Decentralised Anti-windup design approaches with application to Quadrotor UAVs

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Motivation

- Anti-windup (AW) compensators improve performance in both MIMO and SISO systems when saturation occurs.
- However in practice, MIMO AW compensators are:
  - not as simple as SISO AW, the practical consequences of $L_2$ performance may be difficult to interpret.
  - Highly complex, hence problems arise when computational resources are limited.
  - not structured.
- Key features
  - Aim: Design decentralized AW compensators for a class of MIMO systems (eg: Quadrotor UAVs fall in this category).
  - The AW must ensure global stability of the entire nonlinear system.

MIMO Plant Description

$G_D(s) = \text{diag}(G_1(s), G_2(s), \ldots, G_m(s)) \sim \text{diag}(G(s))$

$K_D(s) = \text{diag}(K_1(s), K_2(s), \ldots, K_m(s)) \sim \text{diag}(K(s))$

$G_D$ is a diagonal dynamic part of the plant $G$
$X$ is a non-diagonal but static, invertible matrix. Its inverse $X^{-1}$ can be interpreted as a control allocation matrix.
When saturation inactive, system behaves as $m$ decoupled loops
When saturation occurs, decoupling is destroyed, system experiences windup

Typical Anti-Windup (AW) Structure

The AW compensator, $\Theta(s)$ has the structure and state-space realisation:

$\Theta(s) = [M(s) - I] \sim \begin{bmatrix} A + BF & B \\ F & 0 \end{bmatrix}$

$G$ has right coprime factorisation $G(s) = N(s)M(s)^{-1}(s)$. $F$ is chosen so that $A + BF$ is Hurwitz.

A stable system with $F$ can be obtained by satisfying the LMI described in [2]

Pseudo-Decentralized AW Features

Here, the AW, $\hat{\Theta}(s)$ is driven by virtual signal $\upsilon = v - \chi(v)$.
By performing the stability analysis as described in [3], this LMI is obtained

$H_e \{ \begin{bmatrix} A_{0D} & B_{0D} & B_{0U} & 0 & 0 \\ -L_0X^{-1} & -X^{-1}U & X^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ C_0D & D_0D & D_0U & 0 & -2 \end{bmatrix} \} < 0 \quad (1)$

Result is more stringent than that described in the typical AW above. Note: $H_e[A] = A + A'$. $\hat{\Theta}(s)$ is diagonal.

Channel-by-Channel AW Features

Slightly modified version of figure in typical AW with $\hat{\Theta}(s)$ having the form:

$\hat{\Theta}(s) = \begin{bmatrix} M_0(s) - I \\ 0 \end{bmatrix} X$ \quad (2)

Idea is to design $m$ single-channel AW compensators for each $G_i(s)$ in $G_D(s)$ to ensure stability of the overall nonlinear system.

Using process in [1], a linear programming solution is presented, such that
If diagonal matrices $W > 0$ and $V > 0$ such that $V = X^T W X$, AW compensators can be designed using the LMI in [2]

Result is structured and easy to implement with great practical appeal. Note: $[M_0, N_0] \sim \text{diag}(M(s), \text{diag}(N(s)))$

Quadrotor UAV Test Platform

- System has the same structure as earlier stated MIMO plant
- Practical quadrotor used is a Modified 2014 3DR Quadrotor

Flight Tests Results

- Figure: Pitch angle response: From Left; 1st Nominal response; 2nd Saturated response no AW; 3rd Saturation, decentralized AW; 4th Saturation, channel-by-channel AW

- Note: All plots are not exactly alike because outdoor flight conditions are not constant.

Conclusion

- Two approaches proposed for structured AW design.
- Pseudo-decentralised approach provides a one-step design procedure.
- Channel-by-channel approach allows independent AW design for each channel and combines them safely for the MIMO system.
- Channel-by-channel AW is preferred due to its transparency, flexibility and ease of implementation.
- Flight results for both designs show improved performance during saturation.

References