

Potential of Periodic Networks for Seismic Isolation of Civil Engineering Structures

Mohamed Anis Doufene, Nouredine Bourahla and Ali Bougessi

Abstract: The frequency band gaps concept is commonly employed in many fields of physics. Recently, the concept is extended to other disciplines such as seismic vibrations. This article explores the potential of an approach based on periodic arrays to protect a civil engineering structures from seismic effects. The principle consists of an arrangement of periodic components that act as a barrier to divert or attenuate the propagation of the seismic waves in the frequency range of 0.1-30 Hz. For this purpose, a numerical investigation of two periodic structures (1D and 2D) has been conducted to demonstrate the feasibility of this technique in prevent the propagation of seismic waves in the structure. The obtained numerical results show that under certain conditions, two band gaps have been detected in the low frequency range [7.87 to 12.47Hz and 15.82-21.01Hz] which is suitable for application in earthquake engineering. It is recognized that the models used in this study are very simplistic compared to reality, and therefore more research work is needed to corroborate these results by investigating the transmissibility of the system with different materials and configurations.

Index Terms: Periodic arrays, Bands of frequency gap, Seismic isolation of civil engineering structure, Seismic waves.

I. INTRODUCTION

Phononic crystals are macroscopic composite materials that have a spatial periodicity in one or several directions (1D, 2D and 3D). They are able, through the Bragg reflection, to block the acoustic propagation in certain directions and frequency ranges, when the wavelength is comparable to the structural periodicity. This is known as a band of frequencies gap, which is most often abbreviated as a "band gap"[1].

Over the past two decades, a new category of media with unusual physical behavior affecting the class of phononic crystals has emerged. The composition and configuration of these phononic structures make them capable to block the propagation of waves with wavelengths much larger than the structural periodicity. The origin of this phenomenon is attributed to locally vibrating resonators at low frequencies. The mechanism responsible for this behavior is called Local Resonance, hence the name of Locally Resonant Phononic Crystal. The potential of this mechanism has attracted researchers to use these theoretical bases in the design of

periodic structures for blocking or mitigating seismic waves in civil engineering structures.

In this context, two perspectives were addressed, the first approach rely on isolating a structure using meta-material foundations[2-5]where most of the authors showed experimentally and numerically that the foundation system can efficiently isolate the super-structures; and the second acts directly on the site to divert or stop ground vibration to propagate into a site by implementing periodic networks on the ground [6-9]in which some large-scale experiment showed that a periodic array of boreholes embedded in the soil can deflect the energy of an incoming seismic wave [10].

The aim of the work presented here is to design a periodic media at the scale of civil engineering structures allowing the inhibition of waves having a low frequency range [0.1-30 Hz].

To this purpose two periodic structures with one and two - dimensional periodicities are presented. The first consists of a concrete matrix in which two heavy cores with rubber coating are periodically arranged in both directions of the [x,y] plane. The second structure consists of a one-dimensional periodic arrangement of layers of concrete and rubber. The seismic isolation efficiency of these structures has been assessed in terms of the band gaps range and position. A study of the effect of certain parameters of the elementary cell constituting the periodic network on the band frequency gap is presented, namely the influence of the density of the core, the modulus of elasticity of the elastomer and that of the concrete matrix as well as the filling fraction of the components.

II. BASIC PRINCIPLES AND ASSUMPTIONS FOR THIS STUDY

Before introducing the models, the underlying theory and assumptions are first presented together with the main terms used in this study.

In a periodic network, the passage from one point to another spaced by a period L results in a phase shift of $e^{ik_b L}$. The calculation of the band gaps of a periodic network is limited to the study of an elementary cell by applying periodic boundary conditions given by Eq. (2) below.

According to Bloch's theory [11], solutions of the wave equation with Floquet-Bloch nature can be written as:

$$u(r,t) = e^{i(K.r - \omega t)} \cdot u_k(\vec{r}) \quad (1)$$

Where k denotes the wave vector in the reciprocal space, ω the angular frequency; and $u_k(\vec{r})$ has the same periodicity as elastic parameters and satisfies $u_k(r) = u_k(r+1)$. Then, the following expression can be derived:

$$\begin{aligned} u_k(r+L,t) &= e^{i(K(r+L) - \omega t)} \cdot u_k(r+L) \\ &= e^{iK.L} \cdot e^{i(K.r - \omega t)} \cdot u_k(r) = u(r,t) \cdot e^{iK.L} \end{aligned} \quad (2)$$

Thus an equation with eigen values can be obtained. To each wave vector \vec{k} corresponds a set of eigenvalues, whose solutions $\omega(k)$ constitute the dispersion curves in the network. The solutions being themselves periodic, it is possible to represent the totality of the solutions on a graph reduced to the zone of Brillouin in the reciprocal space [12]. The Brillouin zone is the equivalent of the elementary cell in real space, it is the smallest cell to describe this reciprocal network where the knowledge of the band structure in this reduced Brillouin zone is sufficient to know the set of the propagation modes in the network. For each of these directions, there are frequencies f authorized which are solution of the problem to the eigenvalues.

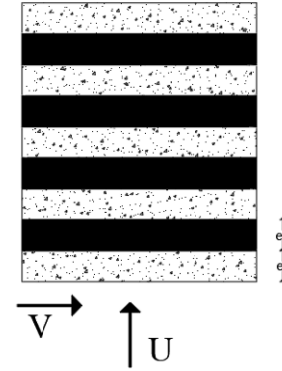
The main property offered by periodic arrays is the possibility of creating band gaps at the band structure. A band gap occurs in this case as a frequency interval where no link is defined between the frequency and the wave vector (it is the unauthorized frequencies) and therefore a wave at these frequencies cannot spread in the middle. The mechanism governing the band gap constitution is based on two phenomena: Bragg reflections due to the periodicity of the network whose waves lengths of the elastic waves, λ propagating in the medium, are on the order of periodicity of the network a or, on the local resonance phenomenon of the elements placed in networks whose waves lengths exceed by far the periodicity of the network $\lambda \gg a$ [13], this is the case for seismic waves.

The main idea is to introduce into each cell a local resonator, the interaction between the modes of vibrations of the resonator and those of the matrix can give rise to band gaps, it is therefore possible to place singular elements possessing resonance properties rather than simple diffusing elements. The appearance of the band of frequency gap is related to the fact that each resonator will trap a part of the energy of the transmitted wave, however the wave interacts with all the resonant elements coupled to each other and also coupled to the host matrix. It is really this association that plays a key role in the presence or not of such a band gap.

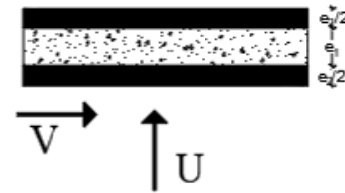
III. NUMERICAL MODELS AND BAND CALCULATION

A. Final Periodic array 1D

The first model is a one-dimensional periodic lattice; the elementary cell of this model (Fig. 1 (b)) consists of succession of two layers: concrete and rubber having thicknesses e_1 , e_2 respectively. This cell is arranged periodically (constant period $a=0.40$) in the direction U , as shown in Fig. 1 (a). The elastic properties of the materials used are given in Table 1.



(a): Periodic array 1D.



(b): unit cell configuration

Fig.1. Periodic array 1D.

This is a one dimensional structure because the periodicity exists only for one direction of space and consequently, k_x is the only component of the wave vector k .

With the configuration mentioned above, a 2D plane strain model is elaborated on COMSOL Multiphysics software [14] to solve for the eigenfrequencies and construct the dispersion curves.

The mesh of the elementary cell and the periodic boundary conditions imposed in the U direction are shown in Fig. 2, where the top boundary conditions $U_i^k(r)$ are related to those at the bottom $U_i^k(l)$. Moreover, for a problem of plane deformations, the components of the displacement vector are U_i^k ($i = u, v$).

$$u_2 = u_1 \cdot e^{i.k_y \cdot a}; \quad v_2 = v_1 \cdot e^{i.k_y \cdot a} \quad (3)$$

Table 1.Properties of used materials.

Material	Density ρ (Kg /m ³)	Elastic modulus E (Gpa)	Poisson's ratio ν
Concrete	2,500	32.1	0.3
Rubber	1,300	$1.37 \cdot 10^{-4}$	0.463
Steel	7,850	210	0.3
Lead	11,600	16.46	0.4

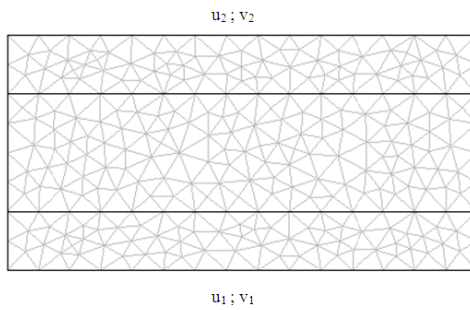


Fig.2. Discretized mesh and periodic boundary conditions.

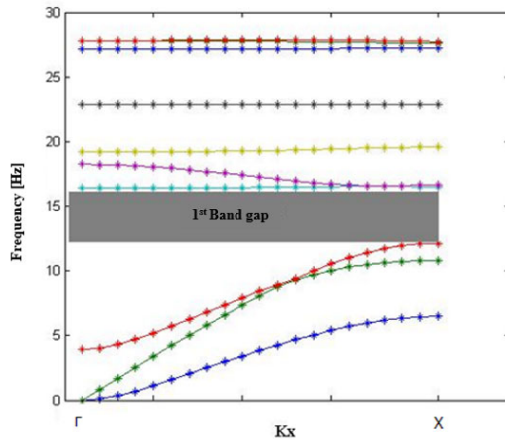


Fig. 3. Dispersion Curves of Model 1 (1D).

Table2. Band gaps obtained for model1.

a (m)	e ₁ (m)	e ₂ (m)	h(m)	Band gap (Hz)
0.4	0.2	0.2	10	[12.4-16.51]

B. Periodic array 2D

In fact, the one-dimensional periodic structures have limited interest for seismic protection applications; hence periodic two-dimensional structures are then produced. The second model studied is a square periodic array, the structural units of this model is made of a concrete matrix containing two resonators consisting of a succession of cylinders and tubes of elastomer and steel. Each of these layers will be managed geometrically in the finite element model by a radius R_i .

These units are arranged periodically in a square lattice (1 m of lattice constant), as shown in Fig. 4.

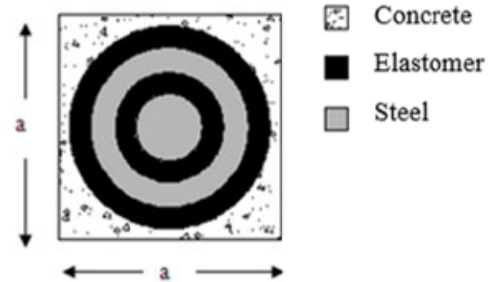


Fig. 4. Periodic array 2D.

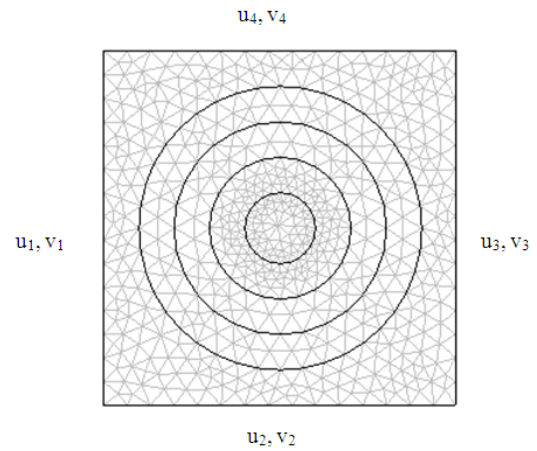


Fig.5. Discretized mesh and periodic boundary conditions.

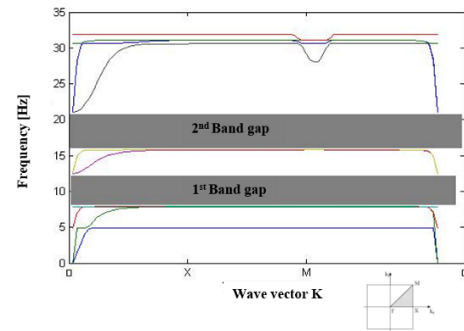


Fig. 6. Dispersion curve of Model 2 (Two-dimensional periodicity).

The mesh results and boundary condition are shown in Fig. 5, where the right and top boundary conditions $U_i^k(r)$ and $U_i^k(t)$ are related to the left and bottom boundary conditions $U_i^k(l)$ and $U_i^k(b)$ through two-phase coefficients e^{ikxa} and e^{iky_a} , respectively.

$$u_3 = u_1 \cdot e^{i.k_x.a} ; v_3 = v_1 \cdot e^{i.k_x.a} ;$$

$$u_4 = u_2 \cdot e^{i.k_y.a} ; v_4 = v_2 \cdot e^{i.k_y.a}$$

(4)

The band of frequency gap of in-plane mode waves is obtained by the COMSOL software. The corresponding dispersion curve is shown in Fig. 6, where the segments ΓX, XM, MΓ on the abscissa represent waves traveling along the direction of 0°, 0°- 45°, 45° respectively, and the vertical coordinate represents the frequency of the wave.

The dispersion curve representing the relationship between the frequency and the wave vector shows for this 2D model two band gap ranges [7.87-12.47] Hz and [15.82-21.01] HZ. The seismic waves (in the two directions x and y) having frequencies which coincide with the band gap frequencies.

Table 3. Band gaps obtained for model 2.

a (m)	R ₁ (m)	R ₂ (m)	R ₃ (m)	R ₄ (m)	Band gap (Hz)
1.0	0.49	0.39	0.29	0.19	[7.87-12.47] & [15.82-21.01]

IV. IDENTIFICATION AND STUDY OF PARAMETERS INFLUENCING THE BAND GAP CHARACTERISTICS

A parametric study on the elementary cell of the second model is presented based on the three characteristics: the lower frequency of the band (LFB), the upper frequency of the band (UFB) and the width of the frequency band (WFB). The factors studied are:

- Elastic modulus E of concrete.
- Elastic modulus of elastomer.
- Core density ρ.
- Geometric configuration including filling fraction F (representing the ratio of the area occupied by the two resonators and the area of the elementary cell; $f = \pi R_1^2 / a^2$.) and the ratios between the thickness of the core and that of elastomer coating for each resonator ($\beta_1 = \frac{R_4}{R_3 - R_4}$ and $\beta_2 = \frac{R_2 - R_3}{R_1 - R_2}$).

The calculation was performed under the same lattice constant.

A. Influence of the core density

Different densities of materials such as concrete, steel and lead were used for the core. Fig. 7 shows that the parameters of the band gap (LFB, UFB and WFB) are very sensitive to the variation of the density of the material constituting the

core. Increasing the core density permits to decrease the range of bang gap frequencies. The system considered can be assimilated to a mass-spring resonator for which the pulsation is written as:

$$f = \sqrt{\frac{K}{M}}$$

As per this formula, it is preferable to use heavy material cores to obtain low frequencies.

One of the heavy density materials is lead (ρ=11,600 Kg/m³). For this value of density, the band gaps obtained cover the ranges 6.70 to 10.80 Hz and 13.60 to 20Hz.

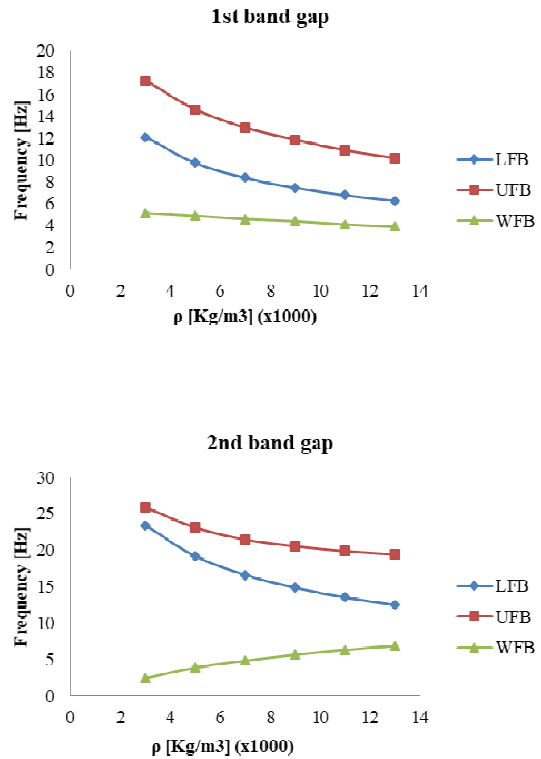


Fig. 7. Influence of core density on frequency band gap.

B. Influence of the coating elastic modulus

The three parameters of the band gap (LFB, UFB and WFB) illustrated in Figs. 8a and 8b increase with the value of the modulus of elasticity of the elastomer. As mentioned above, the system is linked to a mass-spring resonator, where the cores play the role of mass and the elastomer layers are the springs [15]. The increase in the parameters of the band gap is due to the increase of the stiffness of the resonator, the latter is proportional to the modulus of elasticity of the elastomer.

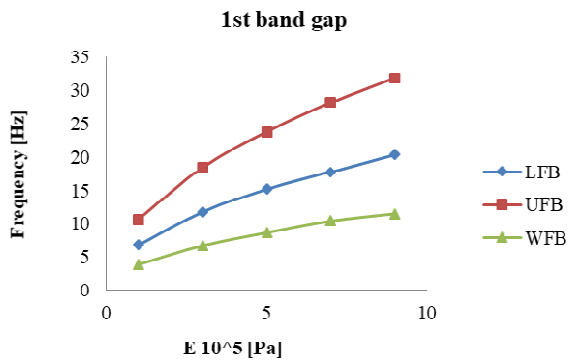


Fig. 8a. Influence of coating elastic modulus on 1st frequency band gap.

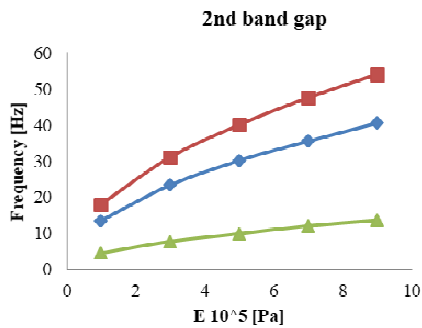


Fig. 8b. Influence of coating elastic modulus on 2nd frequency band gap.

C. Influence of matrix elastic modulus

Fig. 9 shows that the variation in the elastic modulus of the matrix does not entail any modification on the frequency band gap because the elastic modulus of the matrix is much higher than the coating (in order of 10^5). Thus, a small variation in the modulus of elasticity of the concrete has no impact on the band gap.

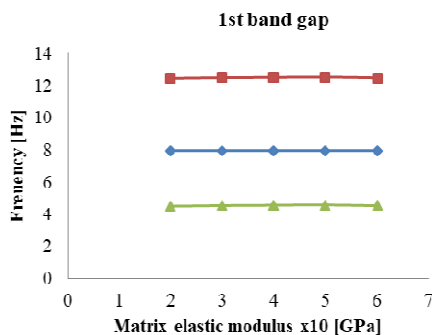


Fig. 9a. Influence of matrix elastic modulus on the 1st frequency band gap.

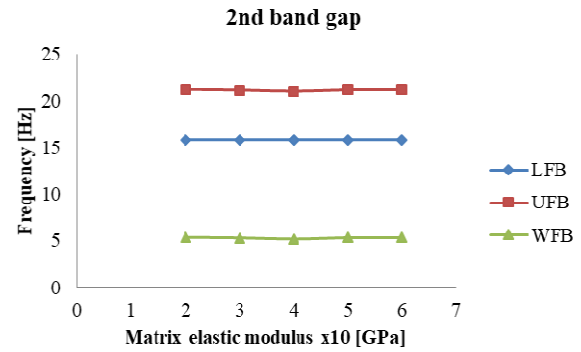


Fig. 9b. Influence of matrix elastic modulus on the 2nd frequency band gap.

D. Influence of filling fraction and parameter α

The filling fraction is defined as: $f = \pi R_1^2 / a^2$ where R_1 denotes the twin resonators radius. The ratios between the thickness of the core and that of the coating elastomer for the two resonators are:

$$\beta_1 = \frac{R_4}{R_3 - R_4} : \text{Ratio of thickness for the first resonator.}$$

$$\beta_2 = \frac{R_2 - R_3}{R_1 - R_2} : \text{Ratio of thickness for the 2nd resonator.}$$

Where R_4, R_3, R_2 and R_1 denote the radius for each layer from the center of cell

A parameters β_1 and β_2 are defined to assess the contribution of these factors on the frequency band gap. Fig. 10a shows that the lower limit (LFB) of the 1st band gap decreases until reaching the value $\beta_{1\min} = 1.85$ corresponding to the lowest frequency for which the 1st band gap can be opened. Beyond this value, the opening of the 1st band gap is at higher frequencies. We also find that the first band gap tends to close for values of β_1 less than 0.5. Fig. 10b shows that the 2nd band gap tends to close with increasing values of β_1 .

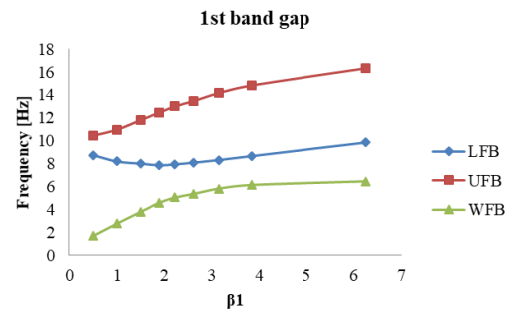


Fig. 10a. Influence of parameter β_1 on the 1st frequency band gap ($\beta_2=1$ and $F=75.43\%$).

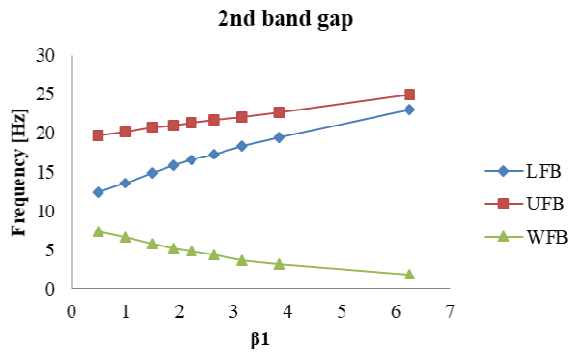


Fig. 10a. Influence of parameter β_1 on the 2nd frequency band gap ($\beta_2=1$ and $F=75.43\%$).

If we consider that this periodic structure is equivalent to a resonator of frequency $f = \sqrt{K / M}$, the increase of the filling fraction tends to increase the mass, therefore, the frequency decreases.

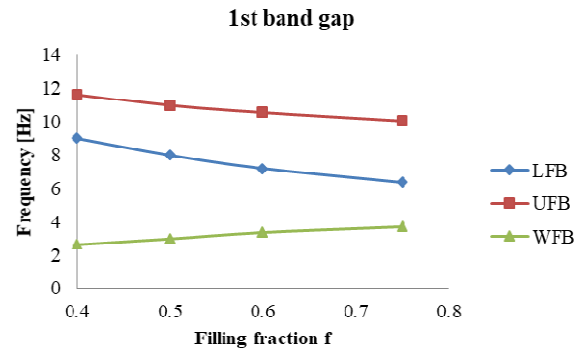


Fig. 12a. Influence of filling fraction on 1st frequency band gap.

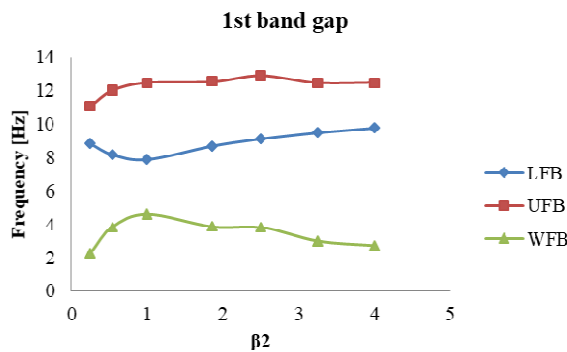


Fig. 11a. Influence of parameter β_2 on the 1st frequency band gap ($\beta_1=1.9$ and $F=75.43\%$).

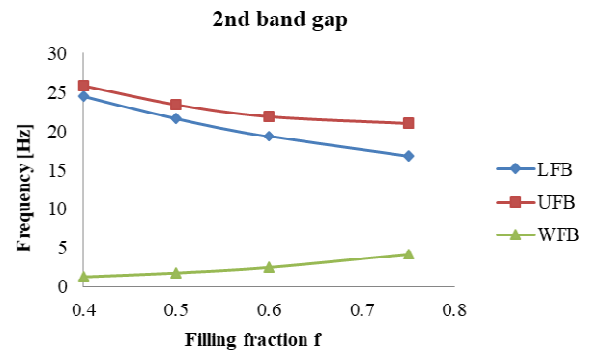


Fig. 12b. Influence of filling fraction on 2nd frequency band gap.

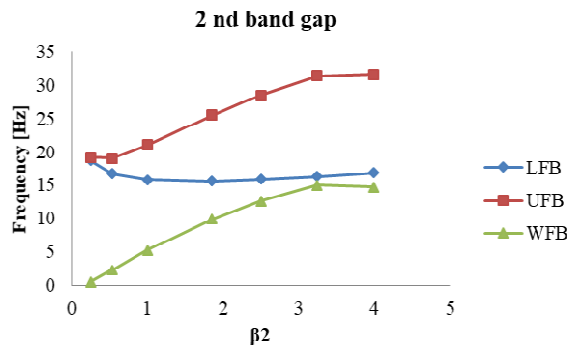


Fig. 11b. Influence of parameter β_2 on the 2nd frequency band gap ($\beta_1=1.9$ and $F=75.43\%$).

Fig. 11 shows that for a value of β_2 near 1, the lower limits of the two band gap tend to their lowest values. These two parameters β_1 and β_2 make it possible to define the geometry of a phononic crystal in an optimal manner.

As illustrated in Fig. 12, when the filling fraction increases, the upper and lower limits (LFB and UFB) of the band decrease while the width of band gap (WFB) increases.

V. CONCLUSION

Application of periodic networks to mitigate earthquake induced vibration is still a challenging task due to the very low frequency range of interest. The main concern of the present work is to explore the potential of this concept to reach frequency band gaps of interest using material properties and network dimensions in the scale of civil engineering structures. The numerical results obtained from 1-D and 2-D models showed that for characteristics of real materials it is possible to produce a low frequency gap from 7.87 to 21.03 Hz. Parameters characterizing the material properties and the geometric configuration of the cells have been varied within the range of common materials to identify the trend and the limits of variation of the low frequency band, the upper frequency band and the width of the frequency band. It has been found that an elementary cell consisting of a high density core coated with a soft material

having a low elastic modulus and a maximum filling fraction produces a wider band gap in the low frequency range.

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