A Hybrid Fuzzy Approach to Bullwhip Effect in Supply Chain Networks

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1. Introduction

Today all small and medium size enterprises, companies and even countries (either in private, public or military domain) in the national and international business area are continuously performing activities to provide capabilities for satisfying customer needs (i.e., demand) those indeed include many sophisticated interrelated functions and processes such as decision making, management, new product development, production, marketing, logistics, finance, quality control and etc. which, all together compose dynamic, complex and chaotic structures called supply chain networks (SCNs). These complex structures with all interrelated functions have to be designed and managed perfectly pointing us to the well-known term SCN management (SCNM). Due to the complex information flow in these systems; which consists of cumulative data about costs parameters, production activities, inventory systems and levels, logistic activities and many other related processes, we may unwaveringly express that the performance of a successful SCN directly related to the constant, accurate and appropriate demand information flow as this vital flow of information inarguably influences all decision making processes in all stages of SCNs. A well-known phenomenon of SCNs called the “Bullwhip or Whiplash Effect” (BWE) is the variability of the demand information between the stages of the SCN and the increase in this variability as the demand data moves upstream from the customer to the following stages of the SCN engendering undesirable excess inventory levels, defective labor force, cost increases, overload errors in production activities and etc. From 1952 till now many studies have been done about BWE. However very few of them interested in fuzzy and neuro-fuzzy system (NFS) approaches to BWE such as Carlsson and Fuller (1999, 2001, 2002, 2004) and Efendigil et al. (2008).

Making accurate and appropriate estimation about future in decision making process is the leading activity providing bases for almost every managerial applications including SCNs. Demand forecasting and decision making are among the key activities that directly affect the SCN performance. To smoothen the undesirable variabilities of demand through the stages of SCN due to the chaotic nature of SCN system, appropriate demand forecasting is vital. As demand pattern varies due to the field of activity and architecture of SCNs, determining the appropriate forecasting model and adequate order/demand decision process for system interested in is snarl. As the nature of forecasting and decision making contains uncertainty or vagueness of the human judgment, they perfectly fit for the
applications of fuzzy logic (FL) (Kahraman, 2006), artificial neural networks (ANNs) and; more specifically, the combination of these two complementary technologies (i.e.; NFS). The FL; which was introduced by Zadeh in 1965 with his pioneer work “Fuzzy Sets”, can simply be defined as “a form of mathematical logic in which truth can assume a continuum of values between 0 and 1” (http://wordnetweb.princeton.edu/, 2009). On the contrary to crisp (discrete) sets which divide the given universe of discourse in to basic two groups as members and nonmembers, FL has the capability of processing data using partial set membership functions which makes FL a strong device for impersonating the ambiguous and uncertain linguistic knowledge (Kahraman, 2006). The advantage of approximating system behavior where analytic functions or numerical relations do not exist provide opportunity to fuzzy set theory for becoming an important problem modeling and solution technique which also bring along the usage of FL successfully in many fields of scientific researches, industrial and military applications such as control systems, decision making, pattern recognition, system modeling and etc. (Ross, 2004). Due to the perfect harmony of forecasting nature and fuzzy set theory, studies related to fuzzy forecasting is pretty much in the literature (see Kahraman, 2006). Fuzzy regression (FR) forecasting models are also among the successful applications fuzzy forecasting models. Contrary to the enormous literature about determining the appropriate forecasting and order/production decision models in SCNs, relatively few of them interested in fuzzy or neuro-fuzzy approaches. The aim of this chapter is to carry out a literature review about the BWE, to provide a brief overview about FL, NFS, FR forecasting model and to introduce the proposed conjoint hybrid approach made up of an ANFIS based demand decision process together and FR forecasting model.

2. Basic literature

In this section at first, a basic review of literature about BWE is given and the fuzzy approach related studies about BWE is overviewed after that.

2.1 Bullwhip literature

The first academic research on BWE grounds on Jay W. Forrester (1958, 1961). In his pioneer work Forrester, using a simple four echelon SCN simulation (retailer, wholesaler, distributor and factory) discovered the existence of the ‘demand amplification’ which later denominated as BWE (Lee et al., 1997a, 1997b). He argued about the causes and suggested same ideas to control the BWE. He concluded that the decision making process and time delays in each phase of SCN and the factory capabilities could be the main reasons of the demand amplification through the chain from the retailer to the factory (upstream through the chain) as; any increase in customer demand at any point of time causes increases in retailers demand from the wholesaler, the wholesalers demand from distributor and in the same manner, the distributors demand from the factory. But in each, the amount of the demand accrual rate amplifies not only by taking account the real demand increases but also possible future increases causing inessential excessive inventory levels. Forrester also analyze the effect of advertising factor and saw that it also influences the system by engendering the BWE. He, as a solution, emphasized on the importance of knowledge about the system and suggested that the key fact for handling the BWE is to understand the whole SCN system.
Burbidge (1961); thought his study was about production and inventory control, also interested in demand amplification. In 1984, he concluded that an increase in demand variability would occur in every transfer of demand information if demands are carried over a series of inventories using “stock control ordering”. This definition is accepted to be the “first thorough definition” of BWE (Miragliotta, 2005).

Like Forrester, Sterman (1984, 1989a, 1989b) also focused on the existence and causes of BWE. He used an experimental four-stage SCN role-playing simulation that simulates the beer distribution in a simple SCN which is then became a well-known SCN simulation game that successfully depicts the notion of system dynamics; “The Beer Distribution Game”, widely used for teaching the behavior, concept and structure of SCN. The model was so simple but despite to its simplicity, it successfully showed the impact of the decision process in each echelon on the demand variability. Main objective is to govern echelons by achieving desired inventory and pipeline levels minimizing the total cost.

Participants of the game try to govern each echelon based on the information available for making ordering decision in each echelon. In other words, the real demand of customer only known by retailer who directly gets the customer orders and other echelons only have the demand information of the predecessor echelons those placed their demand directly to them. Game begins with the customer demand from the retailer, who tries to fill customer order from his/her own inventory if available. If demand exceeds the inventory, retailer placed his/her order to wholesaler. And in the same manner the demand and distribution processes go on through the SCN system of the game till the factory level where beers produced to meet the demand of distributor. The decision process in each echelon is based on the actual and desired inventory levels, current and expected demand; and finally, the desired and real level of items in pipeline.

Sterman; by analyzing the decision methodology of the participants, found out that, participants; instead of focusing on system delays and nonlinearities, focus on their current and target inventory levels ignoring the amount of orders placed but not received which then cause the demand and inventory enlargements that raises upstream from customer to factory. He also concluded that “anchoring and adjustment” heuristic (which is used for simulating the demand decision process of each echelon) is inconsequent as this heuristic is lack of sensibility to delays and repercussions of SCN system or; as to generalize, lack of “System Thinking” (Sterman 2000). With his simple beer SCN simulation game, he exposed general characteristics of SCN dynamics and; as Forrester, emphasized on irrational decision making process (via “misperception of feedback”) which is one of the main causes of and reason for the rise of BWE.

Forrester’s model also used by Towill (1991, 1992) and Wikner et al.(1991). Using the Forrester’s model with additional quantitative measures, Towill analyzed the S.C. systems by applying system dynamics models. System dynamics defined by Towill (1993a, 1993b) as “A methodology for modeling an redesign of manufacturing, business and similar systems which are part man, part machine”. He concluded that one of the reasons of demand amplification is time delays relevant to ‘value added’ or ‘idle’ operations. With an industrial example, he showed that via integration of decision mechanisms in SCN systems improvement could be achieved (both for demand amplifications and stock levels through the system). He also mentioned that this is still the case when MRP II capacity planning is conjoined to JIT flow shop control.
Wikner, Towill and Naim (1991); taking Forrester three echelon model as base, compared several methods of resolving dynamic performance of distribution systems. Though they suggested that Forrester’s model is “far from optimal”, they use it to evaluate their proposed systems. In the study authors tried to gain improvement by;

• eliminating echelons,
• altering decision rules for providing improvement,
• abating delays,
• arranging system ordering parameters and,
• constructing a smooth information flow.

In conclusion, they emphasized on the importance of smooth, better information flow through the whole chain and reducing delays, as these solutions have dominant impact for BWE reduction rather than improvement of ordering system.

Later Towill (1993a, 1993b) showed the influence of servo theory and cybernetics on the system dynamics and via examination he suggested that the input-output analysis is important for model building in system dynamics.

An important analysis in BWE history is made by Lee et al. in 1997 which than would light the way to many other studies (including their following studies) specially related to causes, quantification and also handling tools of the phenomenon (1997a, 1997b, 2002, 2004). Focusing on the operational causes of the problem and proving the existence by documentary evidences provided from several companies from different sectors (such as their well-known cases P&G and Hewlett-Packard), they declared four major causes and triggers of BWE as i.) demand signal processing (forecast updating), ii.) rationing game, iii.) order batching, iv.) price fluctuation.

Lee et al. (1997a) after proposing sources of BWE also proposed activities that can be used to mitigate the impact of these sources as summarized in the following table. Differently from Sterman and Forrester who generally declared that irrational behaviors of decision makers in SC is the main reason of BWE; the study of Lee; demonstrated that BWE is an outcome of the strategic interactions among rational SCN members; i.e., their attitudes inside the SCN constitution (see Table 1).

Though they are outnumbered relatively to others, researches about quantification of BWE can be considered as another category in the research area of this phenomenon. In general, most preferred system for quantifying BWE is computing demand variance or standard deviation ratios of two subsequent stages of SCN for their ability of easily capturing and displaying the scale of BWE. But studies that used cost parameters are also attract attention. Among the studies which tried to quantify BWE Metters’ (1997) and Chen’s (1998, 1999, 2000a, 2000b) studies are the remarkable ones. From the cost-profit perspective of quality management, Metters quantified BWE using costs arisen from BWE through the chain. Simulating a two-staged SCN model, he focused on demand variance, forecast errors and demand seasonality. Analyzing the model under several circumstances, he showed the effect of BWE on profitability and demonstrated that profit improvement can be achieved via BWE reduction. As this study directly shows the monetary impact of BWE on company profitability, it deservedly captured considerable attention from the managerial point of view.

Chen et al. (1998, 1999, 2000a, 2000b) studied the effects of forecasting, lead times and information sharing on BWE quantified as a ratio of demand variances of two consequent stages of simple SCN system. They showed order variances in the upstream echelon will be amplified if upstream echelons demand decisions are renewed systematically using the monitored values of predecessor downstream echelons orders periodically and even
Causes of Bullwhip | Information Sharing | Channel Alignment | Operational Efficiency
---|---|---|---
Demand Forecast Update | -Understanding system dynamics  
-Use of point-of-sale data (POS)  
-Electronic data interchange (EDI)  
-Computer-assisted Ordering (CAO) | -Vendor-managed inventory (VMI)  
-Discount for information sharing  
-Consumer direct | -Lead time reduction  
-Echelon-based inventory control
Order Batching | -EDI  
-Internet Ordering | -Discount for truck-load assortment  
-Delivery appointments  
-Consolidating  
-Logistic outsourcing | -Reduction in fixed cost of ordering by EDI or electronic commerce  
-CAO
Price Fluctuation | -Continuous replenishment program  
-Everyday low cost | -Everyday low price  
-Activity-based costing |  
Shortage Gaming | -Sharing sales, capacity and inventory data | -Allocation based on past sales |  

Table 1. A Framework for Supply Chain Coordination Initiatives (Lee, 1997a)

thought the customer demand data is available for all echelons (i.e. centralized demand information), the forecasting technique and inventory system used is unique in each echelon through whole chain, the BWE will exist (1998, 2000a). In brief, Chen et al. constructed a two stage SCN model in which moving average technique is used for analyzing the unknown demand pattern essential for the inventory system that is operated (i.e. order-up-to policy) and developed a lower bound (a function of demand correlation, lead time and number of observations) on order variances placed by retailer concerning customer demand and developed their findings to multistage models. Despite the drawbacks described above and model simplicity, the study of Chen et al. introduced same executive overlook to quantified BWE adducing the effects of forecasting.

Later authors analyzed the effects of exponential smoothing forecasting technique on BWE for i.d.d. and linear trend demand cases (2000b). The study was very similar to their previous one. This time, the forecasting method used to predict the future demand of customer by the retailer was exponential smoothing. As a result of their study, they conclude following managerial insights:

- the size of demand variability directly influenced from the forecasting technique used to predict future demand variances and from the form of the demand pattern,
- BWE occurs when retailer updates the order-up-to point according to the periodically computed forecast values,
- The longer the lead time greater the demand variability,
- Smothering the demand forecast with more demand information will decrease BWE.
Gavirneni et al. (1999), Cachon et al. (2000), Kefeng et al. (2001) are the others who looked at the problem from the point of information sharing and its value. Gavirneni et al. betrayed the importance of information sharing in inventory control using, uniform and exponential demand patterns. Cachon and Fisher examining a simple SCN with two stages and stochastic stationary demand, compared the value of information sharing between the case in which only demand information available and the case in which both demand and inventory information are available. Their research results from their model showed that there is no remarkable difference between the analyzed cases. Later in his study of US industrial level data in 2005, Cachon et al. absorbed that; contrary to general understanding of BWE, demand variability does not always increase as one move up though the stages of SCN because of production smoothing attitude of manufacturers arisen from marginal costs and seasonality. Kefeng et al. analytically examined the improvement of coordination and appropriate forecasting in SCN. They presented their results for non-stationary, serially correlated demand and stationary one-lag demand before and after collaboration. The outcomes of the study showed that even under non-trendy and non-seasonal demand BWE exists and also the adaptation of forecasting method to the demand pattern and information sharing notably reduces BWE. So, they keynoted the importance of effective communication between the stages of SCN and consistent forecasting.

Kimbrough et al. (2002) looked thorough to SCN and BWE from a different perspective. They analyzed effectiveness of artificial agents in a beer game simulation model and investigated their ability of mitigating BWE through the system. They found out that agents have the effective ability of playing beer game. The study exposed that agents are capable of finding optimal policies (if there exists) or good policies (where analytical solutions are not available) that eliminates BWE tracking demand pattern under the assumptions of the model. Kimbrought and his coauthors study was important as it brought a different perspective to the solution of the problem from the point of computer aided decision models such as artificial intelligence and NF systems.

Towill, with other researchers such as Disney, Dejonckheere and Geary, have made several more important studies from the control theory approach (CTA) related to BWE which also are served as basis to many other researches (Towill et al. 2003; Dejonkheere et al. 2002, 2003, 2004; Disney et al. 2003a, 2003b, 2004, 2006).

From 2003 up till recent years other than Towill’s, Dejonkheere’s, Disney’s and Geary’s studies there is a remarkable increase in the research of BWE. Among these most considerable ones can be summarized as follow.

Aviv (2003), Alwan et al. (2003), So et al. (2003), Zhang (2004), and Liu et al. (2007) analyzed the phenomenon using stationary demand modeling the process as an ARMA type. Modeling demand as first order ARMA process, Aviv performed an adaptive replenishment policy, Alwan et al., Zhang and Liu et al. analyzed the forecasting procedures displaying the effects of moving average (MA), exponentially weighted moving average (EWMA) and minimum mean squared error (MMSE) forecasting models and, So et al. focused on lead times in a simple two phased model. Later Zhang (2005); again modeling customer demand as first order AR (i.e., AR(1)) process and using MMSE forecasting model, showed that delayed demand information reduces BWE.

Machuca et al. (2004) and Wu et al. (2005) studied on the effects of information sharing to BWE. Machuca et al. focused on the usage of EDI in SCN systems. A simple definition of EDI is given by The American Standards Institute as “the transmission, in a standard syntax, of unambiguous information of business or strategic significance between computers of
independent organizations”. As the smooth, correct and on-time information sharing is essential for SCN systems, usage of EDI provides rapid inter-organization coordination standardizing electronic communication (i.e., exchange of routine business data computer to computer), lead time reduction reducing the clerical process and reduction in the inventory costs due to the improvement of trading partner relationship, expedited supply cycle and enhanced inter-organizational relationship. Based on the idea that usage of EDI reduces the information delays, Machuca et al. analyzed the SCN system both as a whole and for individual echelons and showed that a reduction in BWE and related cost (especially costs driven by inventory) can be achieved with the usage of EDI, thought it did not completely eliminate the BWE in SCN systems.

Wu et al. (2005) used the beer game and analyzed the phenomenon from information sharing together with organizational learning perspective. Thought the study looked at the problem only in managerial view, the outcomes displayed that when organizational training and learning combined and coordinated thought data sharing and communication reduction in order oscillation could be achieved. Makui et al. (2007) used a well known mathematical term; the Lyapunov exponent in their study and quantified BWE in terms of this exponent and; differently from the study of Boute et al. (2007) that importance of lead times in order smoothing, expressed the negative effect of lead times in terms of LPE. Based on the Chen et al.’s (1998) work, Makui et al. quantified and measured BWE for centralized and decentralized information cases in a two echelon SCN model and exposed the results with a simple numerical example. Authors’ stated that the Lyapunov exponent; which may use for quantification of the irregularities of non-linear system dynamics, may also be use for quantifying BWE if LPE is sensed as a factor for expanding an error term of a system.

Like Makui et al. (2007) Hwarng et.al (2008) also used Lyapunov exponents in his work for quantify system chaos in SCNs and similar to BWE discovered the “chaos-amplification”. The study; different from the previous recognized acknowledgment that points the main cause of system variability as the external unpredictable conditions, showed that exogenous factors such as demand together with related endogenous factors such as lead times and information flow may also generate chaotic behavior in SCN system. Based on this findings, Hwarng concluded that for effective management in chaotic SCN systems, the interactions between exogenous and endogenous factors have to be understood as well as the effects of various SCN factors on the system behavior for reducing system chaos and inventory variability.

Sohn et al. (2008), Wright et al. (2008), Saeed (2008), Sucky (2009) and Reiner et al. (2009) are the other researches who investigated mainly the effects of forecasting on BWE in their researches. Sohn et al.; using Monte Carlo simulation that simulates various conditions of market environment in SCN, aimed to suggest the appropriate information sharing policy together with appropriate forecasting method for multi-generation products of high-tech industry via which, customer satisfaction and net profit would be maximized considering the factors such as seasonality, supplier’s capacity and price sensitivity of multi-generation products. Thought the study does not directly related to BWE, the research area and finding set a light to forecasting methods appropriate for specific information policies in SCNs for the cases such as the environmental factors like seasonality and price sensitivity exists. Wright et al. (2008) expanded Stermans model and investigated BWE under different ordering policies and forecasting methods (Hold’s and Brown’s Method) separately and in combination. Based on the results Wright concluded that, forecasts which are made in
conjunction with appropriate ordering policy decreases BWE and showed that Holt’s or Brown’s forecasting method might provide more stability in SCN when they are combined with slow adjustment of stock levels and rapid adjustment of supply line levels.

Saeed (2008) also worked on SCN stability in terms of forecasting. He suggested that if trend forecasting is applied to SCN systems as in derivative control, remarkable performance improvements in stability could be achieved. To support this idea, he constructed a SCN model in which, a classical control mechanism is implemented and used the forecasts of stock of inventory to demonstrate the use of trend forecasting as a policy tool in SCN.

Sucky (2008) differently from previous studies, analyzed BWE taking into account the network structure of SCNs and the risk of pooling effect (which can be sensed as a special case of portfolio effect; see Ronnen, 1990). Using a simple three staged SCN, he revealed that BWE may be overestimated by assuming a reasonable SCN and risk of pooling effect could be utilized and also; like Dejonckheere et al. (2003), concluded that order-up-to systems generally generate BWE, depending on the statistical correlation of the demand data.

2.2 Fuzzy approaches to bullwhip effect
Since pioneer work of Zadeh (1965) “Fuzzy Sets” in which FL was introduced many studies have been done related to this brilliant subject. Though studies about FL are extremely high, its application to SCNs and especially to BWE is narrow. The first application of FL approach to BWE topic; due to our knowledge, appears with the works of Carlsson & Fuller (1999, 2001, 2002, 2004). Authors built a decision support system describing four BWE driving factors of Lee et al. (1997a, 1997b); i.) demand signal processing, ii.) rationing game, iii.) order batching, iv.) price variations. Using an ordering policy with imprecise orders, they showed that BWE can significantly be reduced with centralized demand information and fuzzy estimates on future sales. But the study successfully sorted out the complexity of the phenomenon using fuzzy numbers.

Wang et al. (2005, 2007) developed a fuzzy decision methodology for handling SC uncertainties and determining appropriate strategies for SC inventories. In the study, fuzzy set theory is used to model SC uncertainties and a fuzzy SC model is proposed to evaluate SC performance. Though the study does not directly related to BWE; by improving SC performance against SC uncertainties, due to the proposed inventory policy and cost reductions demand variability indirectly reduced.

Zarandi et al. (2008) proposed a fuzzy agent-based model for reduction of BWE. In the study demand data, lead times and ordering quantities are considered as fuzzy and BWE is simulated and analyzed in fuzzy environment. A genetic algorithm module added added fuzzy time series forecasting model is used to estimate the future demand and a back-propagation neural network is used for defuzzification of the output. The simulation of BWE in fuzzy environment showed that the phenomenon still exists in fuzzy domain and genetic algorithm module added time series model perform successfully.

Another important study in recent literature about BWE is made by Efendigil, Önüt and Kahraman (2008). The study provides a comprehensive analysis for the first level of SC modeled by artificial intelligence approach. In the study both neural networks and ANFIS is used for demand forecasting only in retailer level with a real-world case study. The inputs for the demand forecast are unit sales price, product quality and effect of promotions, holidays and special cases. The study showed that hybrid forecasting models perform successfully for demand forecasting in SCNs.
Balan et al. (2009) again used soft computing approach to handle BWE. With a discrete time series single input single output model (SISO) model BWE is measured and via application of soft computing BWE is reduced. This study also showed that the application of FL and ANNs in SCN provides successful result in reduction of BWE.

3. Fuzzy, neuro-fuzzy systems and fuzzy regression forecasting model

As pointed out before, since pioneer work of Zadeh (1965), FL has been successfully applied to many fields of science and engineering including SCNs. In dynamic complex nature of SCNs, demand forecasting; which sound basis for decision making process as mentioned before, is among the key activities that directly affect the performance of the system. As the demand pattern varies from system to system, determination of the appropriate forecasting model that best fits the demand pattern is a hard decision in management of SCNs. Most importantly, the usage of proper demand forecasting model that is adequate for the demand pattern is an important step for smoothing BWE in SCN systems. In this section, a brief overview about FL, NFS, FR forecasting model are encapsulated.

3.1 Fuzzy logic

On the contrary to many cases that involves human judgment, crisp (discrete) sets divide the given universe of discourse in to basic two groups; members, which are certainly belonging the set and nonmembers, which certainly are not. This delimitation which arises from their mutually exclusive structure enforces the decision maker to set a clear-cut boundary between the decision variables and alternatives. The basic difference of FL; which was introduced with the pioneer work of Zadeh; “Fuzzy Sets”, in 1965, is its capability of data processing using partial set membership functions. This characteristic; including the ability of donating intermediate values between the expressions mathematically, turn FL into a strong device for impersonating the ambiguous and uncertain linguistic knowledge. But the main advantage of fuzzy system theory is its ability “to approximate system behavior where analytic functions or numerical relations do not exist” (Ross, 2004, pg.7). Palit et al.(2005) give a basic definition of FL from mathematical perspective as a nonlinear mapping of an input feature vector into a scalar output. As fuzzy set theory became an important problem modeling and solution technique due to its ability of modeling problems quantitatively and qualitatively those involve vagueness and imprecision (Kahraman, 2006, pg.2), it has been successfully applied many disciplines such as control systems, decision making, pattern recognition, system modeling and etc. in fields of scientific researches as well as industrial and military applications.

Differently from the classical sets that can be defined by characteristic functions with crisp boundaries, fuzzy sets can be characterized by membership functions providing to express belongings with gradually smoothed boundaries (Tanaka, 1997). Let \( A \) be a set on the on universe \( X \) with the objects donated by \( x \) in the classical set theory. Then the binary characteristic function of subset \( A \) of \( X \) is defined as follow;

\[
\mu_A(x) : X \to \{0,1\}
\]
such that

$\mu_A(x) = \begin{cases} 
1 & x \in X \\
0 & x \notin X 
\end{cases}$ \quad (2)

But fuzzy sets the characteristic functions; differently from the crisp sets whose characteristic function is defined binary (i.e., 0 or 1), are defined in the interval of [0,1] (Zadeh, 1965). From this point, fuzzy set $\tilde{A}$ in the universe set $X$ with the objects $x$ and membership function $\mu_{\tilde{A}}$ is defined as follow;

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid \forall x \in X \}$$ \quad (3)

where $\mu_{\tilde{A}}(x): X \to [0,1]$.

If the fuzzy set is discrete then it can be represented as;

$$\tilde{A} = \sum_{k=1}^{n} \frac{\mu_{\tilde{A}}(x_k)}{x_k} , \forall x_k \in X , k = 1,2,\ldots,n$$ \quad (4)

And if the fuzzy set is continuous then it can be denoted as;

$$\tilde{A} = \int_{X} \frac{\mu_{\tilde{A}}(x_k)}{x_k} , \forall x_k \in X$$ \quad (5)

The two vital factors for building an appropriate fuzzy set gets through the determination of appropriate universe and membership function that fits the system to be defined. The membership functions are the main fact for fuzzy classification. The highest membership grade value 1 represents full membership while the lowest membership value 0 have the meaning that the defined object have no membership to the defined set. Frequently used membership functions in practice are triangular, trapezoidal, Gaussian, sigmoidal and bell curve (the names are given according to the shapes of the functions). To give an example, trapezoidal membership function is specified by parameters $\{a, b, c, d\}$ as:

$$\text{trapezoid}(x; a, b, c, d) = \begin{cases} 
\frac{x-a}{b-a} & ; \quad a \leq x \leq b \\
\frac{d-x}{d-c} & ; \quad b \leq x \leq c \\
0 & ; \quad x \geq d \text{ or } x \leq a 
\end{cases}$$ \quad (6)

where $a < b \leq c < d$ denoting the $x$ coordinates of the trapezoidal membership function. The function reduces to triangular membership function when parameter $b$ and $c$ are equal. Similar to triangular function, control of the function can be maintained by adjusting parameters.

As fuzzy set theory provides a way to represent vagueness in linguistics in a mathematical manner, fuzzy if-then rules or the if-then rule-based form can simply be defined as schemes for capturing relative and imprecise natured knowledge; just like human knowledge. These provide a way of expressing knowledge in way of elastic nature language expressions in the general form of “IF X THEN Y”. Here X is the antecedent (premise) and Y is the consequent (conclusion) (Ross, 2004, pg.148) with the linguistic values defined by fuzzy sets. The
conclusion described by consequent is come into being when premise described by the antecedent is true (i.e. when input fulfills the rule). The consequent of fuzzy rule are generally classified into three categories as crisp, fuzzy and functional consequent (Yen et al., 1999).

• Crisp consequent: Let z be a non-fuzzy numeric value or symbolic value then the crisp consequent can be expressed in the form: “IF...THEN y=z”;

• Fuzzy consequent: Let A be a fuzzy set then fuzzy consequent can be expressed in the form: “IF...THEN y=Å”;

• Functional consequent: Let \( z^i \) be a constant for \( i=0,1,2,...,n \) then functional consequent can be expressed in the form: “IF \( x_1 \) is \( A_1 \) AND \( x_2 \) is \( A_2 \) AND… \( x_n \) is \( A_n \) THEN \( y=0 \sum_{i=1}^{n} z^i x_i \)”.

The antecedent of the rule may use three logical connectives which are “AND” the conjunction, “O” the disjunction and “NOT” the negation.

Zadeh (1965) adduced that fuzzy systems can be used to illustrate the human reasoning process as human understanding and reasoning take place in the fuzzy environment in general. Taking this prevision into account, fuzzy (or approximate) reasoning can simply be defined as a path for deducing conclusions from incontestable knowledge and fuzzy rules. Defining input variables and required output together with the function that will be used for transferring crisp domain to fuzzy domain; the required fuzzy reasoning procedure can be achieved.

Fuzzy if-then rules and fuzzy reasoning compose bases for the most popular and cardinal computing tool called fuzzy inference systems (FIS) which, as general, perform mapping from a given input knowledge to desired output using fuzzy theory. This popular fuzzy set theory based tool have been successfully applied to many military and civilian areas of including decision analysis, forecasting, pattern recognition, system control, inventory management, logistic systems, operations management and so on. FIS basically consist of five subcomponents (Jang, 1993); a rule base (covers fuzzy rules), a database (portrays the membership functions of the selected fuzzy rules in the rule base), a decision making unit (performs inference on selected fuzzy rules), fuzzification inference and defuzzification inference. The first two subcomponents generally referred knowledge base and the last three are referred to as reasoning mechanism (which derives the output or conclusion).

The input (corresponding to system state variables) of FIS; either fuzzy or crisp, generates generally fuzzy output (corresponding to signal). Fuzzification is the comparison of the crisp input with the membership functions of the premise part to derive the membership values. If the required output value is crisp, then the fuzzy output is to be defuzzified. Ross (2004, pg.99) define this process as “the conversion of a fuzzy quantity to a precise quantity”. For basic concepts of fuzzy sets and related basic definitions see Bellman et al. (1970), Tanaka (1997 pg.5-44), Klir et al. (1995) and Ross (2004, pg.34-44).

3.2 Neuro-fuzzy systems

NFs; which also known as hybrid intelligent systems, can simply be defined as the combination of two complementary technologies: Artificial neural networks (ANNs) and FL. This combined system has the abilities of deducing knowledge from given rules (which come from the ability of FIS), learning, generalization, adaptation and parallelism (which come from the abilities of ANN). So these hybrid systems cover the frailty of both FL (i.e., no ability of learning, difficulties in parameter selection and building appropriate membership function, etc.) and ANN (i.e., black box, difficulties in extracting knowledge, etc.) and became a robust technology using both systems powerful abilities.
Simply, ANNs are mathematical information processing systems which are constituted based on the functioning principles human brains in which neurons in biological neural systems correspond to nodes and synapses correspond to weighted links in ANN (Maduko, 2007). Hecht-Nielsen (1990) described a neural network as; “a parallel, distributed information processing structure consisting of processing elements (which can possess a local memory and can carry out localized information processing operations) interconnected via unidirectional signal channels called connections”. As ANNs are computational models constituted of many interconnected neurons, the basic processing element of the ANNs are neurons and their way of interconnection also effect the ANN structure in addition to learning algorithm type, activation functions and number of layers. Using logical connections (weighted links) neurons in ANNs get the input from adjacent neurons with the input strength effected by the weight and; using the weighted input broadcasted from the adjacent neurons produce an output with the help of an activation function and broadcast the activation as an input; only one at a time, to other neurons (Fausset, 1994 pg. 3-25). In the input layer neurons receive input that is given to the system, contrarily the output layer neurons broadcast the ANN output to external environment while neurons in the hidden layers act as a black box providing links for the relation between the input and output (Choy et al., 2003a, 2003b).

The usage of hybrid NFS is rapidly increasing in many areas both civilian and military domain such as process controls, design, engineering applications, forecasting, modular integrated combat control systems, medical diagnosis, production planning and etc. This multilayer fuzzy inference integrated networks use neural networks to adjust membership functions of the fuzzy systems. This structure provides automation for designing and adjustment of membership functions improving desired output by extracting fuzzy rules from the input data with the trainable learning ability of ANNs and also overcomes the black box structure (i.e., difficulties of in understanding and explaining the way it deducts) of learning process of ANNs. Many studies have been made using different architectures of these hybrid systems, but among those architectures FL based neurons (Pedrycz, 1995); neuro-fuzzy adaptive models (Brown et al., 1994) and ANNs with fuzzy weights (Buckley et al., 1994) can be considered as noteworthy ones. A general NFS is constituted of three to five layers. The first layer represents the input variable, second layer are consists of input membership functions, the third layer or the hidden layers represents the fuzzy rules, the fourth and fifth layers represent the output membership function and output respectively (Jang, 1993; Wang, 1994).

3.2.1 Adaptive neuro-fuzzy inference systems
This system is the implementation of FIS to adaptive networks for developing fuzzy rules with suitable membership functions to have required inputs and outputs (Jang, 1993). In previous sections basic information about fuzzy reasoning and FIS was given. An adaptive network is a feed-forward multi-layer ANN with; partially or completely, adaptive nodes in which the outputs are predicated on the parameters of the adaptive nodes and the adjustment of parameters due to error term is specified by the learning rules (the other node type is named as fix node) (Jang, 1993). Generally learning type in adaptive ANFIS is hybrid learning. This learning model is appropriate for the systems having unsteady nature like SCNs. Jang defined this learning type as the learning that involves parameter updating after each data is given to the system. Due to its flexibility coming from the adaptive networks, ANFIS can design according to the system that it will be used. Using different fuzzy inference system with different IF-THEN rules and different membership functions and also with different network structures distinct types of ANFIS can be derived and extended (Jang
et al., 1997). That is way this powerful system has many field of application. Here, ANFIS is used for demand decision process in a SCN simulation.

3.3 Fuzzy regression forecasting model

Linear regression model that explores the relation between response or dependent variable \( y \) and independent or explanatory variable \( x \), is a successful and commonly used statistical technique use in many fields of science and engineering problems. In linear regression model \( y \) is a function of independent variables and can be written as:

\[
Y = f(x, a) = \theta X = a_0 + a_1 x_1 + a_2 x_2 + ... + a_n x_n
\]

where \( \theta \) is the vector of coefficients which presents the degree of contribution of each variable to output and \( X \) is the matrix of independent. The model is probabilistic as the differences between the observed and estimated values (i.e., output) is assumed to be due to observation errors considering the differences a random variable, and the confidence of estimate is represented by the probability that the estimated values are established between the upper and lower bounds. For the input data in linear regression model, unobserved error term is mutually independent and identically distributed; that is, the application of linear regression model is suitable for the systems in which the data sets observed are distributed according to a statistical model (Wang, 2000; Ross, 2004). But generally, fitting the demand pattern of systems like real SCNs to a specific statistical distribution hard to achieve. The FR model introduced by Tanaka et al. (1982, 1988); in which “deviations reflect the vagueness of the system structure expressed by the fuzzy parameters of the regression model” (i.e. possibilistic), relax and made the model suitable for the declared demand patterns. The model is explained as follow (Ross, 2004).

The fuzzy linear function in the model basically can be formulated as:

\[
\hat{Y} = (c_0, s_0) + (c_1, s_1) x_1 + (c_2, s_2) x_2 + ... + (c_n, s_n) x_n
\]

where \( c_t \) is the central value and \( s_t \) is the spread value, of the \( t \) th fuzzy coefficient; \( \hat{A}_t = (c_t, s_t) \), usually presented as a symmetrical triangular fuzzy number (STFN) with the membership function:

\[
\mu_{\hat{A}_t}(a_t) = \begin{cases} 
1 - \frac{|c_t - a_t|}{s_t} & ; \quad c_t - s_t \leq a_t \leq c_t + s_t \\
0 & ; \quad \text{otherwise} 
\end{cases} \quad \forall t = 1, 2, ..., n
\]

As can be seen from equations the coefficients of the FR model are represented by fuzzy functions. And this representation is fact that relaxes the crisp linear regression model. The usage of triangular membership function for the fuzzy coefficients allows the usage of linear programming for obtaining minimum fuzziness for the output values of the FR model (different membership function require alternative approaches, see Ross, 2004 pg.556). The membership function of output parameter is expressed as follow.

\[
\mu_f(y) = \max \left( \min \left[ \mu_{\hat{A}_t}(a_t) \right] \right) ; \quad \{ a \mid y = f(x, a) \} \neq 0 \\
0 ; \quad \text{otherwise}
\]
And using the membership function expressed we can rewrite \( \mu_{Y}(y) \) as:

\[
\mu_{Y}(y) = \begin{cases} 
1 - \frac{y - \sum_{i=1}^{n} c_{i}x_{i}}{\sum_{i=1}^{n} c_{i}} & ; \ x_{i} \neq 0 \\
1 & ; \ x_{i} = 0, \ y = 0 \\
0 & ; \ x_{i} = 0, \ y \neq 0 
\end{cases}
\]

(11)

The data for the model can be either fuzzy or crisp. In this study input data used to obtain future demand forecast is crisp. So, the following parts of the model express the computation of output for non-fuzzy data. As the aim is to obtain minimum fuzziness for the output parameter, the following linear programming formulation which minimizes the spread of the output parameter, minimum fuzziness for the output can be achieved.

\[
Z = \text{Min} \left\{ ms_{0} - (1 - h) \sum_{i=1}^{m} \sum_{t=0}^{n} s_{t}x_{it} \right\} \\
\text{St} \\
\sum_{i=1}^{n} c_{i}x_{it} - (1 - h) \sum_{t=0}^{n} s_{t}x_{it} \leq y_{i} \\
\sum_{i=1}^{n} c_{i}x_{it} + (1 - h) \sum_{t=0}^{n} s_{t}x_{it} \geq y_{i}
\]

(12)

where \( x_{0i} = 1, \ \forall t = 1, 2, ..., n, \ \forall i = 1, 2, ..., m \) and \( h \in [0, 1] \) which is specified by the designer of the model, defines the degree of belongings as;

\[
\mu_{Y}(y_{i}) \geq h , \ i = 1, 2, ..., m
\]

(13)

The value \( h \) conditions the wide fuzzy output interval. The fuzzy forecast value for period \( t \) expressed will than be computed as:

\[
\hat{F}_{t} = (c_{0}, s_{0}) + (c_{1}, s_{1})x_{1} + (c_{2}, s_{2})x_{2} + ... + (c_{n}, s_{n})x_{n}
\]

(14)

Notice that, If the forecast value is need to be crisp then a defuzzification method must be used.

4. Application of fuzzy hybrid model on supply chain networks

In this chapter, a near beer distribution game extended with ANFIS decision making process and FR forecasting model; which is improved from the base beer game of Sterman (1984, 1989) and its revised version of Paik’s (2003), is used to simulate a two stage SCN for evaluating the impacts of proposed system under relatively medium demand variation which is determined with the demand standard deviation. For each comparison between the base and proposed models, BWE is quantified for each stage as a ratio of standard deviations of subsequent stages to reflect the amount of variability.
The most important issue for a realistic SCN model is modeling the decision making process as the whole system mainly depends on the demand decision of the phases. Parameters and decision variables have to be chosen carefully considering all complex activities included in SCN. In addition, the information that will be used in the decision process has to be appropriate for providing decision accuracy which makes model construction even more difficult. So, for modeling these complex systems same assumptions have to be made for providing simplification which, unfortunately may cause falling short of the reality if those assumptions suggested by not taking consideration of real SCN nature.

Fig. 1. Proposed simulation model structure

For analyzing the system like SCN (due to the purpose of the analysis), simulation approach is the most common choice. Among the simulation models, the most preferred one is the “Beer Game” (or “MIT Beer Game”) model which was founded in 1960s at the Sloan School of Management (Strozzi et al., 2007) and then became well known after Sterman in 1989 (1989a, 1989c). Here, a near beer game simulation model improved from the base beer game model of Sterman and its revised version of Paik’s is used to simulate SCN.

4.1 Supply chain network simulation

The beer distribution or MIT beer game is a role-playing simulation model that represents the beer production and distribution system in a simple SCN which is widely used as a teaching tool for pointing out SCN structure, concept and dynamics. The goal of the game is to govern each stage of the chain (generally consists of two or four stages in which stages simulates the retailer, the wholesaler, the distributor and factor (or producer) respectively) maintaining appropriate inventory levels to meet the desired demand of the predecessor stage and to minimize the total cost avoiding stock-outs taking by supply line into consideration under limited information flow. Due to the rich simulation environment including time delays, cost items, feedbacks and decision rules that successfully represents
the actual decision making process in stock management problems of the real business environment; general characteristic of the beer game fairly illustrates the nature of real world SCNs (Paik, 2003).

The game begins with the demand orders placed from the customer to retailer. Retailer tries to meet the demand from its own inventory upon the availability of the stocks. If demand exceeds the inventory level, retailer place order to wholesaler. Also for maintaining appropriate inventory level for the future customer demand, the ordering decision of the retailer must also comprehend customer demand rate for the upcoming periods. And in the same manner the demand and distribution processes go on through the SCN system of the game till the factory stage where beers produced to meet the demand of distributor. So, in each stage except factory, the participants of the game receives demand orders from downstream stage, tries to meet the demand from its own inventory (actual inventory), ships orders to downstream stage, receives shipments from upstream stage and places orders to upstream stage by taking, future demand from downstream stage, desired inventory level together with shipment and orders that have been placed that have been placed but not received yet into consideration. The only difference in factory (which is the final stage of the game) is that the orders placed from the wholesaler are attempt to be met from either factory inventory or by production made in factory.

Limited information availability, time delays in the information flow and shipments are other important characteristics of the game which increase the complexity of the game and make the game more realistic for reflecting real world applications.

The ordering/production decision process rule in each phase of the model is simple but effective as it takes almost all factors reflecting behaviors of SCN. These factors are (Sterman, 1989; Paik, 2003):

- Current demand: current received orders
- Actual inventory level: current inventory;
- Desired inventory level: adjustment of inventory to expected forthcoming demand for a specified time period which is usually constant and determined by the designer (the adjustment parameter can also be referred to as safety stock constant or safety constant only),
- Actual pipeline orders (actual supply line): total sum of outstanding orders plus shipments in transit,
- Desired pipeline orders (desired supply line): desired rate of outstanding orders and shipments in transit,
- Demand forecast: the expected demand for the forthcoming period; i.e., expected losses.

The decision rule in period \( t \) can be formulated as follow (Sterman, 1989; Paik, 2003):

\[
O_t = \text{Max} \left( 0, \left[ F_t + IC_t + SlC_t \right] \right)
\]

where \( O_t \) is the order quantity, \( F_t \) is the forecast value, \( IC_t \) is the correction of inventory and \( SlC_t \) is the correction of supply line formulated as follow:

\[
IC_t = \theta_i (DInv_t - Inv_t)
\]

\[
SlC_t = \theta_i (SD_t - SA_t)
\]
where $Dinv_i$ is the desired inventory level, $Inv_i$ is the current (actual) inventory level, $SD_i$ is the desired supply line, $SA_i$ is the current (actual) supply line and $\theta_i$, $\theta_{SI}$ are the adjustment parameters of inventory and supply line respectively. Both $\theta_i$ and $\theta_{SI}$ determines “how much emphasis is placed on the discrepancy” between the desired and actual values inventory and supply line (Paik, 2003).

The forecast value used in the decision rule of the game is computed using simple exponential smoothing forecasting model as:

$$F_i = \alpha OI_{i-1} + (1 - \alpha)F_{i-1} \quad 0 \leq \alpha \leq 1$$  \hspace{1cm} (18)

where $OI_{i-1}$ is the actual value of the orders received (incoming orders) in period $t-1$ and $\alpha$ represents the smoothing constant.

The overall decision rule of the model can be rewritten by defining a disturbance term $\varepsilon$ to each period and new parameter $\beta$ as follow;

$$O_i = \text{Max}\left(0, [F_i + \theta_i (AI' - Inv_i - \beta SA_i)] + \varepsilon_i \right)$$  \hspace{1cm} (19)

where $\beta = \frac{\theta_{SI}}{\theta_i}$ and $AI' = DInv_i + \beta SD_i$. For more detail see Sterman (1989), Paik (2003) and Strozzi et al. (2007).

### 4.1.1 Proposed model

As stated before, the main objective here is to analyze the response of BWE to the proposed ANFIS based demand decision process in which the appropriate forecast values that are computed with FR forecasting model in addition to the identical decision variables and parameters used in the base SCN simulation model. The general ANFIS and the used forecasting models architectures and structures have been discussed in previous sections.

The possibilistic FR model is used to predict the appropriate upcoming demand value that will be used in the ANFIS based demand decision process. The demand data structure used in the model (input values) is crisp and the output forecast value is also defuzzified. The fuzzy coefficients of FR model ($A_i$) chosen for the forecasting model are STFN; hereby, the linear programming is used for obtaining minimum fuzziness for the output values of the FR model via minimizing the spread of the output parameter (i.e., the forecast value)(Ross, 2004). As the value $h \in [0, 1]$ (which defines the degree of belongings) conditions the wide fuzzy output interval, similar to the most of previous studies the value of $h$ is taken as 0.5 (Tanaka et.al, 1982, 1988; Wang et.al, 1997; Ross, 2004) As the linear programming formulation states two constraints for each data set, there are $2m$ constraints for each data set. For example if $m = 200$ for the time period $t$, then the simulation model have to solve a 400 constrained linear programming model in each stage to determine the forecast value of the upcoming demand.

Differently from the previous SCN simulations which use beer game to simulate a realistic SCN (with “anchoring and adjustment” heuristic), the proposed model contains an ANFIS based decision process in each phase of SCN to determine order quantities (or, the quantity of production in factory stage) using the forecast values gathered from the selected
forecasting model (i.e., FR) together with inventory and pipeline information which also are the same input used in the base model. Matlab “Fuzzy Logic Tool Box” is used for building the ANFIS structures and via determined input values the same tool box is also used for the solutions. (see Fuzzy Logic Toolbox Users Guide; The Math Works Inc.; 2001). After the performed trials of the simulation, the hybrid method; which is a combination of back propagation and least square estimation (the sum of the squared errors between the input and output), is selected and used for membership function parameter estimation of FIS (Matlab, 2001). The error tolerance is set to zero and ANFIS is trained for each selected forecasting model (fuzzy and crisp) in stage. Data characteristic are chosen as to illustrate a relatively medium variation in demand with demand mean $\mu_d = 50$ and demand standard deviations $10 \leq \sigma_d < 15$.

Among the defuzzification methods, the centroid method is used to defuzzify the output forecast value of the FR model as to obtain crisp demand forecast information for the ANFIS based decision making procedure.

Results gathered from the all simulation runs of the base model (crisp) (which are used for making comparison with the results of the proposed model); for the demand pattern concerned (relatively medium) considering all parameter combinations with and without predefined capacity limit for factory, shows that parameter combination $\alpha = 0.36$, $\beta = 0.34$, $\theta = 0.26$ exposes the minimum $BWE_{TOTAL}$ values where;

$$BWE_{TOTAL} = \frac{BWE_{Cap} + BWE_{IncCap}}{2}$$

In the light of these results, proposed model is evaluated and compared with the base model for the mentioned best parameter combination with and without predefined capacity limit.

### 4.2 Simulation results

Results gathered from the simulation runs of proposed and base model with the best parameter combination with and without capacity limits for factory stage are illustrated in table 2.

<table>
<thead>
<tr>
<th>Parameter Combination</th>
<th>Medium Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.36$, $\beta = 0.24$, $\theta = 0.26$</td>
<td>$BWE_{TOTAL}$</td>
</tr>
<tr>
<td></td>
<td>Percentage of Reduction (%)</td>
</tr>
</tbody>
</table>

Table 2. Results of the base and proposed model

Results show that BWE is reduced approximately 46% using proposed FR and ANFIS based decision approach for demand pattern concerned. The following figures illustrates that proposed model captures rapidly the pattern of the customer orders.
Fig. 2. Order/production decisions with factory capacity of the base and proposed model

Fig. 3. Order/production decisions without factory capacity of the base and proposed model
5. Conclusions and recommendations
SCNs are multi stage complex dynamical systems consist of various involved organizations performing different processes and activities in each and consequent stages which are connected through upstream and downstream linkages to produce value in the form of products and services (Christopher, 1994). Demand forecasting and decision making processes are among the key activities which directly affect the performance of this complex systems. As demand pattern varies due to the field of activity and architecture of system, appropriate forecasting and order decision model determination is a complicated work in SCNs. The variability of the demand information between the stages of SCN and the increase in this variability as the demand data moves upstream from the customer to the consequent stages (i.e., BWE) is a major problem that negatively influencing the stability of SCNs triggering several system defects which directly affects the total performance of SCN.

In this chapter a basic literature review is carried out a about the BWE and fuzzy related studies on BWE. A hybrid fuzzy approach made up of an ANFIS based demand decision process together with FR forecasting model is introduced and analyzes are made to expose the response of this phenomenon to the proposed approach under demand with relatively medium variation in a two stage SCN simulation. A brief overview about the FL, NFS and FR forecasting models is also given.

Results gathered from the simulation runs of the base and proposed models showed that the proposed model easily monitored the demand pattern and provided remarkable decreases in demand variability through the SCN.

Each study with its own purpose contains certain limitations. As Paik pointed out “No research method can guarantee flawless study” (2003, pg.16). Due to the dynamic, chaotic and complex characteristics of SCNs, developing a simulation model that can successfully reflect these specialties is complicated. So future studies can be made by redesigning models using ANNs specially developed for the system using fuzzy cost and lead-time values.

6. References
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