An Efficient Channel Estimator for Frequency Hopping System via Propagator Method

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Abstract
In this paper, the multi-path time delay estimation problem for a Slow Frequency Hopping (SFH) system using the Propagator Method (PM) is considered. Two novel techniques are proposed. The first technique is developed by applying the Propagator Method (PM) in association with the well-known MUSIC algorithm. Based on the proposed technique a highly efficient estimator has been achieved. The second technique is a simple closed-form expression which is obtained by applying PM and Eigen Value Decomposition (EVD) of the projection matrix. The proposed techniques generate estimates of the unknown parameters. Such estimates are based on the observation and/or covariance matrices. Moreover, the PM itself does not require the EVD or Singular Value Decomposition (SVD) of the Cross-Spectral Matrix (CSM) of received signals. As a result, a significant improvement in computational load is achieved. Computer simulations are also included to demonstrate the effectiveness of the proposed methods.

Keywords: Frequency Hopping, Time Delay Estimation, Propagator Method, MUSIC, ESPRIT.

1. Introduction

Different techniques have been used to combat impairments in rapidly varying radio channels. Some of those are channel coding and interleaving, adaptive modulation, transmitter/ receiver antenna diversity, spectrum spreading, and Dynamic Channel Allocation (DCA). Frequency Hopping (FH) is a spectrum spreading technique that can introduce frequency as well as interference diversity. It is very robust and ideal for applications where data reliability is critical [1], [2]. Code Division Multiple Access (CDMA) can be achieved via Frequency Hopping (FH) if the bandwidth is divided into a number of frequency sub-bands. Fast frequency hopping systems (FFH) change frequency at a significantly higher rate than the information rate. Slow Frequency Hopping systems (SFH) change frequency at a rate comparable with (or slower than) the information rate.

Typically, the FH system model is considered as a narrow band system. It was shown in [3], [4] that the flat fading model is not valid for a FH system whose bandwidth is comparable with the coherence bandwidth of the multi-path channel. Therefore, the time delay estimation becomes significant when the received signal in a SFH system at a high data rate is frequency selective. The multi-path time delay estimation problem for a SFH system was studied using Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) [5], Rank-Revealing Triangular Factorization [10], and Signal Parameter Estimation via Cayley-Hamilton Constraint algorithm (SPECC) [6], [7].

In this paper, we address the problem of estimating the multi-path time delay parameters using the Propagator Method (PM) [8]; first in conjunction with the well-known MUSIC algorithm [9], second with Eigen value decomposition of the projection matrix. It is well known that the computational load of the PM-based method is significant. Moreover, this method does not involve Eigen Value Decomposition (EVD) or Singular Value Decomposition (SVD) of the Cross-Spectral Matrix (CSM) of received signals. The propagator is a linear operator which only depends on steering vectors and can easily be extracted from the data set. An estimated propagator from the data set of the first sensor is used to construct the orthogonal projection matrix which represents the noise subspace. The MUSIC algorithm is applied to the projection matrix to estimate the frequencies.

This paper is structured as follows. In Section II, the system model and the problem formulation is presented. The development of the highly efficient method is presented in Section III. In Section IV, the development of the second method is presented. In Section V, the performance of the PM is illustrated using MATLAB simulations. A comparison with the previous work such as LS-ESPRIT and TLS-ESPRIT [7] is
made. Finally, some concluding remarks were given in Section V1.

2. Problem Formulation

Signal multi-path occurs when the transmitted signal arrives at the receiver via multiple propagation paths. Each of these paths may have a separate phase, attenuation, delay and Doppler frequency associated with it. The discrete Multi-path channel is modeled as baseband. The channel is modeled as a tapped delay line, given by:

$$h(t) = \sum_{i=1}^{P} \beta_i \delta(t - \tau_i(t))$$

where, $\beta_i$ and $\tau_i$ are the complex channel gain and the associated time delay of the $i^{th}$ multi-path respectively. The received signal is given by:

$$y(t) = h(t) * s(t)$$

where $s(t)$ is the transmitted signal. In this paper, we consider the baseband model of the Frequency Hopping received signal in a multipath environment of the sample version form

$$y^{(n)}(kT) = \sum_{i=1}^{P} \beta_i e^{-j2\pi f_i T} s(kT - \tau_i; b_n) + w^{(n)}(kT)$$

where $y^{(n)}(kT)$ is the received signal in the $n^{th}$ hop, $T$ is the sampling period, $f_i$ is the frequency in the $n^{th}$ hop, and $b_n$ is the sequence of the transmitted bits. $s(kT; b_n)$ is the transmitted baseband signal and $w^{(n)}(kT)$ is the white Gaussian noise parameter. Parameter $P$ denotes the total number of multi-path considered in the model.

Channel gain, time delay, and the transmitted bit sequence are unknown. However, the hop frequency is known. The problem addressed in this paper is the estimation of the time delays only based on the received signal. When the time delays are estimated, two separate Maximum Likelihood (ML) problems can be considered to estimate $\beta_i$ and $b_n$. Here, three assumptions are made [7]:

1. The first block of symbols is fixed for all hop
2. Time delay remain constant or very slow
3. The frequency does not change within a packet.

Therefore, the discrete time version of (1) is given by:

$$y^{(n)}(k) = \sum_{i=1}^{P} \beta_i e^{-j2\pi f_i k\tau_i} s_i(k) + w^{(n)}(k)$$

where $s_i(k)$ is the delayed version of the transmitted signal through the $i^{th}$ multi-path. Clearly, this part of data is constant among all hops and independent of $n$.

3. Development of a Highly Efficient Method

The Propagator Method (PM) in association with the well-known MUSIC algorithm (PM with MUSIC) will be used to develop a high resolution estimator. Let the $K$ samples in (2) given by:

$$y_k = [y_k(0) \quad y_k(1) \quad ... \quad y_k(K-1)]$$

We collect two subsets of received hop frequencies $\{f_{pi}\}$ and $\{f_{qi}\}$ each at least of size $N$

$$f_{pi} = f_{qi} + \Delta f, i = 1,2, ..., N$$

Let

$$Y_p = [y_{p1}^T \quad y_{p2}^T \quad ... \quad y_{pN}^T]^T \quad and $$

$$Y_q = [y_{q1}^T \quad y_{q2}^T \quad ... \quad y_{QN}^T]^T$$

It is easy to show that $Y_p$ and $Y_q$ can be written as:

$$Y_p = AS + W_p$$

$$Y_q = A\Phi S + W_q$$

where

$$A = \begin{bmatrix}
    e^{-j2\pi f_{p1}\tau_1} & \quad \ldots \quad & e^{-j2\pi f_{p1}\tau_P} \\
    \vdots & \quad \ddots \quad & \vdots \\
    e^{-j2\pi f_{PN}\tau_1} & \quad \ldots \quad & e^{-j2\pi f_{PN}\tau_P}
\end{bmatrix}$$

$$S = \begin{bmatrix}
    \beta_1 \\
    \vdots \\
    \beta_P
\end{bmatrix} = \begin{bmatrix}
    s_1(1) \\
    \vdots \\
    s_P(1)
\end{bmatrix}$$

and $W_p$ and $W_q$ are corresponding noise matrices, and matrix $\Phi$ is defined as:

$$\Phi = diag\left( e^{-j2\pi \Delta f \tau_1}, \ldots, e^{-j2\pi \Delta f \tau_P} \right)$$

The parameters $\beta_i$ are the amplitudes of the respective multi-path. We collect the sub-matrices calculated by (4) in matrix $X$ as:

$$X = \begin{bmatrix}
    Y_p \\
    Y_q
\end{bmatrix}$$

$$X = A[I \Phi]S + [W_p \quad W_q]$$

$A$ is partitioned into two sub-matrices $A_1$ and $A_2$ of size $P \times P$ and $(N-L) \times P$, respectively. Thus, one can define a propagator matrix $P_X$ that is satisfying the following condition:

$$P_X^H A_1 = A_2$$

where $(.)^H$ denotes Hermitian transpose and the dimension of the matrix $P_X^H$ is $(L-P) \times P$. Similarly, the received data matrix $X$ is divided into two sub-matrices $X_1$ and $X_2$ with dimensions $P \times (N-L)$ and $(L-P) \times (N-L)$, respectively. The Propagator matrix $P_X$ can be easily estimated by:

$$P_X = arg \min \| X_2 - P_X^H X_1 \|_F^2$$

$$= (X_2 X_1^H)^{-1} X_1 X_2^H$$

(5)
where \( \| \cdot \|_F^2 \) denotes the Euclidean norm. Matrix \( E_X \) can be defined as:

\[
E_Y = [\bar{P}_Y \quad -I]^T
\]  

(6)

where \( I \) is the identity \((L-P) \times (L-P)\) matrix. Clearly, here

\[
E^H A_{L1} = P^H A_{L1} - A_{L2} = 0
\]

(7)

In a noisy channel the columns of matrix \( E \) are not orthonormal. By introducing an orthogonal projection matrix \( Q_X \) which represents the noise sub-space (the columns of the matrix become orthonormal), so that:

\[
Q_X A_{L} = 0
\]

where

\[
Q_X = E^H X (E^H_X E_X)^{-1} E_X^H
\]

The MUSIC like search algorithm [9] is applied to estimate the frequencies using the estimation function:

\[
\hat{\phi}_M (e^{j\omega}) = \frac{1}{A_{L}^H Q_X A_{L}}
\]

(9)

4. Development of a Closed Form Method

The sub matrices calculated in (4) are gathered in matrix \( Y \) as:

\[
Y = [Y_1^T \quad Y_2^T]^T
\]

(9)

Now, \( Y \) is partitioned as follows:

\[
Y = [Y_1^T \quad Y_2^T]^T
\]

where \( Y_1 \) and \( Y_2 \) contain the first \( P \) and last \( 2N-P \) rows of \( Y \) respectively. The least square solution for the propagator based on the Direct Matrix (DM) is:

\[
\hat{\bar{P}}_Y = \arg \min \| Y_2 - P_1^H Y_1 \|_F^2
\]

\[
= (Y_1 Y_1^H)^{-1} Y_1 Y_2^H
\]

(10)

Based on the estimated propagator in (10), the matrix \( E_Y \) of size \( 2N \times P \) can be defined as:

\[
E_Y = [I^T \quad \hat{\bar{P}}_Y^T]^T
\]

(11)

Now, \( E_Y \) is partitioned as follows:

\[
E_Y = [E_1^T \quad E_2^T]^T
\]

where \( E_1 \) and \( E_2 \) contain the first \( N \) and last \( N \) rows of \( E_Y \) respectively. It is easy to show that:

\[
Q_Y A = \Phi A
\]

(12)

where the orthogonal projection matrix \( Q_Y \) is given by:

\[
Q_Y = E^H Y (E^H_Y E_Y)^{-1} E_Y^H
\]

It is obvious that the \( P \) Eigen-values of \( Q_Y \) are correspond to the \( P \) diagonal elements of \( \Phi \). The multi-path delay parameters are given by:

\[
\bar{\tau}_i = \frac{-\text{angle}(\phi_i)}{2\pi\Delta f}
\]

(13)

5. Simulation Results

To examine the performance of the proposed algorithms, three experiments have been performed. These experiments are carried out using a computer program written in MATLAB. Extensive computer simulations are demonstrated based on this program. In the aforementioned experiments, a three multi-path model \((P=3)\) has been demonstrated. Furthermore, different parameters such as transmission frequency, channel gain, etc., have been set. For instance, the transmission frequency is confined to range from 1899 to 1929 MHz. The uplink frequency ranges for the personal communication system. Seventy-Five frequencies with a 400 KHz frequency separation among carriers have been considered. The multi-path delays are set to be 0.1, 0.4, and 0.9 µs, respectively.

In addition, various assumptions are used in the experiments. For example, the header part in each packet consists of four Quadrature Phase-Shift Keying (QPSK) symbols. The symbol period is assumed to be 4µs. Twenty packets are assumed to be available at the receiver. The hop frequencies form arithmetic series with nineteen possible maximum frequencies in each set. In the first experiment, the channel gain parameter is assumed to be a complex random and exponentially decaying with respect to the time delays. Ultimately, one can run the MATLAB program using the aforementioned parameters and assumptions. Fig. 1 shows the performance of the proposed techniques, the Least Squares ESPRIT (LS-ESPRIT) [6] along with the Total Least Squares - ESPRIT (TLS-ESPRIT) [7]. We considered 500 independent realizations with normalized MSE defined as:

\[
MSE = E \left\{ \sum_{i=1}^{N_2} \sum_{j=1}^{P} \left( \bar{\tau}_i - \hat{\tau}_i \right)^2 \right\}
\]

(9)

In Fig. 1, it can be observed that the Propagator Method in association with the well-known MUSIC algorithm (PM with MUSIC) shows a high resolution estimator.
To examine the proposed algorithm, another experiment has been performed. Here, the multi-path gain parameters are assumed to be complex Gaussian in nature. Under this environment, one can observe that the PM method in conjunction with the MUSIC is still robust, see Fig. 2. The scenario is observed under one thousand different realizations. Moreover, Fig. 2 displays that the closed-form structure (PM with EVD) is demonstrating a reasonable performance compare with the existing reference algorithms such as LS-ESPRIT and TLS-ESPRIT.

In the final experiment, the algorithms behavior has been demonstrated when the number of available packets at the receiver (data acquisition) is change. Here, the SNR is equal to 15 dB. Fig. 3 indicates that when the number of available packets is 15 or more the Propagator Method in association with the well-known MUSIC algorithm achieves significant performance compared to the other methods. Conversely, the closed-form structure (PM with EVD) is representing a rational performance with existing reference algorithms.

6. Conclusion

Channel estimation for Frequency Hopping systems has been widely investigated. In this paper, two techniques for the multi-path delay estimation of frequency hopping system are proposed. The results obtained based on using these techniques outperform those obtained by other existing methods. The multi-path delays can be estimated either by an observation matrix or through the covariance matrix of the received data matrix. The PM method in conjunction with the MUSIC search achieves outstanding performance, especially at lower SNRs. However, the closed-form structure is demonstrating a reasonable performance compared to the existing reference algorithms such as LS-ESPRIT and TLS-ESPRIT. Based on the MUSIC-like algorithm, this work can be extended using a Fourth-Order Cumulants algorithm (FOC) to estimate the time delay parameters.

References


