\partial x_i}\right) - \overline{u_i' u_j'}\frac{\partial \bar{u}_i}{\partial x_j} - \nu_f \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}. \tag{4}$$

MACROSCOPIC CONTINUITY AND MOMENTUM EQUATIONS

A velocity no-slip condition at the wall is imposed. The properties of the spatial average operator allow us to write the doubly averaged mass conservation

$$\langle \frac{\partial \bar{u}_i}{\partial x_i} \rangle_f = \frac{\partial \langle \bar{u}_i \rangle_f}{\partial x_i} = 0 \tag{5}$$

and macroscopic momentum equations

$$\frac{\partial \langle \bar{u}_i \rangle_f}{\partial t} + \frac{\partial \langle \bar{u}_i \rangle_f \langle \bar{u}_j \rangle_f}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \langle \bar{P} \rangle_f}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu_f \frac{\partial \langle \bar{u}_i \rangle_f}{\partial x_j}\right) - \frac{\partial \langle \overline{u_i' u_j'} \rangle_f}{\partial x_j} - \frac{\partial \langle \delta \bar{u}_i \delta \bar{u}_j \rangle_f}{\partial x_j} + \underbrace{\langle \left(-\frac{\delta \bar{P}}{\rho}\delta_{ij} + \nu_f \frac{\partial \bar{u}_i}{\partial x_j}\right) n_j \delta_\omega \rangle_f}_{-\overline{F}_{\phi_i}}. \tag{6}$$

In eq. (6) $\overline{F}_\phi$ is the drag force applied by the fluid flow on the solid inclusions, $-\partial\langle \delta \bar{u}_i \delta \bar{u}_j \rangle_f / \partial x_j$ represents dispersion effects and $-\partial \langle \overline{u_i' u_j'} \rangle_f / \partial x_j$ has to be modeled.

MACROSCOPIC KINETIC ENERGIES BALANCE EQUATIONS

Based on the formal commutativity of both average operators, one can write :

$$\langle \overline{u_i u_j} \rangle_f = \langle \bar{u}_i \rangle_f \langle \bar{u}_j \rangle_f + \langle \delta \bar{u}_i \delta \bar{u}_j \rangle_f + \langle \overline{u_i' u_j'} \rangle_f. \tag{7}$$

Three different kinetic energies are then built, depending on the scale under interest. Averaged turbulence kinetic energy is defined by $\langle \bar{k} \rangle_f = \langle \overline{u_i' u_i'} \rangle_f$. Two kinetic energies are defined using the mean velocities, say the macroscale and sub-filter mean kinetic energies $\overline{E}^M = \frac{1}{2}\langle \bar{u}_i \rangle_f \langle \bar{u}_i \rangle_f$ and $\langle \overline{E}^m \rangle_f = \frac{1}{2}\langle \delta \bar{u}_i \delta \bar{u}_i \rangle_f$. Balance equations for those three energies are derived [9]

$$\frac{D\langle \bar{k} \rangle_f}{Dt} = \underbrace{-\frac{\partial}{\partial x_i}\langle \delta \bar{k} \delta \bar{u}_i \rangle_f}_{I} \underbrace{- \frac{\partial}{\partial x_i}\langle \overline{\left(\frac{P'}{\rho} + k'\right)u_i'} \rangle_f}_{II} \underbrace{+ \frac{\partial}{\partial x_i}\left(\nu_f \frac{\partial \langle \bar{k} \rangle_f}{\partial x_i}\right)}_{III} \underbrace{- \langle \overline{R}_{ij} \rangle_f \frac{\partial \langle \bar{u}_i \rangle_f}{\partial x_j}}_{IV} \underbrace{- \langle \delta \overline{R}_{ij} \frac{\partial \delta \bar{u}_i}{\partial x_j} \rangle_f}_{V} \underbrace{- \langle \bar{\varepsilon} \rangle_f}_{VI}, \tag{8}$$

where
I : dispersion;
II : turbulent diffusion;
III : molecular diffusion;
IV : macroscale shear production: PM;
V : sub-filter production: PSF;
VI : averaged viscous dissipation.

$$\frac{D\langle \overline{E}^m \rangle_f}{Dt} = \underbrace{-\frac{1}{\rho}\frac{\partial \langle \delta \bar{P} \delta \bar{u}_i \rangle_f}{\partial x_i}}_{I} \underbrace{+ \frac{\partial}{\partial x_i}\left(\nu_f \frac{\partial \langle \overline{E}^m \rangle_f}{\partial x_i}\right)}_{II} \underbrace{- \langle \nu_f \frac{\partial \delta \bar{u}_i}{\partial x_j}\frac{\partial \delta \bar{u}_i}{\partial x_j} \rangle_f}_{III} \underbrace{- - \frac{\partial}{\partial x_j}\langle \delta \overline{R}_{ij} \delta \bar{u}_i \rangle_f}_{IV} \underbrace{+ \langle \delta \overline{R}_{ij} \frac{\partial \delta \bar{u}_i}{\partial x_j} \rangle_f}_{V} \underbrace{+ \langle \bar{u}_i \rangle_f \overline{F}_{\phi_i}}_{VI} \underbrace{- \langle \delta \bar{u}_i \delta \bar{u}_j \rangle_f \frac{\partial \langle \bar{u}_i \rangle_f}{\partial x_j}}_{VII} \underbrace{- \frac{\partial}{\partial x_i}\langle \delta \overline{E}^m \delta \bar{u}_i \rangle_f}_{VIII}, \tag{9}$$

where

I :	pressure-velocity correlation;
II :	diffusion;
III :	wake dissipation: $\langle \overline{\varepsilon}_w \rangle_f$;
IV :	turbulent diffusion;
V :	opposite of sub-filter production $-PSF$;
VI :	drag ;
VII :	transfer from macroscale mean motion;
$VIII$:	dispersion.

$$\frac{D\overline{E}^M}{Dt} = \underbrace{-\frac{1}{\rho}\frac{\partial}{\partial x_i}\left(\langle \overline{P} \rangle_f \langle \overline{u}_i \rangle_f\right)}_{I} + \underbrace{\frac{\partial}{\partial x_i}\left(\nu_f \frac{\partial \overline{E}^M}{\partial x_i}\right)}_{II}$$
$$\underbrace{-\nu_f \frac{\partial \langle \overline{u}_i \rangle_f}{\partial x_j}\frac{\partial \langle \overline{u}_i \rangle_f}{\partial x_j}}_{III} \underbrace{-\frac{\partial}{\partial x_j}\langle \overline{u}_i \rangle_f \langle \overline{R}_{ij} \rangle_f}_{IV}$$
$$\underbrace{+\langle \overline{R}_{ij} \rangle_f \frac{\partial \langle \overline{u}_i \rangle_f}{\partial x_j}}_{V} \underbrace{-\frac{\partial}{\partial x_j}\langle \overline{u}_i \rangle_f \langle \delta\overline{u}_i \delta\overline{u}_j \rangle_f}_{VI}$$
$$\underbrace{+\langle \delta\overline{u}_i \delta\overline{u}_j \rangle_f \frac{\partial \langle \overline{u}_i \rangle_f}{\partial x_j}}_{VII} \underbrace{-\langle \overline{u}_i \rangle_f \overline{F}_{\phi_i}}_{VIII}, \qquad (10)$$

where

I :	pressure-velocity correlation;
II :	diffusion;
III :	macroscale dissipation;
IV :	turbulent diffusion;
V :	opposite of macroscale shear production: $-PM$;
VI :	dispersion;
VII :	transfer to sub-filter mean motion;
$VIII$:	drag.

Energy transfers between scales are summerized in Fig. 1. Subfilter mean kinetic energy $\langle \overline{E}^m \rangle_f$ is mainly fed by the macroscale mean kinetic $\overline{E}^M$ energy through the drag force applied by the fluid flow on the solid inclusions.At subfilter scale, energy is mainly tansferred form $\langle \overline{E}^m \rangle_f$ to $\langle \overline{k} \rangle_f$ through the so called subfilter production. Turbulent energy is finally dissipated into heat. Moreover, a part of the energy transferred between scales is directly dissipated into heat by way of wake and macroscale dissipations.

DERIVATION OF THE TURBULENCE MODEL

Macroscopic shear production

Macroscopic Reynolds tensor $\langle \overline{R}_{ij} \rangle_f$ is usually modeled by [7, 8, 9]:

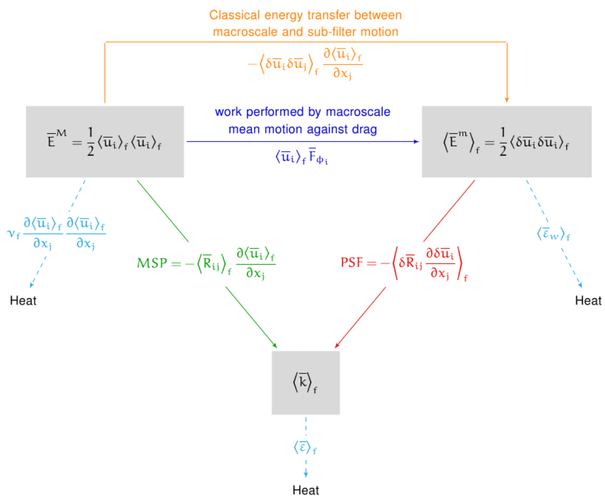

FIGURE 1. Description of energy transfer for flows in porous media.

$$-\langle \overline{R}_{ij} \rangle_f = \nu_{t\phi}\left(\frac{\partial \langle \overline{u}_i \rangle_f}{\partial x_j} + \frac{\partial \langle \overline{u}_j \rangle_f}{\partial x_i}\right) - \frac{2}{3}\langle \overline{k} \rangle_f \delta_{ij}, \quad (11)$$

where the macroscopic turbulent diffusion coefficient is given by

$$\nu_{t\phi} = C'_\mu \frac{\langle \overline{k} \rangle_f^2}{\langle \overline{\varepsilon} \rangle_f}, \; C_\mu = 0.09. \quad (12)$$

Diffusion and dispersion

Following [1, 5, 7, 8, 9] turbulent diffusion in eq. (8) is modeled by

$$-\frac{\partial}{\partial x_i}\langle \overline{\left(\frac{P'}{\rho} + k'\right) u'_i} \rangle_f = \frac{\partial}{\partial x_i}\left(\frac{\nu_{t\phi}}{\sigma_k}\frac{\partial \langle \overline{k} \rangle_f}{\partial x_i}\right). \quad (13)$$

Dispersion is modeled by a gradient approximation

$$-\frac{\partial}{\partial x_i}\langle \delta\overline{k} \delta\overline{u}_i \rangle_f = \frac{\partial}{\partial x_i}\left(\mathscr{D}^{\overline{k}}_{ij}\frac{\partial \langle \overline{k} \rangle_f}{\partial x_j}\right). \quad (14)$$

One can find in [3] general expressions for $\mathscr{D}^{\overline{k}}_{ij}$.

Pressure-velocity correlation, dispersion and turbulent diffusion are modeled collectively as

$$-\frac{1}{\rho}\frac{\partial \langle \delta\overline{P} \delta\overline{u}_i \rangle_f}{\partial x_i} - \frac{\partial}{\partial x_i}\langle \delta\overline{E}^m \delta\overline{u}_i \rangle_f - \frac{\partial}{\partial x_i}\langle \delta\overline{R}_{ij} \delta\overline{u}_i \rangle_f =$$
$$\frac{\partial}{\partial x_i}\left(\frac{\nu_{t\phi}}{\sigma_E}\frac{\partial \langle \overline{E}^m \rangle_f}{\partial x_i}\right) + \frac{\partial}{\partial x_i}\left(\mathscr{D}^{\overline{E}^m}_{ij}\frac{\partial \langle \overline{E}^m \rangle_f}{\partial x_j}\right). \quad (15)$$

Macroscopic turbulent viscosity is defined by $\nu_{t\phi} = C_\mu \langle \overline{k} \rangle_f^2 / \langle \overline{\varepsilon} \rangle_f$, $C_\mu = 0.09$ and $\sigma_k = 1$.

Friction factor

Since wake dissipation is tightly linked to velocity gradient, which is concentrated at walls, it is clearly related to friction. Based on a dimensionnal argument, we assume $\langle \overline{\varepsilon}_w \rangle_f \propto {u_f^3}/{D_h} \propto {\langle \overline{u}_z \rangle_f^3 f_p^{3/2}}/{D_h}$. We thus propose

$$\frac{f_p}{f_p^*} = \left(\frac{\langle \overline{\varepsilon}_w \rangle_f}{\langle \overline{\varepsilon}_w \rangle_f^*} \right)^{2/3}, \tag{16}$$

where f_p^* and $\langle \overline{\varepsilon}_w \rangle_f^*$ are representative values of the friction factor and wake dissipation for the flow under consideration. For instance, for flows in channels, representative values correspond to fully developped flow limit.

Dissipation rate and sub-filter production

Sub-filter production PSF and viscous dissipation $\langle \overline{\varepsilon} \rangle_f$ are modeled by

$$\frac{PSF}{PSF^*} = \left(\frac{\langle \overline{k} \rangle_f}{\langle \overline{k} \rangle_f^*} \right)^a \left(\frac{\langle \overline{E}^m \rangle_f}{\langle \overline{E}^m \rangle_f^*} \right)^b . \tag{17}$$

$$\frac{\langle \overline{\varepsilon} \rangle_f}{\langle \overline{\varepsilon} \rangle_f^*} = \left(\frac{\langle \overline{k} \rangle_f}{\langle \overline{k} \rangle_f^*} \right)^n , \tag{18}$$

where a, b and n are constants and PSF^*, $\langle \overline{k} \rangle_f^*$, $\langle \overline{E}^m \rangle_f^*$ and $\langle \overline{\varepsilon} \rangle_f^*$ are representative values of turbulent kinetic energy, subfilter mean kinetic energy and dissipation for the flow under consideration.

Wake dissipation

Wake dissipation is induced by the velocity deviation gradients. It is related the presence of walls. One can then presume that evolution of wake dissipation results from a competition between production due to the friction with the wall (represented by the drag) and dissipation by viscous effects. The following transport equation is then postulated for $\langle \overline{\varepsilon}_w \rangle_f$

$$\begin{aligned} \frac{D\langle \overline{\varepsilon}_w \rangle_f}{Dt} =& \frac{\partial}{\partial x_i} \left[\left(\nu_f + \frac{\nu_{t\phi}}{\sigma_{\varepsilon_w}} \right) \frac{\partial \langle \overline{\varepsilon}_w \rangle_f}{\partial x_i} \right] \\ &+ \frac{\partial}{\partial x_i} \left(\mathscr{D}_{ij}^{\overline{\varepsilon}_w} \frac{\partial \langle \overline{\varepsilon}_w \rangle_f}{\partial x_j} \right) \\ &- \frac{C_{\varepsilon_{w,2}}}{\tau_{w_1}} \langle \overline{\varepsilon}_w \rangle_f + \frac{C_{\varepsilon_{w,1}}}{\tau_{w_2}} \langle \overline{u}_z \rangle_f \overline{F}_{\phi_z}, \end{aligned} \tag{19}$$

where τ_{w_1} et τ_{w_2} are characteristic time scales and $C_{\varepsilon_{w,1}}$ et $C_{\varepsilon_{w,2}}$ are constants. By analogy with local $\overline{k} - \overline{\varepsilon}$ models, we choose $\tau_{w_1} = \tau_{w_2} = {\langle \overline{E}^m \rangle_f}/{\langle \overline{\varepsilon}_w \rangle_f}$.

APPLICATION TO STRATIFIED POROUS MEDIA

In this section, we propose a turbulence model for flows in stratified porous media (Fig. 2). The turbulent flow is statistically steady and oriented in the z direction. Reynolds number is given by $Re = {\langle \overline{u}_z \rangle_f D_h}/{\nu_f}$. In such media, $\langle \overline{u}_z \rangle_f$ is the bulk flow velocity. No eddies larger than the pore size can thus exist. There is no macroscopic velocity gradient and the macroscopic shear production vanishes. In such configurations, medium study can be reduced to a unit cell study.

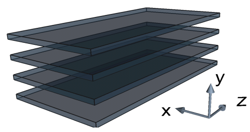

FIGURE 2. Example of stratified porous medium: description of a porous medium composed of flat plates.

In the configuration under study, equations (8), (9) and (19) can be simplified :

$$\begin{aligned} \frac{D\langle \overline{k} \rangle_f}{Dt} =& \frac{\partial}{\partial z} \left[\left(\nu_f + \frac{\nu_{t\phi}}{\sigma_k} + \mathscr{D}_{zz}^{\overline{k}} \right) \frac{\partial \langle \overline{k} \rangle_f}{\partial z} \right] \\ &+ PSF^* \left(\frac{\langle \overline{k} \rangle_f}{\langle \overline{k} \rangle_f^*} \right)^a \left(\frac{\langle \overline{E}^m \rangle_f}{\langle \overline{E}^m \rangle_f^*} \right)^b - \langle \overline{\varepsilon} \rangle_f^* \left(\frac{\langle \overline{k} \rangle_f}{\langle \overline{k} \rangle_f^*} \right)^n , \end{aligned} \tag{20}$$

$$\begin{aligned} \frac{D\langle \overline{E}^m \rangle_f}{Dt} =& \frac{\partial}{\partial z} \left[\left(\nu_f + \frac{\nu_{t\phi}}{\sigma_E} + \mathscr{D}_{zz}^{\overline{E}^m} \right) \frac{\partial \langle \overline{E}^m \rangle_f}{\partial z} \right] + \langle \overline{u}_z \rangle_f \overline{F}_{\phi_z} \\ &- PSF^* \left(\frac{\langle \overline{k} \rangle_f}{\langle \overline{k} \rangle_f^*} \right)^a \left(\frac{\langle \overline{E}^m \rangle_f}{\langle \overline{E}^m \rangle_f^*} \right)^b - \langle \overline{\varepsilon}_w \rangle_f , \end{aligned} \tag{21}$$

$$\begin{aligned} \frac{D\langle \overline{\varepsilon}_w \rangle_f}{Dt} =& \frac{\partial}{\partial z} \left[\left(\nu_f + \frac{\nu_{t\phi}}{\sigma_{\varepsilon_w}} + \mathscr{D}_{zz}^{\overline{\varepsilon}_w} \right) \frac{\partial \langle \overline{\varepsilon}_w \rangle_f}{\partial z} \right] \\ &- \frac{C_{\varepsilon_{w,2}}}{\tau_{w_1}} \langle \overline{\varepsilon}_w \rangle_f + \frac{C_{\varepsilon_{w,1}}}{\tau_{w_2}} \langle \overline{u}_z \rangle_f \overline{F}_{\phi_z} . \end{aligned} \tag{22}$$

Asymptotic states

Far downstream, profiles reach non-evolving levels and shapes. This asymptotic state corresponds to the fully developped flow limit and is denoted "$\cdot_\infty$". It depends on the Reynolds number of the flow. Correlations based on averaged fine-sale simulations are proposed for turbulent asymptotic states in stratified media. The friction factor asymptotic value f_{p_∞} may be given by the correlations available in the literature [6]. We propose the following correlations:

$$\langle \overline{\varepsilon}_w \rangle_{f,\infty} = C_w \frac{\langle \overline{u}_z \rangle_f^3}{2D_h} \times f_{p_\infty}^{3/2}, \tag{23}$$

$$\langle \overline{u}_z \rangle_f \overline{F}_{\phi_z}^{\infty} = \frac{\langle \overline{u}_z \rangle_f^3}{2D_h} f_{p_\infty}, \tag{24}$$

$$PSF_\infty = \langle \overline{\varepsilon} \rangle_{f,\infty} = \frac{\langle \overline{u}_z \rangle_f^3}{2D_h} f_{p_\infty} \left(1 - C_w f_{p_\infty}^{1/2}\right), \tag{25}$$

$$\langle \overline{k} \rangle_{f,\infty} = c_k \times \langle \overline{u}_z \rangle_f^2 f_{p_\infty}/8, \tag{26}$$

For quasi-parallel flows, $\langle \overline{E}^m \rangle_f$ has the structure of a dispersion contribution. Hence for channel flows we propose

$$\langle \overline{E}^m \rangle_{f,\infty} = 2\mathscr{D}_z^{A*} \times \langle \overline{u}_z \rangle_f^2 f_{p_\infty}/8. \tag{27}$$

For plane channel flows, we find $C_w = 3$, $c_k = 1.82$. A detailed description of $\mathscr{D}_z^{A*}$ may be found in [3]. Expressions (23) to (27) are compared with reference results on Fig. 3. Reference results are spatially averaged fine scale simulations.

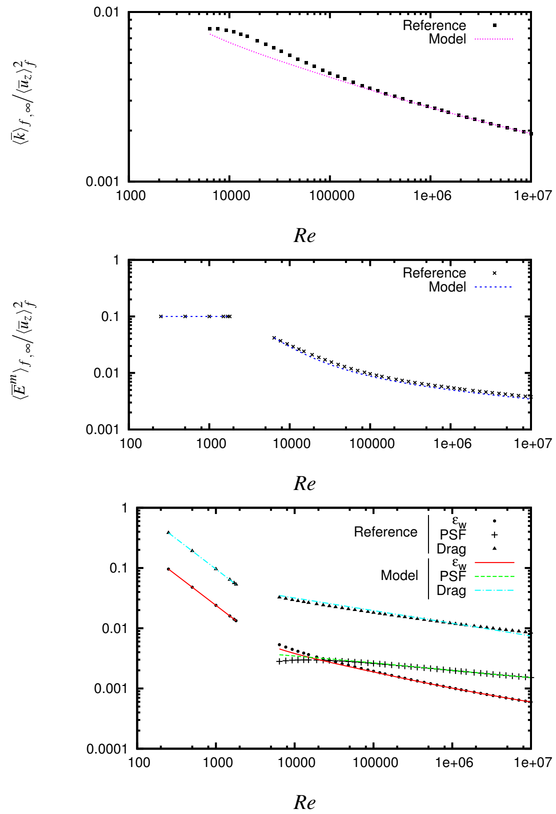

FIGURE 3. Asymptotic states: evaluation of expressions (23) to (27) for flows in plane channels. The source terms $\langle \overline{\varepsilon}_w \rangle_{f,\infty}$, PSF_∞ and $\langle \overline{u}_z \rangle_f \overline{F}_{\phi_z}^{\infty}$ are adimensionned by $\langle \overline{u}_z \rangle_f^3 / 2D_h$.

Model constants

For flows in stratified porous media, the present model should be consistent with asymptotic states. Characteristic scales are then given by $f_p^* = f_{p_\infty}$, $\langle \overline{\varepsilon}_w \rangle_f^* = \langle \overline{\varepsilon}_w \rangle_{f,\infty}$, $PSF^* = PSF_\infty$, $\langle \overline{k} \rangle_f^* = \langle \overline{k} \rangle_{f,\infty}$, $\langle \overline{E}^m \rangle_f^* = \langle \overline{E}^m \rangle_{f,\infty}$ and $\langle \overline{\varepsilon} \rangle_f^* = \langle \overline{\varepsilon} \rangle_{f,\infty}$. Moreover, in order to recover reference asymptotic states, we impose :

$$C_{\varepsilon_{w,2}} = C_{\varepsilon_{w,1}} \times \frac{\langle \overline{u}_z \rangle_f \overline{F}_{\phi_z}^{\infty}}{\langle \overline{\varepsilon}_w \rangle_{f,\infty}}. \tag{28}$$

For such flows, only $\mathscr{D}_{zz}^{\overline{k}}$, $\mathscr{D}_z^{\overline{E}^m}$ and $\mathscr{D}_{zz}^{\overline{\varepsilon}_w}$ are needed. Following [7], dispersion coefficients are modeled by means of thermal dispersion coefficient $\mathscr{D}_{zz}^P$ presented in [3] and Lewis numbers

$$\mathscr{D}_{zz}^{\overline{k}} = \mathscr{D}_z^{\overline{E}^m} = \mathscr{D}_{zz}^{\overline{\varepsilon}_w} = \nu_f \mathscr{D}_{zz}^{P*}(Pr = 1). \tag{29}$$

It has been shown in [3] that dispersion strongly predominates over turbulent diffusion. Hence turbulent diffusion is neglected in eqs. (21) and (22).

Optimal values of the model constants a, b, n and $C_{\varepsilon_{w,1}}$ have been determined for turbulent flows in channels. Formally, the $\langle \overline{k} \rangle_f - \langle \overline{E}^m \rangle_f$ system can be approached by a damped oscillator. A frequential analysis of the reference oscillations leads to $a = b = 1/4$, $n = 1/3$. A numerical optimization procedure has been run to determine an optimal value for $C_{\varepsilon_{w,1}}$. Eq. (19) is solved for $C_{\varepsilon_{w,1}} \in [0.1, 10]$ and the results are compared with reference results. The spatially integrated error is calculated for several test cases. The average integrated error is minimal for $C_{\varepsilon_{w,1}} \in [4, 5.5]$. We choose $C_{\varepsilon_{w,1}} = 5$.

RESULTS AND DISCUSSION

In order to assess the present macroscopic model, steady unidirectional turbulent flows entering into porous medium composed of plane channels shall be investigated from both microscopic and macroscopic points of view. From the microscopic point of view, fine-scale simulations are carried out with FLICA-OVAP CFD code [4] using the low-Reynolds $\overline{k} - \overline{\varepsilon}$ Chien model [2]. At the inlet, velocity, TKE and viscous dissipation profiles are flat. By applying spatial average to fine-scale simulation results, we get reference evolutions for macroscale quantities. Spatially averaged quantities at the inlet provide channel inlet boundary conditions hereafter denoted "$\cdot_0$". Between inlet and asymptotic state, large scale oscillations of spatially averaged physical quantities are observed [10]. From the macroscopic point of view, equations (20), (21) and (19) are solved with the closure relationships (16), (17) and (18). The macroscopic turbulence model has been implemented in a 1D unsteady code. Numerical scheme based upon MUSCL formulation is used. Asymptotic states are given by eqs. (23) and (27). Results of our present model are compared with spatially averaged fine-scale simulations on Figs. 4 to 5. This test case is characterized by $Re = 1.\times 10^5$; $\langle \overline{k}_0\rangle_f / \langle \overline{k}\rangle_{f,\infty} = 3$; $L_0 = \langle \overline{k}_0\rangle_f^{3/2} / \langle \overline{\varepsilon}_0\rangle_f = D_h/10$.

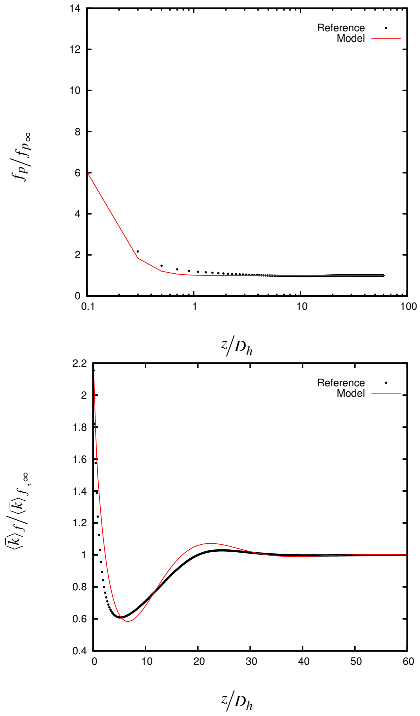

FIGURE 4. Macroscopic turbulence model results for turbulent flow in stratified porous media composed of flat plates. Comparison with reference results. Evolution of f_p/f_{p_∞} and $\langle \overline{k}\rangle_f / \langle \overline{k}\rangle_{f,\infty}$.

CONCLUSION

A macroscopic turbulence model has been proposed for flows in porous media. A two-scale analysis highlighted energy transfers between the mean motion and tubulence embedded in a porous medium. Averaged energies equations have been derived and closure relationships have been determined. The model have been tested for unidirectional turbulent flows in stratified porous media. Reference results are accurately recovered, as well as asymptotic values. Further investigations are needed to find out the applicability of the present model to more general cases (laminar flows, other geometries, mixing grid, exit flows from porous media).

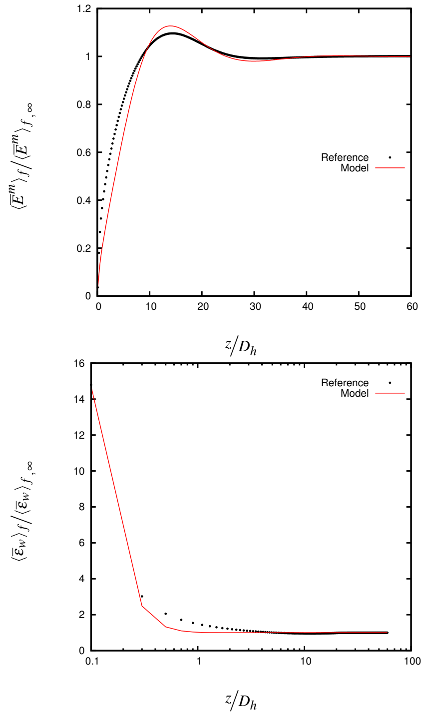

FIGURE 5. Macroscopic turbulence model results for turbulent flow in stratified porous media composed of flat plates. Comparison with reference results. Evolution of $\langle \overline{E^m}\rangle_f / \langle \overline{E^m}\rangle_{f,\infty}$ and $\langle \overline{\varepsilon}_w\rangle_f / \langle \overline{\varepsilon}_w\rangle_{f,\infty}$.

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