Modelling and control of manipulators with flexible links working on land and underwater environments
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SUMMARY
In this study, modelling and control of a two-link robot manipulator whose first link is rigid and the second one is flexible is considered for both land and underwater conditions. Governing equations of the systems are derived from Hamilton’s Principle and differential eigenvalue problem. A computer program is developed to solve non-linear ordinary differential equations defining the system dynamics by using Runge–Kutta algorithm. The response of the system is evaluated and compared by applying classical control methods; proportional control and proportional + derivative (PD) control and an intelligent technique; integral augmented fuzzy control method. Modelling of drag torques applied to the manipulators moving horizontally under the water is presented. The study confirmed the success of the proposed integral augmented fuzzy control laws as well as classical control methods to drive flexible robots in a wide range of working envelope without overshoot compared to the classical controls.

KEYWORDS: Flexible manipulators; Underwater robots; Fuzzy + Integral control.

1. Introduction
From 1980s till now, the modelling and control of lightweight, flexible-link manipulators have been challenging research topics with the objective of improving the robot performance. In comparison to the conventional heavy and bulky robots, flexible-link manipulators have the real advantages of larger work volume, higher operation speed, greater payload-to-manipulator-weight ratio, smaller actuators, lower energy consumption, better manoeuvrability and better transportability. In short, less effort, faster response, low cost, low inertia power, safer working moves flexible arms in front of the rigid ones, this stimulated the use of flexible structures for space and underwater systems. However, application of flexible links cause elastic deformation and vibration, and manipulator carrying different payloads becomes more difficult to control.

Previous researches in the area of flexible manipulators especially concentrated with the material properties in manipulator design,2,3 modelling and analysis of structural deflections and vibrations in the manipulator due to link and joint flexibilities,4–6 development of control schemes for reducing the pre-mentioned problems7–9 and design of compliant devices for performing specific assembly tasks requiring flexibility at the end-effectors.

Studies on the dynamic modelling and control strategies have been carried out by many researchers and some well-documented reviews are present in the literature. A previous review in this work was carried out by Gaultier10 in 1989. Benosman and Vey11 carried out a partial survey on the control aspect of flexible multi-link manipulators. And Dwivedy and Eberhard12 showed the dynamic analysis of flexible manipulators.

The mathematical models of the manipulators are generally derived from energy principle. While for a simple rigid manipulator, the rigid arms store kinetic energy by virtue of their moving inertia and store potential energy by virtue of their position in the gravitational field. On the other hand, the flexible arms store potential energy by virtue of the deflections of its links, joints or drives. The pioneer work is of Book’s4–5 who developed non-linear equations of motion for flexible manipulator arms consisting of rotary joints that connect pairs of flexible links. In most applications Euler–Bernoulli beam is applied but to get the exact solution of such systems is not easy to apply practically and the infinite dimensional model imposes severe constraints on the design of controllers as well. So, other mathematical methods such as finite elements,13 assumed modes14–6 or lumped parameter methods14–16 are also suggested.

In the control of flexible manipulators, the control scheme differs to four subcategories according to the point where the control is applied. Some studied the end-effector regulation problem, some tried the end-effector to rest motion in a desired fixed time, some studied joint-trajectory tracking and finally some concentrated on end-effector trajectory tracking.11 From the very basic proportional–derivative and proportional–integral–derivative or sliding mode control or direct strain feedback control to novel methods adaptive control, self-tuning control, feed-forward control, fuzzy control and neural networks methods were widely used to maintain better performance. The researchers, who derived the dynamic equations according to Hamilton Principle, gathered the quasi-static equations and designed hybrid-position-force controller.17 Although new control methods had evolved, the classical PID controllers are still widely used in all domains. Dogan and Iftar, have studied the modelling and control of a manipulator whose first link rigid
and second one flexible and carrying a payload at the tip. They have also designed PD and PD like controllers with a double scaled controller that is based on a single perturbation method.  

For the complex non-linear systems, the dynamic model of the system is not always possible to obtain, in that case, fuzzy control is a suitable tool which does not require the system’s dynamic parameters. Scientists worked on different fuzzy logic algorithms for the control of flexible manipulators. Classical and novel control methods are used together with fuzzy control in order to improve manipulator systems performance and behaviour in various environments.  

As there exist a wide literature on land manipulators with flexible links, few have done on underwater flexible manipulators. Since most of the deep undersea cannot be reached by human divers and because of inconvenient, dangerous deep-sea environments, the use of underwater manipulators has become vital tools for underwater remotely operated vehicle (ROV) operations. An underwater manipulator could be used to inspect and maintain the components of nuclear power plants because of reducing radiation exposure to human operators. When looked to the studies in this area, Farbrother and Staycey have studied the hydrodynamic forces acting on the arms mounted to the underwater vehicles and have showed their effects with robust control method. Rivera and Hinchev have stated that a manipulator under the sea is subjected to hydrodynamic loads such as drag and hydrostatic forces due to the current and way of water under the sea. Goheen studied the modelling methods for underwater vehicle dynamics. Scientists had studied different control methods, fuzzy control and integral augmented fuzzy control. This augmented control scheme ensures carrying different payloads effectively in its working space. The research on underwater manipulators carried out till now have totally used rigid links. Therefore, the main objective of this paper falls in two categories; first, to study and give the dynamic characteristics of a two link land and underwater manipulator whose first link is rigid and the second one is flexible, and second, to compare the control performances of the manipulators for both environments using classical control methods, fuzzy control and integral augmented fuzzy control. This augmented control scheme ensures carrying different payloads effectively in its working space. The manipulator addressed in this study, is considered moving in horizontal plane in both environments.  

This paper is organised as follows. In Section 2, dynamic equations of motion for a two link manipulator whose first link is rigid and the second one is flexible are developed for a land robot carrying different payloads. In Section 3, the dynamic equations for a two-link underwater manipulator, whose first link is rigid, and the second one, whose link is flexible, are derived. Section 4 presents a classical P controller, fuzzy controller and an integral augmented fuzzy controller based on the tip deflection feedback for both environments, and their performance is compared. Finally, some conclusions are outlined in Section 5.

Fig. 1. Schematic figure of two link rigid-flexible manipulator.

2. Kinematics and Dynamics

2.1. Modelling the two-link land manipulator

The schematic representation of the two-link manipulator whose first link is rigid and the second one is flexible is given in Fig. 1. The rigid link is clamped on a motor at point 0 and another motor is attached to the tip of the first link. The flexible link is clamped on the motor at one end and carries a payload at the other end. \( I_{b1} \) and \( I_{b2} \) denote the moment of inertia of the rotor of each motor, and \( \beta_1 \) and \( \beta_2 \) denote viscous friction coefficient of each joint. The torque developed by each motor is symbolized with \( \tau \) and the transverse displacement of the flexible link from its rigid-body shape at time \( t \) and at a spatial point \( x \) is symbolized with \( z(x, t) \).

Hamilton’s Principle is used to obtain the equation of motion and the corresponding boundary conditions.

\[
\int_{t_1}^{t_2} \left( \delta T - \delta V + \delta W_{nc} \right) \, dt = 0, \quad \delta r_i = 0. \tag{1}
\]

System’s kinetic energy, potential energy and the work done by non-conservative forces are added to Hamilton’s equation:

\[
- \int_{t_1}^{t_2} \left\{ \int_0^{L_2} \left[ \frac{\partial}{\partial \dot{\theta}} \left[ L_1^2 \dot{\theta} + x^2 (\theta + \phi) \right] + L_1 x (2 \dot{\theta} + \dot{\phi}) \cos(\phi) - L_1 (2 \dot{\theta} + \phi) z(x, t) \sin(\phi) \right. \right. \\
+ L_1 z_t (x, t) \cos(\phi) + x z_t (x, t) \right] dx + \frac{\partial}{\partial t} \left[ (I_{b1} + I_{b2}) \dot{\theta} \right. \\
\left. + I_{b2} \phi + \frac{1}{3} M_1 L_1^2 \dot{\theta} + m_a L_1 \dot{\theta} \right] - (\tau_1 - \beta_1 \dot{\theta}) \left. \right) \, dt \\
+ \left( \int_0^{L_2} \left[ \frac{\partial}{\partial x} \left[ \int_0^{L_2} \left[ \frac{\partial}{\partial x} \left[ \int_0^{L_2} \left[ \frac{\partial}{\partial \theta} \right] \right] \right] \right] \right) \, dx \\
+ \left( \int_0^{L_2} \left[ \frac{\partial}{\partial \theta} \right] \left[ \left( \frac{\partial}{\partial \theta} \right) \right] \right) \, dx \\
+ \left[ \frac{\partial}{\partial \theta} \right] \left[ \left( \frac{\partial}{\partial \theta} \right) \right] \, dx \\
\right)
\]
$$+ L_1 \dot{\theta}(\varphi + \psi)z(x, t) \cos(\varphi) + L_1 \dot{\psi}z(x, t) \sin(\varphi)] \, dx$$
$$\quad \frac{\partial}{\partial t} [I_h (\varphi + \psi)] - (\tau_2 - \beta_2 \psi) \, \varphi$$
$$\quad + \int_0^{L_2} \rho_2 \left[ L_1 \dot{\theta} \cos(\varphi) + x(\dot{\varphi} + \psi) + z(x, t) \right] \, dx$$
$$\quad + \int_0^{L_2} \rho_2 [L_1 \dot{\theta}(\varphi + \psi) \sin(\varphi)] \, dx$$
$$+ \int_0^{L_2} \frac{\partial^2}{\partial x^2} [E I_a z_{xx}(x, t)] \, dz(x, t)$$
$$\quad + \left( m_f \frac{\partial}{\partial t} [L_1 \dot{\theta} \cos(\varphi) + L_2 (\varphi + \psi) + z(L_2, t)] \right)$$
$$+ (E I_a z_{xxx}(L_2, t) \frac{\partial}{\partial z}(L_2, t) + E I_a z_{xxx}(0, t) \frac{\partial}{\partial z}(0, t)$$
$$\quad + E I_a z_{xxxx}(L_2, t) \frac{\partial}{\partial z}(L_2, t)$$
$$\quad - E I_a z_{xxx}(0, t) \frac{\partial}{\partial z}(0, t) \right) \, dz = 0.$$  \hspace{1cm} (2)

Here \( \dot{\theta}, \varphi, z(x, t) \) and \( z_x(L_2, t) \) are the independent variables of the system. Since the difference of \( z(x, t) \) and \( z_x(t) \) at the intersection point of the first link and the flexible link is zero, \( \delta z(0, t) = 0 \) and \( \delta z_x(0, t) = 0 \). Then, the factors of \( \delta \theta, \delta \varphi, \delta z(x, t) \) and \( \delta z_x(L_2, t) \) should be zero in order to satisfy the Eq. (2). If the factor of \( \delta z(x, t) \) in Eq. (2) is zero, then by deriving the partial derivations in this equation, the Eq. (3) which symbolizes the dynamics of flexible arm is obtained below.

$$z_{tt}(x, t) + \frac{E I_a}{\rho_2} z_{xxxx}(x, t) = -x(\dot{\theta} + \dot{\varphi}) - L_1 \cos(\varphi) \ddot{\theta}$$
$$\quad - L_1 \theta^2 \sin(\varphi).$$  \hspace{1cm} (3)

First three boundary conditions are given in Eq. (4).

$$z(0, t) = z_x(0, t) = 0, \quad z_x(L_2, t) = 0.$$  \hspace{1cm} (4)

Since the tip of the flexible arm is free, the variation of \( z_s(L_2, t) \) is also free, then the fourth boundary condition of flexible arm at \( x = L_2 \) is obtained in Eq. (5) as

$$z_{xx}(L_2, t) = \frac{m_p}{E I_a} z_{tt}(L_2, t) + \frac{m_p}{E I_a} \left[ L_1 \ddot{\theta} \cos \varphi - L_1 \dot{\theta} \dot{\varphi} \right]$$
$$\quad \times \sin \varphi + (\dot{\theta} + \dot{\varphi}) \dot{L}_2 + L_1 \dot{\theta} \sin(\dot{\varphi} + \psi) \right].$$  \hspace{1cm} (5)

To express the behaviour of the flexible link, partial differential equation of flexible arm dynamics is suppressed with an infinite ordinary differential equation by using the differential eigenvalue problem. The parameter \( z(x, t) \), which represents the displacement of flexible link, can be expressed in the interval \( (0 \leq x \leq L_2) \) as the infinite sum of \( \phi_i(x) \), representing the mode shapes of the flexible link, and \( \eta_i(t) \), representing the displacement of each point on the flexible link due to time:

$$z(x, t) = \sum_{i=1}^{\infty} \phi_i(x) \eta_i(t).$$  \hspace{1cm} (6)

Finally, Eq. (2), addressing the dynamics of flexible link, can be written as below:

$$a \ddot{\eta}(t) + a \Lambda \eta(t) + (\dot{\theta} + \dot{\varphi}) \gamma^a + [(L_1 \cos(\varphi)) \ddot{\theta}$$
$$\quad + L_1 \theta^2 \sin(\varphi)] \gamma^b = 0.$$  \hspace{1cm} (7)

Here \( a \) and \( \gamma \) shows the symbolic representations of integrations in the deriving equations and \( \Lambda \) is a constant in the form of an infinite matrix. They are given in the Appendix. The Eq. (6) is given below in vector–matrix form with Eq. (8) for the two mode shapes of the flexible link. The expressions of matrix entries are also given in the Appendix:

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \dot{\varphi} \\ \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix}.$$  \hspace{1cm} (8)

2.2. Modelling of two link underwater manipulator

There are four different forces acting on the manipulator under the water; buoyant force, drag force, lift force and the gravity force. The drag and the lift forces are defined according to the moving object’s velocity vector:

$$F_D = \frac{1}{2} C_D \rho AV^2,$$  \hspace{1cm} (9)

$$F_L = \frac{1}{2} C_L \rho AV^2.$$  \hspace{1cm} (10)

The values of drag and lift forces depend on the geometry of the object. The value of lift force at a cylindrical surface moving horizontally with a very low speed, that can never produce a turbulence, is too small than that of the drag force. Drag force acts in the horizontal plane which is in the opposite direction to the velocity vector. On the other hand, lift force is on the outer way of horizontal plane but perpendicular to the velocity vector. Since the movement occurs in the horizontal plane, the term of the lift force does not appear in the dynamic equations. Moreover, both the gravity and the buoyant forces are not included in the equations; they are opposite to each other and outward direction from the horizontal plane. Then, only \( M_D \) drag torque contribution comes from underwater environment, which is formed because of \( F_D \) drag force. This torque is developed on the opposite direction of the links angular velocity:

$$M_D = \frac{1}{2} C_D \rho h \int_0^L x V_s^2 \, dx.$$  \hspace{1cm} (11)

Equation (2) was the one defining the mathematical model of two-link land manipulator. By adding the hydrodynamic
torque to this equation, the system is adapted to two link underwater manipulator, which is given in Eq. (12):

\[
- \int_{t_1}^{t_2} \left\{ \int_0^{L_2} \left[ \rho_2 + m_p \delta_{\theta}(x - L_2) \right] \frac{\partial}{\partial t} \left[ L_1^2 \dot{\theta} + x^2(\dot{\theta} + \dot{\varphi}) + L_1 z_t(x, t) \cos(\varphi) + L_1 z_i(x, t) \cos(\varphi) - L_1 (2 \dot{\theta} + \dot{\varphi}) \times z(x, t) \sin(\varphi) + x z_t(x, t) \right] dx + \frac{\partial}{\partial t} \left[ (h_1 + h_2) \dot{\theta} \right. \\
+ \left. h_2 \dot{\varphi} + \frac{1}{3} M_1 L_2^2 \dot{\theta} + m_a L_2^2 \dot{\varphi} \right] - (\tau_1 - \beta_1 \dot{\theta} - M_{D1}) \right\} \delta \theta \\
+ \left\{ \int_0^{L_2} \left[ \rho_2 + m_p \delta_{\theta}(x - L_2) \right] \frac{\partial}{\partial t} \left[ L_1 \dot{\theta} (\dot{\theta} + \dot{\varphi}) \sin(\varphi) \right. \\
\left. + L_1 \dot{\theta} z_i(x, t) \sin(\varphi) + L_1 \dot{\theta} (\dot{\theta} + \dot{\varphi}) \cos(\varphi) \right] dx \right\} \delta \varphi \\
+ \left\{ \int_0^{L_2} \rho_2 \frac{\partial}{\partial t} \left[ L_1 \dot{\theta} \cos(\varphi) + x (\dot{\theta} + \dot{\varphi}) + z_i(x, t) \cos(\varphi) \right. \\
\left. + x(\varphi) + z_i(x, t) \right] dx + \left\{ \int_0^{L_2} \rho_2 [L_1 \dot{\theta} (\dot{\theta} + \varphi) \sin(\varphi) ] dx \right\} \delta z(x, t) \\
+ \left\{ \int_0^{L_2} \frac{\partial^2}{\partial x^2} [EI_{u} z_{xx}(x, t)] dx \right\} \delta z(x, t) \\
+ \left\{ m_p \frac{\partial}{\partial t} [L_1 \dot{\theta} \cos(\varphi) + L_2 (\dot{\theta} + \varphi) + z_i(L_2, t)] \\
+ m_p [L_1 \dot{\theta} (\dot{\theta} + \varphi) \sin(\varphi)] \right\} \delta z(L_2, t) \\
+ \left\{ -EI_u z_{xx}(L_2, t) \delta z(L_2, t) + EI_u z_{xx}(0, t) \delta z(0, t) \right. \\
\left. + EI_u z_{xx}(L_2, t) \delta z_i(L_2, t) - EI_u z_{xx}(0, t) \delta z_i(0, t) \right\} dt = 0. \\
\right.
\]

Here, \( M_{D1} \) is drag torque when the flexible link fixed to the first link and forced to rotate with the first link together around the point O with \( \dot{\theta}_1 \) angular velocity; \( M_{D2} \) is drag torque if only the flexible link rotates around the point A with \( \dot{\varphi} \) angular velocity when the first link is stationary; \( M_{D1} \) is calculated for two different configurations of the two links which depends on the comparison of the distances \( L_{bo} \) and \( L_1 \), the distance between origin O and the end point of the flexible link consecutively:

- \( L_{bo} \geq L_1 \) (Fig. 2)

\[
M_{D1} = M_{D1a} + M_{D1b} \\
M_{D1} = \frac{1}{8} C_D \rho h_1 L_1^2 V_{ao}^2 + \frac{1}{8} C_D \rho h_2 (L_{bo}^2 V_{bo}^2 - L_1^2 V_{ao}^2). \\
\]

(13)

- \( L_{bo} < L_1 \) (Fig. 3)

\[
M_{D1} = \frac{1}{8} C_D \rho h_1 L_1^2 V_{ao}^2 + \frac{1}{8} C_D \rho h_2 (L_1^2 V_{ao}^2 - L_{bo}^2 V_{bo}^2). \\
\]

(14)
$M_{D2}$ is calculated in the same way in each case as;

$$M_{D2} = \frac{1}{8} C_D \rho h_2 L_2^2 V_{ba}^2$$  \hspace{1cm} (15)

$V_{ao}$: Linear velocity at point A of first link rotated around
O with $\dot{\theta}_1$ angular velocity. This velocity vector is
perpendicular to the first link at point A.

$V_{bo}$: Linear velocity formed when the flexible link is fixed
and rotated together with the first link around point
O with $\dot{\theta}_1$ angular velocity. This velocity vector is
perpendicular to the OB line at point B.

3. Control of Land and Underwater Flexible Manipulators
The common approach in the control of flexible manipulators
is that the manipulators should orient its tip to the destination
as quickly as possible without any deflection. In the proposed
case, while driving a payload to its destination, each arm’s
final angular positions ($\theta_r, \phi_r$) are given as a reference input
to the controller. Each instantaneous angular positions ($\theta, \phi$)
and the tip deflection ($\eta$) is calculated using numerically
implemented control scheme and then plotted in time domain
to see the merit of the controller strategy.

3.1. Classical controller
Proportional (P) and Proportional + derivative (PD)
controllers are utilized to drive both rigid and flexible
links of land robot with and without payload. Obtained
simulation result is used for benchmarking with fuzzy control
performance. Proportional action provides an instantaneous
response to the control error as given below:

$$\tau(s) = K_p E(s).$$  \hspace{1cm} (16)

PD control is preferred for fast response which is shown in
the following equation:

$$\tau(s) = (K_p + K_{D}s) E(s).$$  \hspace{1cm} (17)

3.2. Fuzzy logic + Integral controller
In general, a fuzzy logic controller consists of a set of
linguistic conditional statements that are derived from human
operators, and which represent expert’s knowledge about the
system being controlled. The fuzzy logic comprised of four
principal components: a fuzzification interface, a knowledge
base, a decision-making logic and a defuzzification interface.
The fuzzification interface transforms crisp measured
data into suitable linguistic values. The knowledge base
consists of a database and a fuzzy control rule base.
The decision-making logic has the capability of simulating
human decision-making based on fuzzy concepts. The
defuzzification interface is utilized to yield a non-fuzzy
control action from an inferred fuzzy control action. When
the system is controlled with only fuzzy logic controller, for
large payloads, an overshoot is observed. For this reason,
an integral controller is accompanied with fuzzy controller
with weighting values $w_F$ and $w_I$ in order to prevent the
overshoots. In this study, the best $w_F$ and $w_I$ for each link
are 0.5, 1 and 1.1 respectively. This both control application
is depicted in Fig. 4.

While developing the fuzzy logic controllers, the error $e(t)$
and the rate of error $de/dt$, are the inputs of the fuzzification interface.Triangular membership functions are used for each
input of the fuzzy logic controllers and simple rule tables
are defined by taking into account the specialist knowledge
and the experience that are shown in Tables I and II. The

| Table I. Fuzzy Rule table for the rigid link. |
|-----------------|--|--|--|--|--|
| de             | NB | NS | Z  | PS | PB |
| $e$            |    |    |    |    |    |
| NB             | DUMIN1 | DUMIN1 | DUMIN1 | DUMIN1/4 | DUMAX1 |
| NS             | DUMIN1 | DUMAX1/16 | 7*DUMAX1/2 | DUMIN1/8 | DUMAX1 |
| Z              | DUMIN1 | DUMAX1/16 | 0 | DUMAX1 | 3*DUMAX1 |
| PS             | DUMIN1 | DUMAX1/4 | 2*DUMAX1 | DUMIN1 | 2*DUMAX1 |
| PB             | DUMIN1 | DUMAX1/4 | 2*DUMAX1 | DUMAX1 | DUMAX1 |
Table II. Fuzzy rule table for the flexible link.

<table>
<thead>
<tr>
<th></th>
<th>NB</th>
<th>NS</th>
<th>Z</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>DUMIN2</td>
<td>DUMIN2</td>
<td>DUMIN2</td>
<td>3*DUMIN2</td>
<td>DUMIN2/2</td>
</tr>
<tr>
<td>de</td>
<td>DUMIN2</td>
<td>DUMIN2</td>
<td>2*DUMIN2</td>
<td>3*DUMIN2/2</td>
<td>DUMAX2/8</td>
</tr>
</tbody>
</table>

Table III. Maximum and minimum values of \( e \), \( de \) and \( y \).

<table>
<thead>
<tr>
<th>Rigid link</th>
<th>( E_{MAX1} = 2.2^2 \theta_R/3 )</th>
<th>( E_{MIN1} = -E_{MAX1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEMAX1</td>
<td>0.002</td>
<td>DEMIN1 = -DEMAX1</td>
</tr>
<tr>
<td>DUMAX1</td>
<td>0.08</td>
<td>DUMIN1 = -DEMAX1</td>
</tr>
<tr>
<td>Flexible link</td>
<td>( E_{MAX2} = 0.6^2 \phi_R )</td>
<td>( E_{MIN2} = -E_{MAX2} )</td>
</tr>
<tr>
<td>DEMAX2</td>
<td>0.002</td>
<td>DEMIN2 = -DEMAX2</td>
</tr>
<tr>
<td>DUMAX2</td>
<td>0.02</td>
<td>DUMIN2 = -DEMAX2</td>
</tr>
</tbody>
</table>

Fig. 5. Membership functions for \( e \), \( de \) and \( y \).

output of each fuzzy system \( y_F \) is derived using the standard Zadeh–Mamdani’s min–max gravity reasoning method at the defuzzification interface.

As output membership functions, the control output was represented with five membership functions equally spaced as in Fig. 5. The ranges of \( e \), \( de \) and \( y \) values which are used when establishing Fig. 4 are stated in Table III.

4. Numerical Simulation of the System

A simulation program is developed in MATLAB to find the response of the system for any input. And Runge–Kutta algorithm has been implemented for the numerical integration of non-linear ordinary differential equations of the system.

The assumed physical and geometric parameters of the simulated system are given in Table IV.

The system is controlled with three methods for the comparison reasons. First, classical P and PD controls are applied to the system. Simulation studies show that choosing appropriate gain values \( K_p \) and \( K_d \) by trial and error method, it is possible to drive both links to the reference without any steady state error. Therefore integral controller is not necessary to associate with classical P and PD strategy. P control simulation is performed selecting proportional gain of rigid and flexible link by trial and error method as 0.0066 and 0.0004, respectively. For PD control, \( K_p \) 0.08 and 0.03, and \( K_d \) 0.2 and 0.1 values are chosen for each link simulation. In Figs 6 and 7, the comparison of P and PD control for the land robot under different payload (0–0.25 kg) conditions are given.

In P control with and without a payload causes a significant delay in both links while reaching their reference. However, in PD control, change in payload does not cause an important delay, but an overshoot is inevitable at each link. For the case of underwater robot, both control method faces up with significant delay in settling time because of the viscous damping effect of the environment. This case of P and PD control for the underwater manipulator without and with payload are given in Figs 8 and 9.
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Fig. 7. Comparison of the P and PD controller performance for the land robot with a payload of 0.25 kg.

Fig. 8. Comparison of the P and PD controller performance for the underwater robot without a payload.

Fig. 9. Comparison of the P and PD controller performance for the underwater robot with a payload of 0.25 kg.

Fig. 10. Comparison of the P, PD and Fuzzy + I controller performance for the land robot without payload.

Fig. 11. Comparison of the P, PD and Fuzzy + I controller performance for the land robot with a payload of 0.25 kg.

In Figs 10 and 11, comparison of P, PD and Fuzzy + Integral control for land robot are given. Under PD and Fuzzy + Integral control strategy, both links reach to the reference without an error. Although PD control gives a quick response for the no payload condition, a 10° of overshoot is observed for the first link and a 6° of overshoot is observed for the flexible link with the payload. But Fuzzy + Integral controller shows no overshoot for both situations. The behaviour of both links under the water for PD and Fuzzy + Integral control are similar to each other because of the viscous effect of the environment. This phenomenon is presented in Figs 12–13.

In order to show the merit of Fuzzy + Integral controller over PD strategy, working ranges of each link are chosen as 40°, 80° and 120°. Simulation studies were performed for both environments with and without payload and the performance of land robot can be depicted from the Figs 14 and 15. It is clear that the increase in reference angle has a little effect on settling time of PD control but as the reference angle becomes larger, the settling time of Fuzzy + Integral also gets larger for land robot without payload. Meanwhile, for the case of payload, PD control ends up with an increasing
performed analysis and performance at all. The study confirmed the success for the drag forces acting on the manipulators moving on a two-link robot manipulator. The modelling is done for the underwater robot with a payload of 0.25 kg. Carrying a payload does not affect the Fuzzy + Integral performance at all.

5. Conclusion and Discussion

This paper has studied the dynamic analysis and performances of the P, PD and Fuzzy + Integral control on a two-link robot manipulator. The modelling is done for the drag forces acting on the manipulators moving horizontally under the sea with the flexible link which is the originality of this study. The study confirmed the success of the proposed integral augmented fuzzy control laws as well as classical control methods. According to the proposed Fuzzy + Integral method, the integral action takes place when the overshoot likely to happen under the control of fuzzy logic. And by choosing the appropriate weighting constants, it is proved that driving the robot in both the environments without overshoot can be achieved. Also for the underwater conditions, Fuzzy + Integral control and PD control gives similar responses compared to the conditions on land one. Finally, from the simulations it is concluded that P control alone is not sufficient to control this kind of flexible robots.

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Appendix

\[
m_{11} = (I_{a1} + I_{b1}) + \frac{1}{3} M_1 L_1^2 + m_a L_1^2 + M_2 \left[ L_1^2 + \frac{1}{3} L_2^2 + L_1 L_2 \cos \phi \right] + m_p \left[ L_1^2 + L_2^2 + 2 L_1 L_2 \cos \phi \right] - 2[I_1 \sin \phi] a \phi \eta.
\]

\[
m_{12} = (I_{a1} + I_{b1}) + M_2 \left[ \frac{1}{3} L_2^2 + \frac{1}{2} L_1 L_2 \cos \phi \right] + m_p \left[ L_2^2 + L_1 L_2 \cos \phi \right] - [I_1 \sin \phi] a \phi \eta.
\]

\[
m_{13} = I_{a2} + M_2 \left[ \frac{1}{3} L_2^2 + \frac{1}{2} L_1 L_2 \cos \phi + \frac{1}{2} L_1 L_2 \cos \phi \right] + m_p \left[ L_2^2 + L_1 L_2 \cos \phi \right] - I_1 \sin \phi a \phi \eta.
\]

\[
m_{14} = a^a + L_1 \cos \phi a^b, \quad m_{21} = m_{12},
\]

\[
m_{22} = I_{a2} + \frac{1}{3} m_2 L_2^2 + m_p L_2^2, \quad m_{23} = I_{a2} + \frac{1}{3} M_2 L_2^2 + m_p L_2^2,
\]

\[
m_{24} = a^a,
\]

\[
m_{31} = I_{a2} + M_2 \left[ \frac{1}{3} L_2^2 + \frac{1}{2} L_1 L_2 \cos \phi \right] + m_p \left[ L_2^2 + L_1 L_2 \cos \phi \right] - I_1 \sin \phi a \phi \eta.
\]

\[
m_{32} = I_{a2} + \frac{1}{3} M_2 L_2^2 + m_p L_2^2, \quad m_{33} = I_{a2} + \frac{1}{3} M_2 L_2^2 + m_p L_2^2,
\]

\[
m_{34} = a^a, \quad m_{41} = \gamma^a + L_1 \cos \phi \gamma^b, \quad m_{42} = \gamma^a,
\]

\[
m_{43} = \gamma^a, \quad m_{44} = a,
\]

\[
\gamma_j^a = \int_0^{t_2} x \phi_i(x) \, dx
\]

\[
\gamma_j^b = \int_0^{t_2} \phi_j(x) \, dx
\]

\[
a_{ij} = \int_0^{t_2} \phi_j(x) \phi_i(x) \, dx = \frac{m_p}{\rho_2} \phi_i(L_2) \phi_i(L_2), \quad i \neq j,
\]

\[
a_i^a = \rho_2 \gamma_j^a + m_p L_2 \phi_i(L_2),
\]

\[
a_i^b = \rho_2 \gamma_j^b + m_p \phi_i(L_2).
\]
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Fig. 14. Comparison of PD and Fuzzy+I controller performance of the land robot for different reference positions without payload.

\[ f_1 = \beta_1 \dot{\theta}_1 - \left[ \frac{1}{2} M_i + m_P \right] L_i L_2 \dot{\theta}_1 (2 \dot{\psi} + \dot{\psi}) \sin \varphi \]

\[-L_i \dot{\psi} (2 \dot{\theta}_1 + \dot{\psi}) \cos \varphi a \hat{\eta} - 2 L_i (\dot{\theta}_1 + \dot{\psi}) \sin \varphi a \hat{\eta} \]

\[ f_2 = \beta_2 \dot{\psi} + \left[ \frac{1}{2} M_i + m_P \right] L_i L_2 \dot{\theta}_2 \sin \varphi + L_2 \dot{\theta}_2 \cos \varphi a \hat{\eta} \]

\[ [f_3, f_4]^T = a \Lambda \eta(t) + L_i \dot{\theta}_2 \sin \varphi a \hat{\eta} \]

Detailed derivation of each expression can be found in ref. [18].

Abbreviations: $\beta_i$; viscous friction factor; $I_i$; moment of inertia of the cross; $\delta_\eta$; dirac delta operator; sectional area of the beam; $\mu_i(t)$; deformation coordinates; $\dot{\theta}_i$; Angular positions of rigid link; $\rho_i$; density of water; $\varphi_i$; Angular positions of flexible link; $\rho_i$; mass of each link per length;

Fig. 15. Comparison of PD and Fuzzy+I controller performance of the land robot for different reference positions with payload.
\( \vec{r}_i(x, t) \): position vector of the point \( x \) on; \( \phi_i(x) \): infinite mode shapes; each link; \( \tau_i \): Torque applied to each link; \( C_D \): drag coefficient; \( L_1 \): length of the rigid link; \( C_L \): buoyant coefficient; \( L_2 \): length of the flexible link; \( A \): characteristic area; \( M_1 \): the mass of the rigid link; \( V \): velocity flow; \( M_2 \): the mass of the flexible link; \( K_p \): proportional gain; \( m_a \): mass of the first motor; \( K_D \): derivative gain; \( m_r \): payload; \( K_I \): integral gain; \( E \): young modulus of elasticity; \( \varepsilon \): Error; \( I_{ho} \): moment of inertia of the motors at each joint; \( \alpha \): Constant for the first link equations.

References