Analysis of a Multiple-Token Contention Scheme for Broadband Wireless Access Networks

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Abstract—In this paper, we present a mathematical analysis to evaluate the contention phase of a wireless system in which each contending station transmits multiple copies of the bandwidth reservation/request packet to increase the probability of successful transmissions using multiple slots randomly selected from the pool of available slots at the given moment. Here, we define this type of systems as a multi-token reservation scheme with uniform random backoff (MT-UNI). The proposed MT-UNI mechanism can be used in various broadband wireless access systems including IEEE 802.16, PRMA, C-TDMA and others. The simulation results show that the proposed performance formulae are very accurate, and the MT-UNI mechanism can improve the performance of the system at low load conditions. In addition, when each station uses 2 reservation tokens, the MT-UNI scheme seems to produce the near-optimal performance.

I. INTRODUCTION

Over the past many years, a wide range of Media Access Control (MAC) protocols have been developed for many different wireless systems. Many of the developed protocols tend to organize the channel access structure into two phases: the contentious reservation phase and the reserved access phase. This type of protocols and variants are widely used in various broadband wireless access systems including Packet Reservation Multiple Access (PRMA) [1], C-TDMA [2], IEEE 802.16 and others [3]-[6].

PRMA and C-TDMA [1]-[2] are essentially a combination of TDMA and slotted ALOHA schemes. In PRMA, when a user terminal has a burst of traffic to transmit, it contends for the uplink channel by attempting to transmit a packet in the first available slot according to a permission probability similar to the slotted ALOHA scheme. The state of each uplink slot (i.e., idle or reserved) is periodically broadcast to the user terminal via a feedback channel. After this first transmission, the terminal waits for the outcome from the base station that shall arrive after a round-trip delay (RTD). If the attempt is successful, the same slot in the subsequent frames will be reserved for this terminal. If the attempt fails because two or more contending terminals transmit their packets on the same slot, all involved terminals must retransmit.

In the original PRMA protocol, contention was based on the permission probability scheme. Recently, a modified PRMA scheme, called PRMA with random contention, was proposed in [3]. Instead of relying on the permission probability to access the first available slot, each contending terminal randomly selects an available slot to use from the current pool of idle slots. The results in [3] showed that the PRMA with random contention scheme could provide higher channel utilization efficiencies for the case of low traffic conditions, and lower average packet delays for the case of high traffic conditions. On the other hand, the results in [4] suggested that each contending terminal may transmit copies of the same packet in multiple available slots while waiting for the positive outcome of the sending packet to improve the successful transmission probability under a low traffic condition. An interesting question to ask here is how the PRMA scheme with multiple request packets (PRMA-HS) [4] would perform if the random contention scheme were used instead of the permission probability approach.

A contention-based MAC scheme is also used in the emerging IEEE 802.16 standard for broadband wireless metropolitan area networks. For the uplink transmissions in IEEE 802.16, when a client station that is classified as the “best effort” station has data to transmit, the station first needs to send the bandwidth request packet to the base station using the allocated contention slots. The base station can then piggyback the assigned uplink slot information for this client station via the downlink channel. The bandwidth request packets from many contending stations may collide at a certain value of collision probability. The performance of this standard random access scheme in IEEE 802.16 has been widely evaluated and analyzed [7]-[8]. However, an interesting open question to explore here is whether the probability of successful bandwidth request transmissions would increase if each client station sends several bandwidth request packets over several contention slots instead of sending one bandwidth request.

In this work, we present an accurate mathematical analysis to evaluate the contention phase of a wireless system in which each contending station transmits multiple copies of the bandwidth reservation/request packet to increase the probability of successful transmissions using multiple slots randomly selected from the pool of available slots at the given moment. Here, we define this type of system as a Multi-Token reservation scheme with uniform (UNI) random backoff (MT-
In this work, we consider a wireless system in which there are dedicated downlink and uplink channels. The uplink channel is divided into TDMA slots and some of which are allocated for the contention-based uplink bandwidth requests or data, while the others are allocated for the scheduled uplink transmission. To initiate a burst of uplink traffic, a client station randomly selects one of the available non-reserved or contention slots in the uplink channel to transmit the first data or bandwidth reservation packet. This transmission may collide with a bandwidth request packet from other stations. However, if the base station successfully receives this first packet without collision, it then reserves a dedicated uplink slot and informs this station to use this particular uplink slot thereafter. Note that this general system model can be found in various broadband wireless systems including IEEE 802.16, PRMA [1], C-TDMA [2] and others, e.g., [3]-[6]. Here, we focus on the contention phase of such wireless systems. In particular, there are $N$ slots currently available for contention, and there are $M$ client stations actively contending to transmit bandwidth reservation/request packets.

In practice, the base station may allocate a relatively large portion of the uplink channel bandwidth for contention, with respect to the number of client stations actively contending to transmit the bandwidth reservation packets. In such situations, having each contending client station transmitting multiple copies of the bandwidth request packet using multiple randomly selected contention slots over each uplink TDMA frame can improve the probability of successful bandwidth request transmission. Here, we refer to this improved scheme as a Multi-Token reservation scheme with uniform (UNI) random backoff, or MT-UNI. In the MT-UNI scheme, when a station needs to send a bandwidth reservation/request packet, it randomly chooses $T$ slots from $N$ available slots and repeatedly transmits the bandwidth reservation/request packet using the selected slots. If the base station successfully receives one of the transmitted bandwidth reservation/request packets from each client station, the base station then reserves an uplink slot and informs this client via the downlink channel or the feedback channel. If the base station receives multiple bandwidth reservation/request packets from the same station, it simply disregards the duplicated packets. In the sequel, we present the analytical work to accurately evaluate the performance and gain of the MT-UNI scheme.

## III. MATHEMATICAL ANALYSIS

In this section, we analyze performance of the MT-UNI scheme in terms of the average number of successful requests for a particular TDMA frame. A goal here is to determine the average number of stations that successfully transmit over this given TDMA frame. Note that a station is assumed to have a successful transmission when at least any other contending stations do not choose one of its selected slots. Here, we provide two approaches to derive the average number of successful terminal over a given frame as a function of the number of contending stations, available slots, and reservation tokens. The formulae provided here can be used to determine the optimum number of reservation tokens that should be used by each contingning station as a function of the number of available slots and contending stations in order to maximize the probability of successful transmissions at a given moment. For brevity of the presentation, much of the derivation detail will be neglected.

### A. Recursive Formula

First, let’s define $P_{\text{MT-UNI}}(k; t_1, t_2, \ldots, t_M, B_1, B_2, \ldots, B_M, N)$ as the probability that $k$ contending stations successfully transmit provided that the successful stations are those specified with the status bit ($B_i$) equal to 1, the $i^{th}$ station has $t_i$ remaining contention tokens, and there are currently $N$ available slots and $M$ contending stations. After some analytical work, this probability can be computed by the following equation:

$$
P_{\text{MT-UNI}}(k; t_1, t_2, \ldots, t_M, B_1, B_2, \ldots, B_M, N)
=\prod_{i=1}^{M}(1-p_i)\times P_i + \sum_{i=1}^{M} p_i \prod_{j=i+1}^{M}(1-p_j)\times P_j(i)
=\sum_{i=1}^{M} \sum_{j=i+1}^{M} \prod_{k=i}^{M}(1-p_k)\times P_k(i, j) + \cdots + \prod_{k=M}^{M} P_k(1, 2, \ldots, M)
$$

where $p_i$ is the probability that $i^{th}$ station transmits on a given slot which can be computed as $p_i = \frac{t_i}{N}$. In addition, $P_0$ denotes the probability that no one chooses this particular slot, $P_j(i)$ denotes the probability that this particular slot is only
chosen by the $i^{th}$ station, $P_s(i,j)$ denotes the probability that this particular slot is chosen by the $i^{th}$ station and the $j^{th}$ station, $P_a(m,1,2,\ldots,M)$ denotes the probability that all stations choose this particular request slot, and so on. After some analytical work, these probabilities can be expressed as follows:

$$P_s(i,j) = P[k \left| t_1, t_2, \ldots, t_{M-1}, t_j, B_1, B_2, \ldots, B_{M-1}, B_j, N - 1 \right|]$$

$$P_a(1) = P[k \left| t_1, t_2, \ldots, t_{M-1}, t_j, B_1, B_2, \ldots, B_{M-1}, B_j, N - 1 \right|]$$

$$P_a(2) = P[k \left| t_1, t_2, \ldots, t_{M-1}, t_j, B_1, B_2, \ldots, B_{M-1}, B_j, N - 1 \right|]$$

$$P_a(3) = P[k \left| t_1, t_2, \ldots, t_{M-1}, t_j, B_1, B_2, \ldots, B_{M-1}, B_j, N - 1 \right|]$$

$$P_a(M-1) = P[k \left| t_1, t_2, \ldots, t_{M-1}, t_j, B_1, B_2, \ldots, B_{M-1}, B_j, N - 1 \right|]$$

Finally, the average number of successful users of the MT-UNI system with $M$ users, $N$ request slots per frame and $T$ tokens can be then calculated as

$$P_{at\text{-con}}[k \left| t_1, t_2, \ldots, t_{M-1}, B_1, B_2, \ldots, B_{M-1}, N \right|] = \sum_{n=1}^{N} n \times P_{at\text{-con}}[k \left| t_1, t_2, \ldots, t_{M-1}, B_1, B_2, \ldots, B_{M-1}, N \right|]$$

$$S_{at\text{-con}}[M,N,T] = \sum_{k=0}^{N} k \times P_{at\text{-con}}[k \left| t_1, t_2, \ldots, t_{M-1}, B_1, B_2, \ldots, B_{M-1}, N \right|].$$

The detail of the derivation is quite complex. Nonetheless, to briefly explain how we can find the desired probabilities in the recursive form given above; let’s consider the following simple examples. In Figure 1, all possible cases and the associated probabilities are provided for the scenario in which there are 2 contending stations, 2 available slots, and each station has 1 remaining contention token. Here, the $i^{th}$ station decides to use a given slot with the probability $p_i(n)$, where $n$ denotes the number of remaining available slots. In the first case, a collision occurs and no one succeeds since both stations choose the same slot. For the second case, the first station uses the first request slot and the second station uses the second slot. In this case the number of successful users is equal to 2. Figure 2 displays the scenarios for the case in which there are 2 contending stations and 3 available slots. It can be observed that, all parts of Figure 1 appear as a component in Figure 2. Note that we can then write the probability in Figure 2 in terms of a function with a smaller number of available slots. This motivates the recursive derivation as in (3).

![Diagram](image1.png)

Fig. 1. All possible scenarios when there are 2 stations and 2 available slots.

![Diagram](image2.png)

Fig. 2. All possible scenarios when there are 3 stations and 2 available slots.
### B. Closed-Form Solution

In this section, we will derive a closed-form formula to conveniently calculate the average number of successful stations $S(M,N,T)$ as a function of $M$, $N$ and $T$. For the case when $T=1$, the analysis is basically similar to the result in [7]-[8], which gives

$$S_{\text{closed-form}}(M,N,1) = M \left( \frac{N-1}{N} \right)^{M-1}. \quad (4)$$

After some derivation work, we found that when $T = 2$ and $N \geq 2$,

$$S_{\text{closed-form}}(M,N,2) = M \left( \frac{N-2}{N} \right)^{M-1} \left[ 2 - \left( \frac{N-3}{N-1} \right)^{M-1} \right]. \quad (5)$$

When $T = 3$ and $N \geq 3$,

$$S_{\text{closed-form}}(M,N,3) = M \left( \frac{N-3}{N} \right)^{M-1} \left[ 3 - 3 \left( \frac{N-4}{N-1} \right)^{M-1} + \left( \frac{N-4(N-5)}{(N-1)(N-2)} \right)^{M-1} \right]. \quad (6)$$

From (4)-(6), we can rewrite $S(M,N,T)$ in the closed form formula as the following equation

$$S_{\text{closed-form}}(M,N,T) = M \left( \frac{N-T}{N} \right)^{M-1} \left[ T \sum_{i=0}^{T-1} (-1)^i \left( \prod_{j=2}^{T} \left( \frac{N-T-j+1}{N-j+1} \right) \right)^{M-1} \right], \quad N \geq T.$$  

It should be pointed out that we can now find the appropriate number of tokens by varying the number of tokens ($T$) and determine the value of $T$ that gives the maximum average number of successful stations. That is, the optimal system performance may be achieved when the number of reservation tokens is appropriately set for the given number of contending stations ($M$) and the number of available slots per frame ($N$).

### IV. RESULTS AND DISCUSSIONS

In this section, we provide numerical results to verify the validity and accuracy of the proposed performance formulas. We also evaluate the performance of the MT-UNI based schemes under various conditions via the proposed formulae. Here, we consider the contention phase of a wireless system in which there are currently $N$ available slots, $M$ contending stations at the current moment, and each station has $T$ contending tokens. In the simulation, each of the $M$ contending stations randomly selects $T$ slots to use from the pool of $N$ available contention slots. The average number of stations that successfully transmit at least one bandwidth reservation packet without collision is then determined.

Figure 3 displays the simulated and calculated results for the average number of successful stations in various scenarios. It is apparent that the simulated results exactly match the calculated results for all cases under considerations. That is, the proposed formulae are valid and very accurate. Figure 4 displays the performance of the MT-UNI scheme for various cases when there are 50 available contention slots and each station uses 1, 2, 3, and 10 reservation tokens. We can see that the MT-UNI scheme can improve the system performance when the number of contending stations is small compared to the number of available slots (a low load condition). For example, in this case, when there are 10 contending stations, MT-UNI with $T = 2$ and $T = 3$ perform better than MT-UNI with $T = 1$. That is, having each station repeatedly transmit the reservation packet using 2 and 3 slots produces better results in this case. In fact, the optimal number of contending tokens to use under a specific load condition can be numerically determined and utilized to maximize performance of the system. In practice, however it may be difficult for each station to accurately know the number of contending station at the given moment. From the provided results, we observe that using 2 contending tokens seems to yield a good overall performance over low load conditions. In other words, when using more than 2 contending tokens the performance improvements at low load conditions seem to be insignificant. That is, to achieve good overall performance, we may design a system that directs client stations to use 2 contending tokens at low load conditions and 1 contending token at high load conditions. Note that in IEEE 802.16, the base station periodically broadcast the contention parameters to client stations. It might as well broadcast the appropriate number of tokens for every station to use.
As an example, we evaluate the contention phase of several PRMA-base schemes: the PRMA scheme with random contention [3], the PRMA-HS scheme [4], and the PRMA scheme with multiple reservation tokens and uniform random contention (PRMA+MT-UNI) introduced in this paper. Figure 5 displays the average number of successful users as a function of the number of users for the PRMA+MT-UNI, PRMA with random contention and PRMA-HS schemes under light load conditions when there are 50 slots available for contentsions. Here, we assume that each station in the PRMA-HS and PRMA+MT-UNI schemes employs the optimal number of reservation tokens. We can see that the PRMA+MT-UNI scheme further improves performance of the recently proposed PRMA scheme with random contention under low load conditions. For the moderate-to-high load conditions, the PRMA+MT-UNI scheme and the PRMA scheme with random contention are equivalent since the optimal number of reservation tokens is equal to one in this case. At extremely high load conditions, PMRA-HS performs better because the permission probability contention approach becomes more efficient than the uniform random slot selection approach. In fact, one may implement a smarter PRMA-MT-UNI system that would direct stations to use the permission probability contention approach at very heavy load conditions.

V. CONCLUSION

This paper considers the contention phase of broadband wireless access systems such as IEEE 802.16, PRMA, and C-TDMA. In the proposed MT-UNI scheme, each station is allowed to transmit several copies of the uplink bandwidth reservation packet using several contention slots. We present the mathematical analysis to accurately evaluate the MT-UNI scheme. The simulation results show that the proposed performance formulae are very accurate. The MT-UNI mechanism can improve the performance of the system at low load conditions. In addition, when each station uses 2 reservation tokens, the MT-UNI scheme seems to produce the near-optimal performance.

REFERENCES