Multilevel turbo coded–continuous phase frequency shift keying (MLTC–CPFSK)

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1. Introduction

Turbo codes are error correction codes that were introduced along with a practical decoding algorithm [1]. The importance of Turbo codes is that they enable reliable communications with power efficiencies close to the theoretical limit predicted by Shannon. Turbo codes are the most efficient codes for low-power applications such as deep space and satellite communications, as well as for interference limited applications such as third generation cellular and personal communication services.

In trellis based structures, to improve the bit error probability, many scientists not only study the channel parameters as in [2,3] but as in [4–7] they have also used multilevel coding as an important band and power efficient technique, since it provides significant amount of coding gain and low coding complexity. Multilevel encoder is a combination of several error correction codes applied to subsets of some signal constellation. The multilevel coding scheme employs, at each signaling interval one or more output bits of each of several binary error-control encoders to construct the signal to be transmitted. An important parameter of a coded modulation scheme is the computational complexity of the decoder. Usually, a kind of suboptimal decoder, called the multistage decoder, is used for multilevel codes [6–8]. Nowadays, there are also many...

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attempts to improve the performance of multilevel turbo-based systems. In [9] the author discussed the impact of interleaver size on the performance of multilevel turbo-based systems. Power and bandwidth efficiencies of multilevel turbo codes are also discussed in [10,11]. In these papers authors have used PSK and QAM whereas we used continuous phase modulation (CPM) as modulation type. On the other hand, CPM modulated turbo coding schemes are studied and proposed in [12,13]. In band-limited channels, such as deep space and satellite communications, continuous phase modulation has explicit advantages, since it has low spectral occupancy property. CPM can be composed of continuous phase encoder (CPE) and memoryless mapper (MM) [14]. CPE is a convolutional encoder producing codeword sequences that are mapped onto waveforms by the MM, creating a continuous phase signal. CPE related schemes have better performance than systems using the traditional approach for a given number of trellis states, since they increase Euclidean distance. Once the memory structure of CPM is assigned, it is possible to design a single joint convolutional code, composed of trellis and convolutionally coded CPM systems as in [14–16].

To improve error performance and bandwidth efficiency, we combine multilevel turbo coding and continuous phase modulation and thus we introduce multilevel turbo coded–continuous phase frequency shift keying (MLTC–CPFSK).

In MLTC–CPFSK transmitter, each Turbo Encoder is defined as a level. Since the number of encoders and decoders are the same, for each level of the multilevel encoder, there is a corresponding decoder, defined as a stage. Furthermore, except the first decoding stage, the information bits, which are estimated from the previous decoding stages, are used for the next stages. This process is known as multistage decoding (MSD).

This paper is organized as follow: continuous phase frequency shift keying described in Section 2. In Section 3, Multilevel turbo encoder and decoder is explained and as example, two level Turbo coded 4-ary CPSK system and three level Turbo coded 8-ary CPFSK system are investigated. The error performance of the proposed system is discussed in Section 4.

2. Continuous phase frequency shift keying

Here, we study on M-ary continuous phase frequency shift keying, M-ary CPFSK, which is an M dimensional form of CPM. Rimoldi derived the tilted-phase representation of CPM in [14], with the information bearing phase given by

$$\phi(t, Y) = 4\pi h \sum_{i=0}^{\infty} Y_i q(t - iT)$$

(1)

The modulation index \( h \) is equal to \( J/P \), where \( J \) and \( P \) are relatively prime integers. \( Y \) is an input sequence of M-ary symbols, \( Y_i \in \{0,1,2,\ldots,M-1\} \). \( T \) is the channel symbol period. \( J \) is generally chosen as 1 and modulation index appears in the form of \( h = 1/P \). \( P \) is a number that can be calculated as 2 to the power of \( A \), number of memories in CPE. The phase response function \( q(t) \), is a continuous and monotonically increasing function subject to the constraints

$$q(t) = \begin{cases} 0 & t \leq 0 \\ 1/2 & t > LT \end{cases}$$

(2)

where \( L \) is an integer. The phase response is usually defined in terms of the integral of a frequency pulse \( g(t) \) of duration \( LT \), i.e., \( q(t) = \int_{-\infty}^{t} g(\tau) d\tau \). For full response signaling \( L \) equals to 1, and for partial response systems \( L \) is greater than 1. Finally, the transmitted signal \( s(t) \) is as

$$s(t, Y) = \frac{2E_s}{T} \cos \left( 2\pi f_1 t + \phi(t, Y) + \phi_0 \right)$$

(3)

where \( f_1 \) is the asymmetric carrier frequency as \( f_1 = f_c - h(M-1)/2T \) and \( f_c \) is the carrier frequency. \( E_s \) is the energy per channel symbol and \( \phi_0 \) is the initial carrier phase. We assume that \( f_1T \) is an integer; this condition leads to a simplification when using the equivalent representation of the CPM waveform.

3. Multilevel turbo CPFSK encoder and decoder structure

In this Section, we explain multilevel turbo CPFSK encoder and decoder structures. As an example, two level Turbo CPFSK scheme is investigated.

3.1. Encoder structure

Multilevel turbo CPFSK encoder and decoder consist of many parallel turbo encoder/decoder levels as in Fig. 1. There is one binary Turbo encoder at every level of multilevel turbo encoder and continuous phase encoder (CPE) is placed after the last level turbo encoder. Here, CPE is used to achieve the phase continuity of the transmitted signals and its state indicates the initial phase of the instant signal which is given in (3) as \( \phi_0 \). Each turbo encoder is fed from one of the input bit streams, which are processed simultaneously. The outputs of these encoders can be punctured and thereafter only the last level output is passed through CPE. Then, these outputs are mapped to M-ary CPFSK signals according to the partitioning rule.

In partitioning, \( x_{k,1} \) is the output bit of the first level turbo encoder where signal set is divided into two subsets. If \( x_{k,1} = 0 \) then the first subset is chosen, if \( x_{k,1} = 1 \) then the second subset is chosen. The \( x_{k,2} \) bit is the output bit of the second level
turbo encoder and divides the subsets into two same as previous levels. This partitioning process continues until the last partitioning level is occurred. At the last level, to provide phase continuity CPE encoder is placed after last level turbo encoder. Therefore, at this level signal set is divided twice and hence the signal, which will be sent to channel, is selected.

In Fig. 2, as an example, we show multilevel turbo coding system for 4-ary CPFSK two Level Turbo Codes (2LTC–CPFSK). In each level, we consider a 1/3 recursive systematic convolutional (RSC) encoder with memory size two. For each level, input bit streams are encoded by the turbo encoders. At turbo encoder outputs, the encoded bit streams to be mapped to M-ary CPFSK signals are determined after a puncturing procedure. The first bit is taken from the first level turbo encoder output, the second bit is taken from second level encoder output and the other bits are obtained in similar way. Following this process, the bits at the outputs of the turbo encoders and continuous phase encoder are mapped to 4-ary CPFSK signals by using set partitioning technique, which is mentioned above. Set partitioning of 2LTC–CPFSK is shown in Fig. 3. Here, if the output bit of the first level turbo encoder is \( x_{k,1} \), then \( u_1^0 \) set, if it is \( x_{k,1} = 1 \), then \( u_1^1 \) set is chosen. The first output bit \{\( x_{k,2,1} \)\} of the CPE determines whether \( u_2^1 \) or \( u_2^0 \) subsets to be chosen and the second output bit \{\( x_{k,2,2} \)\} of the CPE selects the signal, which will be transmitted using previous partitioning steps.

In this example, the remainder \( w_k \) of the encoder can be found using feedback polynomial \( g^{(0)} \) and feedforward polynomial is \( g^{(1)} \). The feedback variable is

\[
w_k = d_k + \sum_{j=1}^{K} w_{k-j}g_j^{(0)}
\]  

and RSC encoders outputs \( x_k^{1,2} \) which called parity data is

\[
x_k^{1,2} = \sum_{j=0}^{K} w_{k-j}g_j^{(1)}
\]  

Where \( A \) indicates the number of half recursive systematic encoder for each level of MLTC encoder and \( \Phi \) shows the level number of overall MLTC encoder.

In our case, RSC encoder has feedback polynomial \( g^{(0)} = 7 \) and feedforward polynomial \( g^{(1)} = 5 \), and it has a generator matrix. Here \( D \) is memory unit.

\[
G(D) = \begin{bmatrix} 1 & 1+D_1+D_2^2 \\ 1+D_1+D_2^2 & 1+D_1+D_2^2 \end{bmatrix}
\]  

3.2. Decoder structure

The problem of decoding turbo codes involves the joint estimation of two Markov processes, one for each constituent code. While in theory, it is possible to model a turbo code as a single Markov process, such a representation is extremely complex and does not lend itself to computationally tractable decoding algorithms. In Turbo decoding, there are two Markov processes, which are defined by the same set of data, hence, the estimation can be refined by sharing the information between the two decoders in an iterative fashion. More specifically, the output of one decoder can be used as a priori information by the other decoder. If the outputs of the individual decoders are in the form of hard-bit decisions, then there is little advantage to sharing information. However, if the individual decoders produce soft-bit decisions, considerable performance gains can be achieved by executing multiple iterations of decoding. The received signal can be shown as

\[
r_k = r_k u_k + n_k
\]
where \( r_k \) is noisy received signal, \( u_k \) is transmitted MLTC–CPFSK signal, \( \rho_k \) is fading parameter and \( n_k \) is Gaussian noise at time \( k \). The maximum a posteriori (MAP) algorithm calculates the a posteriori probability of each bit.

Let 
\[
\gamma^d(s_k \rightarrow s_{k+1}) = \ln \gamma^d(s_k \rightarrow s_{k+1})
\]
\[
\ldots = \ln P[d_k] + \ln P[r_k|x_k]
\]
where, 
\[
\ln P[d_k] = z_k d_k - \ln(1 + e^{z_k})
\]
\( z_k \) is the a priori information which is obtained from the output of the other decoder. For every decoding stage of MLTC–CPFSK, zero and one probabilities \( \{P_{k,0}, P_{k,1}\} \) of the received signals are calculated at time \( k \) and decoding stage \( st \) as below,

\[
\text{Fig. 2. 2LTC–CPFSK system (a) Encoder structure, (b) Decoder structure.}
\]
\[ P_{k,0}^* = \sum_{j=0}^{(M/2)-1} \frac{1}{(v_k - u_{0,j}^*)^2} \] (10a)
\[ P_{k,1}^* = \sum_{j=0}^{(M/2)-1} \frac{1}{(v_k - u_{1,j}^*)^2} \] (10b)

where, \( M \) is the dimension of CPFSK modulation, \( v_k \) is CPFSK demodulator output and \( \{ u_{0,j}^*, u_{1,j}^* \} \) are signal sets which are obtained by the set selector using previous stage turbo decoder output \( s_{d+1} \). In MLTC–CPFSK scheme, each digit of binary correspondence of \( M \)-ary CPFSK signals, matches to one stage from most significant to least significant while stage level \( s_t \) increases. Signal set is partitioned into the subsets due to the each binary digit matching stage depending on whether it is 0 or 1. After computing the one and zero probabilities, are in Eqs. (10a) and (10b), received signal is mapped to \([-1,1]\) range using zero and one probabilities of the received signal as

\[ \zeta^t_k = 1 - \frac{2 \cdot P_{k,0}^*}{P_{k,0}^* + P_{k,1}^*} \] (11)

These probability computations and mapping are executed in every stage of decoding process according to the signal set. In our decoder scheme in Fig. 1, signal set selector operates using (10) and (11). In Thus, Eq. (8) becomes

\[ \gamma^t(s_k \rightarrow s_{k+1}) = \ln P[d_k] - \frac{1}{2} \ln(\pi N_0/E_s) - \frac{E_s}{N_0} \sum_{q=0}^{M/2-1} [\zeta^t_k - (2x^t - 1)]^2 \] (12)

Now let \( \tilde{\alpha}^t(s_k) \) be the natural logarithm of \( \alpha^t(s_k) \),

\[ \tilde{\alpha}^t(s_k) = \ln \alpha^t(s_k) \]
\[ = \ln \left\{ \sum_{s_{k-1} \in A} \exp \left[ \tilde{\alpha}^t(s_{k-1}) + \gamma^t(s_{k-1} \rightarrow s_k) \right] \right\} \] (13)

where \( A \) is the set of states \( s_{k-1} \) that are connected to the state \( s_k \). Now let \( \tilde{\beta}^t(s_k) \) be the natural logarithm of \( \beta^t(s_k) \),

\[ \tilde{\beta}^t(s_k) = \ln \beta^t(s_k) \]
\[ \ldots = \ln \left\{ \sum_{s_{k+1} \in B} \exp \left[ \tilde{\beta}^t(s_{k+1}) + \gamma^t(s_k \rightarrow s_{k+1}) \right] \right\} \] (14)

where \( B \) is the set of states \( s_{k+1} \) that are connected to state \( s_k \), and we can calculate the log likelihood ratio (LLR) by using
where $S_1 = \{ s_k \rightarrow s_{k+1} : d_k = 1 \}$ is the set of all state transitions associated with a message bit of 1, and $S_0 = \{ s_k \rightarrow s_{k+1} : d_k = 0 \}$ is the set of all state transitions associated with a message bit of 0. At the last iteration we make the hard decision by using the second decoder output $A (st, 2)$.

Fig. 4. Performances of 2LTC–CPFSK system for $N = 100$.

Fig. 5. Performances of 2LTC–CPFSK system for $N = 1024$. 
4. Simulation results

The bit error ratio (BER) against signal to noise ratio (SNR) curves of two level turbo coded 4-ary CPFSK system and three level turbo coded 8-ary CPFSK system are obtained for AWGN, Rician (for Rician channel parameter $K = 10 \, \text{dB}$) and Rayleigh channels via computer simulations. The results are shown in Figs. 4–7. Here, frame sizes are chosen 100 and 1024.

\[
\hat{d}_k = \begin{cases} 
1 & \text{if } A(st, 2) \geq 0 \\
0 & \text{if } A(st, 2) < 0 
\end{cases}
\] (16)
For the purpose of comparison we selected two well-known best codes from literature, which are presented in [17] by Naraghi-Pour. In our study, these reference codes are called Ref-1 and Ref-2. Ref-1 is binary trellis coded 4-ary CPFSK scheme with $R_s = 2/3$ coding rate. Our first example, 2LTC–CPFSK system, is very suitable for comparison with Ref-1, since they both have 4-ary CPFSK with $R_s = 2/3$. In the same way, Ref-2 is comparable with our second example system, 3LTC–CPFSK. Ref-2 and 3LTC–CPFSK systems both use 8-ary CPFSK with $R_s = 1$. Our proposed systems have better error performance than Ref-1 and Ref-2 in all channels and SNR values. As an example, the proposed systems have coding gain between 3.1 dB and 8.3 dB for the same channels with a bit error rate of $10^{-4}$ when compared to reference systems as shown in Table 1. For the frame size 1024, at a bit error probability of $10^{-4}$, 2LTC–CPFSK system provides 4.2, 4.3, 5.5 dB coding gains over Ref-1 for AWGN, Rician and Rayleigh channels, respectively. For the same frame size and bit error probability, 3LTC–CPFSK system provides 5.9, 6.9, 8.3 dB coding gains over Ref-2 for AWGN, Rician and Rayleigh channels, respectively. Furthermore, there is only approximately 0.5 dB gain by increasing frame size from 100 to 1024. Thus, in our model, even if small frame sizes are chosen, sufficient bit error probabilities are obtained.

Classical turbo codes require large frame sizes and high number of iterations to obtain better bit error rates. These two major disadvantages of turbo codes are improved by MLTC–CPFSK schemes. As shown in Figs. 4–7 and Table 1, MLTC–CPFSK provides significant amount of coding gain with short frame sizes and a few iterations. While second iteration provides between 0.6–1.2 dB coding gains over first iteration, third iteration provides only 0.1–0.2 dB coding gain over second iteration. Therefore, two iterations are adequate. Thus, proposed system decreases iteration number and provides less sensitivity to frame sizes. Mentioned coding gains above are natural results of using Turbo codes and multilevel scheme. Because, Turbo codes give best performance in the literature and multilevel structure improves this performance.

5. Conclusion

In this paper, multilevel turbo coded CPFSK system is proposed. We use binary turbo codes instead of classical convolutional codes at each coding level in Imai–Hirakawa type multilevel coding scheme. Besides, we combine multilevel turbo codes and CPFSK modulation. Thus we obtain multilevel turbo coded CPFSK systems which has better error performance than the corresponding reference systems. In our scheme, decoding delay can be minimized, since sufficient bit error rate is reached in a few number of iterations. In this case, we have shown that, our scheme provides considerable coding gain up to 8.3 dB. Furthermore, since CPFSK is selected, our MLTC–CPFSK system has also bandwidth efficiency. Thus, our proposed system has important power and bandwidth advantages and it is very suitable for low-power and bandlimited applications such as satellite and mobile radio communications.

References

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