Automation and cybernetics: control of a flexible one-link manipulator

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Introduction
One of the most important requirements in robotics is accuracy, and in order to solve this problem robot structures are built very stiff and rigid. In addition, frequently most of today's analyses and controls of industrial robots are based on the assumption that the robot arm is just a collection of rigid bodies, so that, in order to position the end effector to the commanded location, the joint angles must assume computed values, and the robot is presumed stiff enough so that the end effector will thus be in the intended location. Therefore, most robots are built to be massive and unwieldy.

Nowadays, the use of light materials has increased in fields such as robotics, machine tools or advanced space applications, and with requirements such as large working volume, high mobility and the ability to carry heavy payloads, it is desirable to build a lightweight robot arm. Therefore, the rigid-body-assumption in robotics has to be abandoned. Flexibility effects noticeably limit the performance of the different structural elements and thus must be taken into account both at the design and control level. Modelling and control activities should therefore be carried out together.

The deflection and the vibration of the flexible arm robot present a severe problem to the accuracy and position stability. Even the static deflection of the robot arm has to be taken into account for positioning accuracy; and what is...
more important, the high moving speed of the arm implies the action of very large inertia forces on it; thus its stability becomes a critical problem and a more sophisticated control system must be designed. Therefore the control of flexible manipulators is becoming a critical issue in robotics.

The modelling techniques for elastic structures represent an extension of the modelling techniques for rigid robots combined with structural analysis methods[1]. Generally, a model of the elastic deformations (corresponding to small displacements) superimposed on a rigid body dynamic model (corresponding to the large displacements of the structure) is considered. In this case, two different reference systems are used, one linked to the movement of the rigid solid and the other linked to the elastic deformations.

Many papers have been published on the vibration and control of flexible manipulators. Some of them are concerned with the positioning of manipulators using the information from the potentiometers and strain gauges that are deployed along the manipulator axis[2]. It is generally said that the number of sensors should be a minimum to make the control system simple. In 1984, Cannon and Schmitz[3] published the pioneer experimental work in the area of control of flexible robot arms in which they proposed a method of using endpoint sensing. This method was, later on, successfully applied to the position control of a one-link flexible arm with a tip mass. The commanded tip position stored in the computer memory was compared with the sensor's measurement of actual tip position. The difference was then used as a basis for applying control torque to the arm base via the direct-current servomotor. It was shown that the feedback control using the tip sensing is enough to shift the end-position to the desired position and hold it there.

In this work the dynamic equations for a flexible one-link manipulator moving in the horizontal (x-y) plane are summarized. The payload is simulated attaching additional masses to the arm at any specified locations. The finite element method, based on elementary beam theory, has been employed during the process of formulation. The validation of the dynamic model and the structural analysis of the flexible manipulator is given by the SELSPOT II system (opto-electronic motion analysis system)[4] which uses active light sources for determining actual positions of objects in space. The experimental results show that the performance of the control is satisfactory, even under perturbation action.

System description

The position control strategy of a one-link flexible robot arm rotating in the horizontal plane used here is essentially a feedback of the hub angle with the vibration information given by a simple second order linear model, constructed to simulate the modal behaviour of the structure in order to drive the motor for suppressing the vibrations due to flexibility. In the experiment, a shaft encoder is used as a sensor of the hub angle. It is assumed that perturbations exciting the vibration modes of the link affect the low-reduction motor axis. The flexible one-link robot arm consists of two parts; the hub, which is modelled as a rigid
body, and the arm, with length $l$ and circular cross-section, which is modelled as a flexible beam.

Let us consider a rectangular co-ordinate system $(x, y, z)$, in which the $z$-axis is opposite to the direction of gravity. The rotation of the arm about the $z$-axis is given by a time-dependent variable $\theta(t)$, which defines the joint angle. The configuration, in which the axis of the beam in its undeformed state, parallel to the $x$-axis, is named the home configuration. The differences in position between the deformed state and the undeformed state of the beam in the home configuration are the displacements $U_i$ referring to the home configuration, as shown in Figure 1.

![Figure 1. Flexible one-link manipulator in its home configuration](image)

The flexible beam is modelled by finite element method[5], which is further divided into $n$ beam elements, and then it has $n + 1$ nodal points. The generic $i$th nodal point ($i = 0, 1, 2, \ldots, n$) is associated with the lumped mass, $m_i$, and referred to home configuration. The payload is simulated by the mass attached to the end-point (the $n$th nodal point), $m_a$, as indicated in the figure. Because the motion only occurs in the horizontal plane, it is not necessary to consider the gravitational effect. The dynamic equations for the flexible robot arm[6] can be written as follows:

$$\sum_{j=1}^{n} K_{ij} U_j = -m_i X_i \ddot{\theta} - m_i \dddot{U}_i + m_a X_i \dot{\theta}^2$$  \hspace{1cm} (1)

where $K$ is the $n \times n$ stiffness matrix, $U_i$ is the displacement $U_{ij}$, and $l_i = X_i - X_{i-1}$. Those terms on the right-hand side of equation (1) are the sum of inertia forces acting on the $i$th node.
A torque, $\tau$, can be applied at the hub which has an inertia moment, $I_h$. So, another equation, which stipulates that the total moment due to inertia plus the applied torque equals $I\dot{\theta}$ is:

$$\tau = \dot{\theta} \left[ I_h + \sum_{i=0}^{n} m_i (U_i^2 + X_i^2) \right] + \sum_{i=0}^{n} (m_i X_i \dot{U}_i + 2m_i \dot{U}_i U_i)$$

with the assumption that $U_0$ equals zero.

**Actuator model**

The actuator dynamics [7] is given by the second-order differential equation:

$$\tau_l = K_{eq} V_e - J_a \dot{\theta}_m - B_a \dot{\theta}_m$$

where $\theta_m$ is the rotor position, $J_m$ is the actuator inertia, $B_m$ is the internal viscosity coefficient, $K_m$ is the torque constant, $\tau_m$ is the generated torque, $J_a = J_m/r$, $B_a = (B_m + (K_b K_m / R))/r$, and $K_{eq} = K_m / rR$. The armature resistance is given by $R$, and $K_b$ is the back emf constant. See the block diagram of Figure 2 corresponding to the reduced order system (3), which represents the direct-current (dc) motor together with both the load torque $\tau_l$ and the gear train ratio given by $r$.

\[ V_{(s)} \]
\[ K_m \]
\[ \tau_l \]
\[ + \]
\[ 1 \]
\[ J_m S + B_m \]
\[ + \]
\[ \theta_m(s) \]
\[ k_a \]
\[ r \]

A simple way to identify the actuator dynamics is to fit it to the behaviour of a first-order transfer function, given by:

$$\dot{\theta}_m = \frac{a}{V_e} \left( T s + 1 \right)$$

where $V_e$ is the input voltage of the amplifier; $a = K_{eq} / B_a$; and $T = J_a / B_a$.

**Dynamic equations and feedback control**

From now on, the governing equations of the system, equations (1), (2) and (3), may be written symbolically as:

$$M(\alpha) \ddot{\alpha} + N(\alpha, \dot{\alpha}) \dot{\alpha} + K \alpha = L \mathbf{u}$$

where

$$\alpha = \begin{bmatrix} \theta \\ U_i \end{bmatrix}, \quad M(\alpha) = \begin{bmatrix} (J_a + \sum [m_i (X_i^2 + U_i^2)], m_i X_i) \\ m_i X_i & m_i \end{bmatrix}.$$
Since the resulting equations are non-linear, we linearize them with respect to some reference state $q_s$, and obtain linear time-invariant differential equations. In this representation, it is assumed that $n$ is large enough to describe the dynamic behaviour of the flexible beam. Thus equation (5) can be regarded as the full-order model of the flexible beam but, practically, it is assumed that these equations can be truncated at some finite number $n$. Therefore, as a result of actuator/sensor bandwidth limits, a finite-dimensional model of the flexible-link manipulator that ignores the high-frequency modes is obtained. In general, to study the control of the flexible manipulator it is convenient to represent the dynamic model in the space state:

$$ x = Ax + Bu $$

$$ y = Cx $$

where $A$, $B$, $C$ are constant matrices. Based on this model the state estimator will be constructed, where the state vector is given by $x = [\alpha \ \dot{\alpha}]^T$, and the input $u$ is given by the armature voltage.

It must be noticed that the linearized model of the modal vibration can be assumed to be a second-order system, where the input is the hub angle and the output is the induced vibration. This assumption was necessary here in order to simplify the model and the control strategy, minimizing the number of sensors needed to diminish the oscillation of the tip. An estimate of the tip oscillation is obtained from this model and feedforwarded to the input (see Figure 3).

Since our problem is to regulate (3) and (4) simultaneously by the control $u(t)$, a feedback control law should be sought so as to guarantee the stability of the closed-loop system. In general, state feedback control law for the system governed by the equations system (6) has the form:

$$ u(t) = -Gx = -[g_{rp} \ g_{rd} \ g_{fp} \ g_{fd}]x. $$

(7)
The poles of the feedback system described by equation (6) can be placed arbitrarily by choosing suitable values of the state feedback gain vector $G$. However, the feedback gain vector $G$ should be chosen carefully so as not to exceed the allowable input torque.

**Experimental set-up**

The developed one-link experimental manipulator is illustrated in Plate 1. The arm is a flexible brass beam with a circular cross-section of $d = 3$ mm, which has $l = 0.8$ m, Young's elasticity module $E = 5.8 \times 10^{10}$ N/m$^2$, density $\rho = 8.38 \times 10^3$ kg/m$^3$, and $m_a = 0$. At one end, the arm is clamped on a hub of radius $r_h = 6.5$ mm which is mounted on the vertical shaft of a dc-motor. The position and vibration of the end-point of the arm is controlled simultaneously by this actuator, which has a gear reduction ratio of 35. A shaft encoder attached to the dc-motor is used to measure the hub angle. A Maxon dc-motor 47.065.032-00 is used and an HCT L-1100, which is a high-performance, general-purpose motion control IC from Hewlett-Packard. The parameters of the actuator, experimentally determined by applying a step input, are $a = 0.7592$ rd/s/volt and $T = 0.0711$s.

Simulation results are shown for a flexible arm with previous characteristics, reference angle $\theta_{ref} = 45^\circ$, and number of beam elements $n = 1$. Figure 4, without vibration control, shows the step response of hub angle for the case of a conventional rigid arm controller. As it can be seen from the figure, the rigid-body mode reaches the desired position rapidly. However, the flexible modes are excited and damped slowly. Therefore, Figure 5, with vibration control, shows
The measured hub angle, exhibiting a 3 per cent overshoot in the time response, and the vibration modes are quickly damped.

The experimental implementation that was conducted to validate the dynamic model and to measure the vibration and the end-point position during the motions of the flexible manipulator employs the SELSPOT II motion analysis system (see Appendix). Experimental results are shown in Figure 6, in which the feedback signals are the output of the shaft encoder and its derivative, i.e., without vibration control. The velocity feedback gain $g_{vd}$ is
adjusted on the amplifier such that there is no joint velocity overshoot. One can see that only the experimental behaviour of the system with hub control is very similar to results obtained in simulation.

After experimental modal analysis by applying a PRBS-signal through the actuator, we obtained the dominant characteristic in frequency of the end-point behaviour to construct an observer (whose state variables are the estimates of the state variables of the model); and feedback these signals, position and velocity, by control software. The program is written in C language, and the
sampling period is 2ms. Since the bandwidth of the actuator system is about 30Hz, the overall system can be regarded as a continuous system. Experimental results of the flexible arm with vibration control are shown in Figure 7, where the vibrations are totally eliminated. It is observed that the velocity feedback suppresses the arm vibration. However, an excess value of the displacement feedback gain $g_{fd}$ again promotes the appearance of the vibration even if the servo loop is reinforced with the velocity feedback. Hence, it can be said that there are optimum values of $g_{fd}$ for satisfactory behaviour of the arm. As can be seen from the figures the theoretical results are, in general, in good agreement with the experimental ones, and the performance of the overall system is satisfactory.

Conclusions

A one-link flexible manipulator was developed and instrumented in order to control its vibration modes. The vibration model used here is very simple and permits vibrations on the rod to be estimated using the hub angle. A very cheap system is thus obtained that only senses the motor rotated angle. It is shown that, for the simple case of a one-link manipulator this second-order model is enough for control purposes. The system is powered by a dc-drive with appropriate gear-train, and the control law is implemented by software, increasing flexibility of the control system. The experiments demonstrate the high quality of the link position stabilization achieved with the vibration mode dynamic controller. In the future, to get added robustness and improved performance, feedback of the measured tip position from the SELSPOT II system also will be considered. Finally, a complete observer-based strategy will be implemented and the results will be compared with those presented here.

References


Appendix. SELSPOT II system description

The Selspot II system, as shown in Figure A.1, is an opto-electronic motion analysis system which utilizes active light sources for determining actual positions of objects in space. These positions can be presented in Cartesian co-ordinates and can be calculated into speed and acceleration. This system uses active light sources, such as light emitting diodes (LEDs) or lasers, which are applied
to the points of an object that are of interest. The LEDs are powered by an LED control unit (LCU) which turns on the light sources sequentially.

The camera, of which up to 16 can be used, consists of a detector, analogue pre-amplifier, analogue amplifiers for X and Y position and A/D converters, senses the intensity of light from the light source. The camera will now record the position of the LED and will deliver two 12-bit serial words of position information to the camera interface module (CIM). The CIM converts the serial data to parallel and will also supply an analogue signal.

The parallel data is sent to the Selspot control module (SCM) or to the FIFO buffer module (FBM) which communicates with an external computer. The administrating unit (AU) contains, in addition to the camera interface module, the Selspot control module and the sequencer module (see also Selspot II system manual[4]). Selspot II was developed by Innovision Systems for Selcom AB, Partille, Sweden.