Interfaces with Other Disciplines

The impact of accessibility on the value of information
and the productivity paradox

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Abstract

The value of information systems availability is analyzed in this study through theoretical models of information economics. The article employs the information structure model to assess the values of information systems under various situations, with particular examples of the impact of data accessibility level on the quality of decision-making.

The study centers on the relationship between the information system’s time and content characteristics and the value of the information. It suggests a method to model the utility considerations that lead to the choice of an information system. The entailed models are employed to illuminate certain facets of the productivity paradox.

The results of the analysis indicate that there is a direct relationship between systems accessibility and its informativeness. Consequently, there are some aspects of the “Productivity Paradox” that may be explained by using these results. The article proves a number of theorems and discusses the theoretical and practical interpretation of the results.

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1. Introduction

Despite the ever-increasing importance of the information resource to the organization, evaluating the actual benefit of using information systems remains problematic (Ahituv, 1980, 1989).

Surprisingly, no direct relationship between the investment in information systems and organiza-
tional performance has been found, a phenomenon that has been termed “the productivity paradox” (Brynjolfsson, 1993). The difficulties in forecasting the benefits at the design stage, and evaluating the actual utility during the operating stages, have led to the realization that the existing tools for measuring the utility of information systems are problematic (Jurison, 1997) and do not reflect the full value of the information. New directions for evaluation information clearly need to be developed.

Brynjolfsson and Hitt (1998) claim that the productivity paradox may arise from the fact that information economics area is in the throes of an
By extending the model to cases of no information, this study will describe situations of lack of information, and try through these cases to explain various aspects of the productivity paradox in a normative fashion. It will be shown how technological and environmental factors affect the value of information. A quantitative model will demonstrate how the general informativeness ratio between two systems under complete accessibility is not retained under incomplete accessibility level. The study develops a model to calculate the value of an information system that deals with only a subset of the possible states of nature. This model enables to explain in a normative fashion why it is sometimes preferable to focus on the computerization of only some common states of nature.

1.2. The structure of the article

The next section reviews pertinent literature in the area of information economics, focusing on the productivity paradox. Section 3 consists a focus on the information structure model and the Blackwell Theorem (McGuire and Radner, 1986, Chapter 5). Section 4 describes the process of analyzing the factors that affect the value of the information contained in different data configurations and discusses the basic premises underlying the models. Moreover, It describes the models in light of these underlying premises. Sections 5–7 present theorems and provide examples that illustrate the various aspects of the productivity paradox. The focus of Section 5 is on the diminishing performance of an information system over time due to environmental changes. Section 6 shows how the general informativeness ratio between two information systems under complete accessibility is not retained under incomplete accessibility level. Section 7 delineates how an information system that deals with only a subset of the states of nature is sometimes preferable to an information system that deals with all the states of nature. Section 8 provides a summary and conclusions, and presents the contribution of the study and its significance. Proofs of the theorems and lemmas appear in the appendices.
2. Analytical research into the value of information in the area of information economics

An early mathematical model presenting the relaying of information in a quantitative form was that of Shannon (Shannon and Weaver, 1949). The model distinguished between two situations:

(1) a noise-free system—a univalent fit between the transmitted input data and the received signals;

(2) a noisy system—the transmitted input data (denoting a state of nature) are translated into signals probabilistically.

In assigning an expected normative economic value to information, some researchers made use of Microeconomics and Decision Theory tools (Raiffa, 1968). The combination of utility theory and the perception of a noisy system led to the construction of a probabilistic statistical model that accords to an information system the property of transferring input data (states of nature) to output (signals) in a certain statistical probability (Feltham, 1968; Marschak, 1971, see also the collection of articles on the subject edited by McGuire and Radner (1986)). This model, which delineates a noisy information system, is called the information structure model. It is based on the assumption that the system is noisy but it does not analyze the nature of the noise. In this study, we analyze potential characteristics of the noise and construct a model that attempts to deal with two types of noise:

(1) An information system that produces a no-information signal in a situation of absence of information, that is, a system that informs the user that it cannot produce a meaningful signal under certain situations. The user knows that he or she does not know (for example, when someone asks me for Julia Roberts’ home telephone number I can say immediately that I do not know it. The decision-maker, that is the person requesting the information, knows that he or she has received a no-information signal. Similarly, when an Internet search request is responded with no items).

(2) An information system that produces a random signal in the case of lack of information. The user does not know that he or she does not know (for instance, continuing the previous example, if the answer would be a series of random digits in the form of a telephone number, the decision-maker does not know that he or she does not know).

This study addresses the issue of not knowing. It develops models to represent lack of information, and analyzes general informativeness ratios for different levels of lack of information.

Beyond the problematic nature of empirically measuring the value of information, decision-makers have to consider also the productivity paradox, that is, the basic question of whether the business expenditures on information technology are justified. It is hard to prove the relationship between information technology and the organization’s business achievements (Brynjolfsson, 1993).

Using information technologies entails certain managerial problems. For example, according to Stressmann (1997), in the food industry there is no connection between computerization and productivity. Jurison (1997), for his part, claims that productivity is an important measure of organizational success, but in the knowledge era, much value that is hard to measure is produced from the information. He also maintains that the existing measurement tools are effective for traditional industries but not for information-based business areas. Ahituv and Elovici (1995) tried to resolve the productivity paradox by introducing the time dimension into the information structures model, by presenting the information value of information technology over time.

In the present work, an attempt will be made to resolve several aspects of the productivity paradox: the need for the information system to respond more quickly as the environment changes over time (Section 5); the fact that in certain cases an incomplete level of accessibility detracts from the general informativeness ratio between systems, as for instance when an information system deals with only a subset of the set of states of nature (Section 7).
3. The information structure model and the Blackwell Theorem

The tool employed to investigate the phenomena described later in Section 4 is the information structure model (McGuire and Radner, 1986, Chapter 5, pp. 101–109). This is a general model for comparing information systems based on the rules of rational behavior. According to the information structure model, four factors determine the expected value of information.

1. The a priori probabilities of pertinent states of nature.
2. The information system—a stochastic (Markovian) matrix that transmits states of nature into signals.
3. The decision matrix—a stochastic matrix that links signals with the decision set of the decision-maker.
4. The payoff matrix—a matrix that presents the quantitative compensation to the decision-maker resulting from the combination of a decision chosen and a given state of nature.

The information structure model enables comparison of information systems in terms of their quantitative economic value. An information structure \( Q_1 \) is said to be more informative than an information structure \( Q_2 \) if the expected payoff of using \( Q_1 \) is not lower than the expected payoff of using \( Q_2 \). Two information structures may be compared at several levels:

1. Comparison between two information structures for a given vector of a priori probabilities and a given payoff matrix.
2.1 Comparison between two information structures when the vector of probabilities for the occurrence of states of nature is given and all payoff matrices are possible (a generalization for all possible compensations).
2.2 Comparison between two information structures when the payoff matrix is known in advance and the vector of a priori probabilities of states of nature is not known in advance (a generalization for all possible probabilities).

3. Comparison between two information structures for all vectors of a priori probabilities of the occurrence of states of nature and any possible payoff matrices.

When information structure \( Q_1 \) is more informative than information structure \( Q_2 \) according to level 3 above (irrespective of compensations and a priori probabilities), a general informativeness ratio is defined between them. This ratio and the conditions for its existence are defined in the Blackwell Theorem (McGuire and Radner, 1986, Chapter 5).

Let us now consider a model in which \( S \) is a finite set of \( n \) states of nature: \( S = \{ S_1, \ldots, S_n \} \). Let \( P \) be the vector of a priori probabilities for each of the states of nature: \( P = (p_1, \ldots, p_n) \).

Let

\[
\Pi = \begin{pmatrix}
p_1 & 0 & \ldots & 0 & 0 \\
0 & p_2 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & p_n & 0
\end{pmatrix},
\]

where \( \Pi \) is a square matrix the diagonal of which is the a priori probabilities of obtaining states of nature and the remaining elements are equal to 0.

Let \( A \) be a finite set of \( k \) possible decisions, \( A = \{ A_1, \ldots, A_k \} \).

Let \( U \) be the payoff function: \( U : A \times S \rightarrow \mathbb{R} \) (a combination of state of nature and decision gives a fixed compensation that is a real number).

Let \( Y \) be a finite set of \( m \) signals, \( Y = \{ Y_1, \ldots, Y_m \} \).

A information structure \( Q \) is defined such that its elements obtain values between 0 and 1, \( Q : S \times Y \rightarrow [0,1] \).

\( Q_{ij} \) is the probability that a state of nature \( S_i \) displays a signal \( Y_j \).

\[
\sum_{j=1}^{m} Q_{ij} = 1.
\]

Let \( D \) be the decision function. Like \( Q \), \( D \) is a stochastic (Markovian) matrix, namely, it is assumed that the decision is not necessarily always the same for a given signal.

\( D : Y \times A \rightarrow [0,1] \).

The expected payoff is \( \text{trace}(\Pi \ast Q \ast D \ast U) \), where trace is an operator that sums the diagonal ele-
ments of the square matrix. The objective function for maximizing the expected compensation is
\[
\max_D (\text{trace}(\Pi * Q * D * U)).
\]

When the utility function is linear, that is, the decision-maker is of the type EMV (Raiffa, 1968), a linear programming problem is obtained, where the variables being the elements of the decision matrix \( D \). It can be proved that at least one of the optimal solutions is in a form of a decision matrix whose elements are 0 or 1 (a pure decision rule) (McGuire and Radner, 1986, Chapter 5).

### 3.1. The general informativeness ratio

Given two information systems that deal with the same state of nature and are represented by the information structures \( Q_1 \) and \( Q_2 \), \( Q_1 \) will be considered generally more informative than \( Q_2 \) if its expected payoff is not lower than that of \( Q_2 \) for all a priori probability vectors and any payoff matrix. In terms of the information structure model, if for every possible payoff matrix \( U \) and for every a priori probability matrix \( \Pi \),
\[
\max_D (\text{trace}(\Pi * Q_1 * D * U)) \geq \max_D (\text{trace}(\Pi * Q_2 * D * U)),
\]
then \( Q_1 \) is generally more informative than \( Q_2 \), denoted: \( Q_1 \geq Q_2 \).

The Blackwell Theorem states that \( Q_1 \) is generally more informative than \( Q_2 \) if and only if there is a Markovian (stochastic) matrix \( R \) such that \( Q_1 * R = Q_2 \).

It should be noted that the general informativeness ratio is a partial rank ordering of information structures. There is no necessary rank order between any two information structures. The rank ordering is transitive.

### 4. The model

#### 4.1. Characterizing the impact of information system configuration on performance

In order to analyze the performance of an information system, we first have to try to identify the factors that affect the system’s quality, the system parameters by which they are expressed, and how.

**Components relating to data storage and accessibility**: information systems differ in the volume of data they store. Methods of managing the database and particularly decisions on whether it is to be centralized or decentralized are among the essential questions that have to be addressed in designing the system (Stedman, 1996; Kimball, 1996). The database has much influence on user satisfaction with the information system (Fadlalla and Gorla, 1997).

**Components relating to data transmission**: data transmission is a factor strongly affecting information system performance. Planning the network structure and network capacity is the most important task of the network manager (Halterman, 1996; Menenbloom, 1996). Furthermore, today in the “World-Wide-Web” based digital arena information can be retrieved or accessed anytime and anywhere. Hence, the accessibility level and the fast response of information systems (especially, web based information systems), becomes crucial in order to obtain competitiveness.

### 4.2. Identifying the factors that are affected by the information configuration

#### 4.2.1. Accessibility

**System accessibility** depends, among others, on the system’s complexity, that is the level of its dependency on infrastructure software e.g., a complex operating system, applications and projects for managing different levels of communications, complex databases, backups, routines and the liked. The more complex the system is in terms of these components, the greater the likelihood that one or another of them will break down and impair the system’s accessibility. This enables us to ascribe percentages to system accessibility. **Accessibility of data** is defined in a negative form. Data are not available if they do not exist in the system or if they are so out of date that they serve no useful purpose.

**Level of accessibility**: let \( p \) be the probability that an information system gives an effective
response to a query. An effective response will be obtained if the system is accessible for a period of time sufficient for decision-making and if the system can provide the user with the information required.

Modeling complete inaccessibility decision situations: an information system that deals only with a certain percentage of the queries presented to it, because of data or system accessibility problems.

Modeling partial inaccessibility decision situations: an information system that is capable of dealing with only a subset of the pertinent states of nature, because of data accessibility or system accessibility problems. Moreover, improper analyzing, designing or software development might partially reduce the functionality of information systems or might lead to unexpected results ("bugs"). Such failures can also be modeled sometimes by partial accessibility of information systems.

4.2.2. The critical response time

Response time might be critical in various systems such as in medical systems (monitors) and flight control systems. Let \( t_0 \) be the critical response time of the system, that is, the maximal time permitted from the moment the user sends a request to the system to perform a processing and/or retrieve data until the time the reply becomes ineffective. A reply received after the critical time means in fact that the user did not receive an effective response and remains in a situation of no-information (practically the system is defined as not accessible), similar to his or her initial information before issuing the request.

4.2.3. The effect of the signal produced by the system when it is not accessible (either in partial inaccessibility or in complete inaccessibility situations)

The user is not aware of the non-accessibility situation: in a situation of non-accessibility, the system randomly produces a signal of some sort.

The user knows that the system is not accessible: in a situation of non-accessibility, the system produces a no-information signal indicating that the system is not accessible.

4.3. Typical information structures

4.3.1. A general information structure representing total inaccessibility (lack of information)

Let \( M(Q) \) be an information structure describing a situation of total absence of information for sets of states of nature \( S \) and signals \( Y \) of the information structure \( Q \) such that

\[
\forall i, \ 1 \leq i \leq n, \ \forall j, \ 1 \leq j \leq m, \quad M(Q)_{ij} = P(Y_j|S_i),
\]

\[
\forall i, \ 1 \leq i \leq n, \ \forall k, \ 1 \leq k \leq n, \ \forall j, \ 1 \leq j \leq m, \quad M(Q)_{ij} = M(Q)_{ik}
\]

(Note: The definition is not dependent on the a priori probabilities of the states of nature.)

Example:

\[
0 < a < 1, \quad Y = \{Y_1, Y_2\}, \quad S = \{S_1, S_2\},
\]

\[
M(Q) = \begin{pmatrix}
a & 1-a \\
1-a & a
\end{pmatrix}
\]

Explanation: The interpretation of such an information structure is that it is not possible to know about the event (state of nature) that caused a given signal anything more than can be elicited from the information existing due to knowing the a priori probabilities. Thus, in the above example, the probability assigned to \( S_1 \) after seeing the signal \( Y_1 \) or \( Y_2 \) is identical to the a priori probability of \( S_1 \). This information structure presents total absence of data availability, or, alternatively, it describes an information system that does not provide any added value.

A special type of \( M(Q) \) is \( N(Q) \): an information structure representing lack of information—a random signal with uniform distribution.

Let \( N(Q) \) be an information structure describing a situation of absolute lack of information for a set of states of nature \( S \) and signals \( Y \) of an information structure \( Q \), that is, a random signal with uniform distribution is received in a situation of lack of information (the probabilities of a signal of any type being displayed are equal). \( N(Q) \) represents a situation of absence of information and is described as follows:

\[
\forall i, \ i = 1 \ldots n, \ \forall j, \ j = 1, \ldots, m :
\]

\[
N(Q)_{ij} = P(Y_j|S_i) = 1/m.
\]
For example,
\[ Y = \{Y_1, Y_2\}, \quad S = \{S_1, S_2\}, \quad N(\mathcal{Q}) = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}. \]

4.3.2. An information structure for an information system that identifies no-information situations, and an information structure representing the display of a no-information signal

Let \( \mathcal{Q} \) be an information structure describing an information system. Let \( S = \{S_1, \ldots, S_n\} \) be the set of the states of nature of \( \mathcal{Q} \). Let \( Y = \{Y_1, \ldots, Y_m\} \) be the set of signals of \( \mathcal{Q} \). Let \( Y_{m+1} \) be a no-information signal, which is not part of \( Y \).

Clarification: \( Y_{m+1} \) is a signal that is not displayed during normal system operations, but only when a situation of absence of information is identified. This signal aims to indicate situations in which the system is not accessible for the user and identifies itself as such. That is, the system informs the user that it is not informative. For example: a browser that indicates that a requested site is not available.

Let \( \mathcal{Q}_0 \) be an information structure describing an information system with the addition of a no-information signal. Let \( S = \{S_1, \ldots, S_n\} \) be the set of the states of nature of \( \mathcal{Q}_0 \).

Let \( Y_0 = \{Y_1, \ldots, Y_m, Y_{m+1}\} \) be the set of signals of \( \mathcal{Q}_0 \).

\[ \mathcal{Q}_{0,i,j} = Q_{i,j}, \quad 1 \leq i \leq n, \quad 1 \leq j \leq m, \]

\[ \mathcal{Q}_{0,i,j} = 0, \quad 1 \leq i \leq n, \quad j = m + 1. \]

\( \mathcal{Q}_0 \) will be used to represent the information structure \( \mathcal{Q} \) in the models developed later to present information systems that provide a no-information signal. This is possible because they are identical in their level of informativeness (Ahituv, 1981).

4.3.2.1. An information structure representing the display of a no-information signal. Let \( N_0(\mathcal{Q}) \) be a structure that represents a no-information signal for information structure \( \mathcal{Q} \). Let \( S = \{S_1, \ldots, S_n\} \) be the set of the states of nature of \( \mathcal{Q} \). Let \( Y = \{Y_1, \ldots, Y_m\} \) be the set of signals of \( \mathcal{Q} \). Let \( Y_{m+1} \) be a no-information signal that is not part of the set of signals of \( \mathcal{Q} \). Let \( S \) be its set of states of nature and \( Y_0 = \{Y_1, \ldots, Y_m, Y_{m+1}\} \) the set of signals of \( N_0(\mathcal{Q}) \).

\[ N(\mathcal{Q})_{i,j} = 0, \quad \text{where } 0 \leq j \leq m, \]

\[ N(\mathcal{Q})_{i,j} = 1, \quad \text{where } j = m + 1. \]

Example: \( S = \{S_1, S_2\}, \quad Y = \{Y_1, Y_2\}, \quad Y_0 = \{Y_1, Y_2, Y_3\}; \quad Y_3 \) is the no-information signal, \( N_0(\mathcal{Q}) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \).

Comment: The definitions of \( \mathcal{Q}_0 \) and \( N_0(\mathcal{Q}) \) are given in order to present two extreme situations of accessibility of an information system in which a no-information signal is produced.

4.4. Models for representing the effects of information system variables

4.4.1. Presentation of a model of an information structure that takes into account the combination of system accessibility and information accessibility

Let \( \mathcal{Q} \) be an information structure that describes a normative information system. Let \( p \) be the probability that the system and the information in the system is available.

Example: \( \mathcal{Q}_p \), an information structure representing an information system at an accessibility level of \( p \), is described as follows. When the system produces a random signal in a situation of non-accessibility:

\[ \mathcal{Q}_p = p \ast \mathcal{Q} + (1-p) \ast N(\mathcal{Q}) \]

Suppose,

\[ p = 0.8, \quad N(\mathcal{Q}) = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}, \quad \mathcal{Q} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \]

Then,

\[ \mathcal{Q}_p = 0.8 \ast \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + (1-0.8) \ast \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}. \]

4.4.2. An information system that deals with only a subset of the states of nature, due to data accessibility problems

Let \( \mathcal{Q} \) be an information structure describing an information system. Let \( S = \{S_1, \ldots, S_n\} \) be the set
of the states of nature of $Q$. Let $Y = \{Y_1, \ldots, Y_m\}$ be the set of signals of $Q$. Let $Q_{l,0}$ be an information structure representing an information system that deals with only part of the states of nature with which $Q$ deals. We denote this partial set of states of nature $\{S_1, \ldots, S_k\}$, such that $n > k$, and it is complete accessible (with a probability of 1). The system responds to the complementary set of states of nature with which it cannot deal, with a special signal indicating that the system does not have meaningful signals relating to the given states of nature.

Let $Y = \{Y_1, \ldots, Y_m, Y_{m+1}\}$ be the set of signals of $Q_{l,0}$, where $Y_{m+1}$ is the no-information (no-access) signal.

\[
\forall i, \; i = 1, \ldots, k, \; j = m+1, \quad Q_{l,i,j} = 0,
\]

\[
\forall i, \; i = 1, \ldots, k, \; \forall j, \; j = 1, \ldots, m, \quad Q_{l,i,j} = Q_{i,j},
\]

\[
\forall i, \; i = k+1, \ldots, n, \; j = m+1, \quad Q_{l,i,j} = 1,
\]

\[
\forall i, \; i = k+1, \ldots, n, \; \forall j, \; j = 1, \ldots, m, \quad Q_{l,i,j} = 0.
\]

See Fig. 1.

**Example:**

\[
Q = \begin{pmatrix}
0.9 & 0.1 & 0 \\
0.1 & 0.9 & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

$k = 1$ ($Q_{l,0}$ deals only with a state of nature $S_1$),

\[
Q_{l,0} = \begin{pmatrix}
0.9 & 0.1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

5. The diminishing value of probabilistically accessible information systems over time

In order to calculate the expected value of sampled information (EVSI) (Raiffa, 1968) of an information system that is accessible only at a certain probability, we use a model that presents the probability of accessibility as follows. Let $Q_1$ be an information structure. Let $S = \{S_1, \ldots, S_n\}$ be the set of the states of nature of $Q_1$. Let $Y = \{Y_1, \ldots, Y_m\}$ be the set of signals of $Q_1$. Let $M(Q_1)$ be a structure that represents lack of information. $M(Q_1)$ operates on $S$ as the set of states of nature, and produces $Y$ as the set of signals.

\[
\forall i, \; 1 \leq i \leq n, \quad \forall k, \; 1 \leq k \leq n, \quad \forall j, \; 1 \leq j \leq m,
\]

\[
M(Q_{l,1})_{i,j} = M(Q_{1})_{k,j}
\]

(that is, a structure whose rows are all identical). Suppose that $p$ is the level of accessibility of $Q_1$. We denote the actualization of the information structure $Q_1$ at a level of accessibility $p$ by $Q$, according to the model. $Q$ is the convex combination of $Q_1$ and $M(Q_1)$

\[
Q = p * Q_1 + (1 - p) * M(Q_1).
\]

Theorem 1 below shows that the higher the level of accessibility of an information system, the higher is the level of its general informativeness.

**Theorem 1.** Let $Q$ be an information structure. Let $S = \{S_1, \ldots, S_n\}$ be the set of states of nature of $Q$. Let $Y = \{Y_1, \ldots, Y_m\}$ be the set of signals of $Q$. Let $M(Q)$ be a structure representing lack of information. Let $S$ be its set of states of nature and $Y$ its set of signals.

\[
\forall i, \; 1 \leq i \leq n, \quad \forall k, \; 1 \leq k \leq n, \quad \forall j, \; 1 \leq j \leq m,
\]

\[
M(Q)_{i,j} = M(Q)_{k,j}
\]

(an information structure whose rows are all identical). Let $Q_1$ be a convex combination of $Q$ and $M(Q)$.

\[
Q_1 = p_1 * Q + (1 - p_1) * M(Q),
\]

where $p_1$ is the level of accessibility (certainty) of $Q_1$.

Let $Q_2$ be a convex combination of $Q$ and $M(Q)$.

\[
Q_2 = p_2 * Q + (1 - p_2) * M(Q),
\]

Fig. 1. An information system that deals with only some of the states of nature.
where \( p_2 \) is the level of accessibility (certainty) of \( Q_2 \). Suppose \( 0 \leq p_2 \leq p_1 \leq 1 \). Then \( Q_1 \) is generally more informative than \( Q_2 \).

The proof appears in Appendix A.

From Theorem 1 it is clear that the value of an information system is a function of the system's accessibility and the accessibility of the information in the system. The relationship between the choice of an information structure and the value of the information can be expressed.

Theorem 1 may also be applied to demonstrate the productivity paradox as follows: Let \( Q \) be an information system, and suppose that in the period \( T_0 \), the critical response time required of the system was \( t_0 \). Environmental changes such as new information technologies and faster response times of competitors have entailed that in the period \( T_1 \), the critical response time required of the system became \( t_1 \), where \( t_1 < t_0 \). The system's level of accessibility dropped from \( p_0 \) to \( p_1 \), that is, the information system has become generally less informative, even without any deterioration or change of the system itself. If such changes occur during the time the system is under development, then its productivity is in doubt on the first day it becomes operational.

Example: Let \( Q \) be an information structure representing a strategic information system of investment agency for forecasting the profitability of investment in the stocks market. When a user ask for recommendation about a specific stock, the system has to inform him or her whether the stock is attractive for investment. For the sake of simplicity, suppose that there is only one stock (Stock \( A \)). Table 1 gives the possible states of nature and the signals received from the information system.

In 1999 (during \( T_0 \)), the required system response time was 10 min, and the system was accessible in 90% of the cases. In 2001 (during \( T_1 \)), in the face of ever more keen competition from other investment agencies, the response time required of the system was 5 min, and the system was accessible at a probability of 80%. Furthermore, in 2001 the firm installed an improved and generally more informative information system. Infrastructure upgrading in the form of a larger computer and additional communication lines also contributed to improving the level of accessibility from 80% to 82% at the required 5-minute response time.

Using the comparison of the information system in 1999 and 2001 presented in Table 2, we demonstrate how the value of the information diminishes.

- Though the improvement made during 2001 raised the accessibility of the system and increased the level of performance compared to the previous level, it did not return to the 1999 level.
- The investment in information technology did not bring the system to the previous level of performance. In this case, there is a need to improve the system just in order to survive, and not in order to improve performance.

6. Consistency of the general informativeness ratio at different levels of accessibility

Theorem 2 below deals with two completely accessible information systems operating on the same set of states of nature and having the same number of signals. The two systems produce a random signal in situations of non-accessibility. Initially there is a ratio of general informativeness between them—one is generally more informative than the other. The theorem proves that there is a possibility of not maintaining the same order of

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Table 1
Possible states of nature and signals received from the information system

<table>
<thead>
<tr>
<th>State of nature</th>
<th>Signals (mutually exclusive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_0 ) — Stock ( A ) is profitable</td>
<td>( Y_1 ) — Stock ( A ) is &quot;good&quot; investment</td>
</tr>
<tr>
<td>( S_2 ) — Item ( A ) is not profitable</td>
<td>( Y_2 ) — Item ( A ) is &quot;bad&quot; investment</td>
</tr>
</tbody>
</table>

The system produces a uniformly distributed random signal if the data are not available.
Table 2  
Accessibility probability: Diminishing information value

<table>
<thead>
<tr>
<th></th>
<th>1999</th>
<th>2001</th>
<th>2001 an improved information system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information structure</td>
<td>$Q_{old} = \begin{pmatrix} 0.9 &amp; 0.1 \ 0.2 &amp; 0.8 \end{pmatrix}$</td>
<td>$Q_{new} = \begin{pmatrix} 0.90 &amp; 0.10 \ 0.15 &amp; 0.85 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>Required response time</td>
<td>10 minutes</td>
<td>5 minutes</td>
<td>5 minutes</td>
</tr>
<tr>
<td>Level of accessibility $p$</td>
<td>90%</td>
<td>80%</td>
<td>82%</td>
</tr>
<tr>
<td>The weighted information structure</td>
<td>$Q_1 = \begin{pmatrix} 0.86 &amp; 0.14 \ 0.23 &amp; 0.77 \end{pmatrix}$</td>
<td>$Q_2 = \begin{pmatrix} 0.82 &amp; 0.18 \ 0.26 &amp; 0.74 \end{pmatrix}$</td>
<td>$Q_3 = \begin{pmatrix} 0.828 &amp; 0.172 \ 0.213 &amp; 0.787 \end{pmatrix}$</td>
</tr>
<tr>
<td>The matrix of a priori probabilities for the states of nature</td>
<td>$\Pi = \begin{pmatrix} 0.6 &amp; 0 \ 0 &amp; 0.4 \end{pmatrix}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The payoff matrix</td>
<td>$D = \begin{pmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{pmatrix}$</td>
<td>$D = \begin{pmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{pmatrix}$</td>
<td>$D = \begin{pmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{pmatrix}$</td>
</tr>
<tr>
<td>Optimal decision rule</td>
<td>(A&lt;sub&gt;1&lt;/sub&gt;—invest, A&lt;sub&gt;2&lt;/sub&gt;—do not invest)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max&lt;sub&gt;D&lt;/sub&gt;(trace($\Pi \ast Q \ast D \ast U$))</td>
<td>4.24</td>
<td>3.88</td>
<td>4.116</td>
</tr>
</tbody>
</table>

informativeness ratio when the accessibility is not perfect.

From this theorem it seems that sometimes inaccessibility causes the information system that is seemingly the generally more informative one to become inferior. That can explain a certain aspect of the productivity paradox. For example, installing a new system that was more informative at the design and planning stage will not bring about the expected improvement if the accessibility has not been improved, and since it is dependent on the payoff matrix, in fact the new system is no better in absolute terms.

**Theorem 2.** Let $Q_1$ and $Q_2$ be two information structures operating on the same set of states of nature. Let $S = \{S_1, \ldots, S_n\}$ be the set of states of nature of $Q_1$ and $Q_2$. Let $Y_1 = \{Y_{1,1}, \ldots, Y_{1,m}\}$ be the set of signals of $Q_1$ and let $Y_2 = \{Y_{2,1}, \ldots, Y_{2,m}\}$ be the set of signals of $Q_2$. $Q_1$ is generally more informative than $Q_2$, that is,

$$Q_2 = Q_1 \ast R,$$

where $R$ is a stochastic matrix.

Let $N(Q_1)$, $N(Q_2)$ be an information structures at absolute lack of information, suitable for the states of nature and the signals of $Q_1$ and $Q_2$, respectively. Suppose $N(Q_1) = N(Q_2)$.

Let $p$ be the level of accessibility of the systems $Q_1$ and $Q_2$.

Let us now examine the $j$ columns in which $\sum_{i=1}^{m} R_{ij} > 1$ and in which we denote the minimal element in the $j$ column as $\text{Min}_j R_{ij}$. A sufficient condition for $Q_3 = p \ast Q_1 + (1 - p) \ast N(Q_1)$ being generally more informative than $Q_4 = p \ast Q_2 + (1 - p) \ast N(Q_2)$ is that for each such column $j$ there exists

$$\text{Min}_j R_{ij} - (1 - p) \ast \left( \frac{1}{m} \sum_{i=1}^{m} R_{ij} - \frac{1}{m} \right) \geq 0,$$

where $\text{Min}_j R_{ij}$ is the minimal element of $R$ in the $j$ column and $\sum_{i=1}^{m} R_{ij}$ is the sum of the $j$ column elements.

It should be noted that this is a condition that ensures the existence of a stochastic matrix $R_2$ such that $Q_3 \ast R_2 = Q_4$, as will be proved later.

The proof of Theorem 2 appears in Appendix B.

The result of this theorem is that sometimes an information system that at a level of complete accessibility is generally more informative than another system, might become not generally more informative than the other system under incomplete accessibility (less than 1). This can explain an aspect of the productivity paradox: introducing a new system that is designed to be generally more
informative will not bring about the expected improvement if the accessibility is not improved and, in fact, the new system is no better in absolute terms due to the dependence on the payoff matrix. Indeed, there may even be a worsening of the expected payoff!

**Example:** Consider an information system for managing a firm’s inventory. The firm has an inventory of raw materials on the production floor and in the raw materials warehouse. For the sake of simplicity, let us suppose that there is only one item in the warehouse (Item $A$). Table 3 shows the possible states of nature and the signals received from the information system.

The warehouse manager has to decide whether to order raw materials from the supplier. Assume the information system is represented by $Q$:

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.2 & 0.8 \end{pmatrix}.$$  

The system represented by $Q$ is accessible at a probability of $P = 0.9$. Some time later it is decided to install a more accurate information system, represented by the information structure $Q_1$, which is also accessible at a probability of $P = 0.9$.

$$Q_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  

We shall now observe the geometrical meaning of not maintaining a general informativeness ratio. Let $N(Q)$ be an information structure representing the generation of a random signal in situations of lack of information:

$$N(Q) = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \\ 1 & 1 & 1 \\ 3 & 3 & 3 \\ 1 & 1 & 1 \\ 3 & 3 & 3 \end{pmatrix}.$$

Let $Q_3 = 0.9 * Q + 0.1 * N(Q)$, $Q_2 = 0.9 * Q_1 + 0.1 * N(Q)$. The likelihood matrix of the information structure is constructed by summing the elements of each column separately, dividing each element appearing in the column by the sum of the elements of that column, and substituting the new figure in its place. This is the process of normalizing the sum of the vectors of the columns to 1. The likelihood matrices $Q_1', Q_2', Q_3'$, constructed from the information structures are as follows:

$$Q_3' = \begin{pmatrix} 28 & 1 & 1 \\ 30 & 30 & 30 \end{pmatrix},$$  

$$Q_2' = \begin{pmatrix} 28 & 10 & 10 \\ 30 & 354 & 246 \end{pmatrix},$$  

$$Q_1' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  

The 3-dimensional geometric polygon of each likelihood function (for the three states of nature) is exhibited in Fig. 2. According to condition 4 of the Blackwell Theorem (McGuire and Radner, 1986, pp. 105–109), if an information structure is generally more informative than another one then the polygon derived from the first one contains the polygon derived from the second one. Let us now look at the geometric diagram of the likelihood matrix (Fig. 2), which represent geometrically the fourth condition of Blackwell Theorem (McGuire and Radner, 1986, pp. 105–109).
It can be seen that there is no general informativeness ratio between the structures: the point represented by the second column of $Q_2$ is not contained in the polygon created by $Q_3$ and, conversely, the point represented by the third column of $Q_3$ is not contained in the polygon created by $Q_2$.

According to Theorem 2, a sufficient condition for the existence of a general informativeness ratio between $Q_2$ and $Q_3$, is

$$\min_{j} R_{i,j} - (1 - p) \cdot \left( \frac{1}{m} \sum_{i=1}^{m} R_{i,j} - \frac{1}{m} \right) \geq 0,$$

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.2 & 0.8 \end{pmatrix},$$

where this case $m = 3$, $p = 0.9$ and thus if we look at the matrix $R_2$,

$$R_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.2 & 0.8 \end{pmatrix} - 0.1 \cdot \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}.$$

Closer inspection reveals that $R_{3,2} = -0.1 \cdot \frac{1}{3} \cdot (1.2 - 1) = -0.0066 < 0$ which means that $R_2$ is not a stochastic matrix.

Table 4 displays a comparison of the two information systems $Q_2$ and $Q_3$. Table 4 shows that in certain cases there might even be a deterioration in the system performance. $Q_3$ is not preferable to $Q_2$ (it is not generally more informative), because of the system’s behavior under conditions of lack of information.

This example demonstrates how, due to accessibility problems, introducing a new, improved and generally more informative system might not necessarily lead to an expected improvement. The new system is not better in absolute terms. Indeed, in this particular case there is even a worsening of performance, which can be attributed to the following factors:

- The system is not designed to cope adequately with situations of lack of information.
- The system provides signals indicating situations of lack of information but the user assumes that there is no need to apply judgment or does not know how to make a decision in such situations.
- The user’s decisions are random and are not based on rational behavior.

7. A model of an information system that deals only with a subset of the states of nature

In some cases, there is a question whether there is a need for an overall information system that can deal with all the possible states of nature or whether the less likely states of nature may be ignored. For example, due to a long series of
Table 4
Comparison of two information systems at the same level of accessibility, producing a random signal in situation of lack of information

<table>
<thead>
<tr>
<th>Information structure</th>
<th>Old information system</th>
<th>Improved information system</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q = \begin{pmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0.2 &amp; 0.8 \end{pmatrix} )</td>
<td>( Q_1 = \begin{pmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{pmatrix} )</td>
<td></td>
</tr>
<tr>
<td>( Q_2 = \begin{pmatrix} 28 &amp; 1 \ 30 &amp; 30 \ 1 &amp; 64 \end{pmatrix} )</td>
<td>( Q_3 = \begin{pmatrix} 28 &amp; 1 \ 30 &amp; 30 \ 1 &amp; 64 \end{pmatrix} )</td>
<td></td>
</tr>
<tr>
<td>( Q_3 = \begin{pmatrix} 0.8 &amp; 0 \ 0 &amp; 0.15 \ 0 &amp; 0.05 \end{pmatrix} )</td>
<td>( U = \begin{pmatrix} -60 &amp; -5 \ -10 &amp; -25 \end{pmatrix} )</td>
<td></td>
</tr>
<tr>
<td>( D = \begin{pmatrix} 0.1 \ 1.0 \ 0.0 \end{pmatrix} )</td>
<td>( D = \begin{pmatrix} 0.1 \ 1.0 \ 0.0 \end{pmatrix} )</td>
<td></td>
</tr>
<tr>
<td>( \text{Max}_0(\text{trace}(\Pi * Q * D * U)) )</td>
<td>( -8.75 )</td>
<td>( -8.75 )</td>
</tr>
<tr>
<td>( D = \begin{pmatrix} 0.1 \ 1.0 \ 0.0 \end{pmatrix} )</td>
<td>( D = \begin{pmatrix} 0.1 \ 1.0 \ 0.0 \end{pmatrix} )</td>
<td></td>
</tr>
<tr>
<td>( \text{Max}_0(\text{trace}(\Pi * Q_3 * D * U)) )</td>
<td>( -11.22 )</td>
<td>( -11.44 )</td>
</tr>
</tbody>
</table>

agreement with the Teachers Union, the payroll system of the Ministry of Education contains about 800 formulae for calculating salaries, some of which relate to only a small number of teachers. It has to be decided whether it is worthwhile to computerize all of them. Another example is the accounts archive of a telephone company. Suppose that about 95% of the inquiries relate to accounts sent out in the last year. The question then is whether it is worthwhile to keep all the information in the computerized system, with all the storage, processing and communications costs that it requires, or to consider a partial database but less complex and therefore more accessible. Such question is particularly relevant to the design of large data warehouses.

Theorem 3 below, indicates that sometimes, because of system or data accessibility problems, it is not worthwhile to invest in the development of an information system that deals with all the states of nature. In such cases, it is best to deal only with some of the states of nature, and to give the decision-maker a no-information signal in the other situations (the theorem formulate, in a way, the famous popular 80/20 rule).

Theorem 3
1. Let \( Q \) be an information structure. Let \( S = \{S_1, \ldots, S_n\} \) be the set of states of nature of \( Q \). Let \( Y = \{Y_1, \ldots, Y_m\} \) be the set of signals of \( Q \).
2. Let \( Y_{m+1} \) be a no-information signal that is not part of the set of signals of \( Q \). Let \( Q_0 \) be an information structure with the addition of a no-information signal \( (Y_{m+1}) \). Let \( S = \{S_1, \ldots, S_n\} \) be the set of states of nature of \( Q_0 \). Let \( Y_0 = \{Y_1, \ldots, Y_m, Y_{m+1}\} \) be the set of signals of \( Q_0 \).
3. Let \( N_0(Q) \) be a structure that represents receipt of a no-information signal. Let \( S \) be the set of its states of nature and \( Y_0 \) its set of signals.

\[ N_0(Q)_{i,j} = 0 \text{ when } 0 \leq j \leq m, \]
\[ N_0(Q)_{i,j} = 1 \text{ when } j = m + 1. \]
4. Let $Q_1$ be a convex combination of $Q_0$ and $N_0(Q)$
   
   $$Q_1 = p \ast Q_0 + (1 - p) \ast N_0(Q), \quad 0 \leq p < 1.$$ 

5. Let $Q_2$ be an information structure representing an information system that deals with part of the states of nature dealt with by $Q_0$, that is, the set of its states of nature is $\{S_1, \ldots, S_k\}$, $n > k$, and the system is completely available (at a probability of 1).

   Let $Y_0 = \{Y_1, \ldots, Y_m, Y_{m+1}\}$ be the set of signals of $Q_2$.

   $\forall i, \ i = 1, \ldots, k, \ \forall j, \ j = 1, \ldots, m, \ Q_{2ij} = Q_{ij},$

   $\forall i, \ i = k + 1, \ldots, n, \ \forall j, \ j = 1, \ldots, m, \ Q_{2ij} = 0,$

   $\forall i, \ i = k + 1, \ldots, n, \ j = m + 1, \ Q_{2ij} = 1.$

   Then $Q_1$ is not generally more informative than $Q_2$.

The proof of Theorem 3 appears in Appendix C.

This is another aspect of the productivity paradox: Sometimes you should spend less (in computerizing) in order to achieve more. Sometimes, at least equally good results can be obtained by computerizing the information system such that it is capable of dealing with only some of the states of nature rather than investing in a comprehensive full scale information system.

Example: Let us consider an information system for managing an organizational information center. The information items are classified into three categories: Class A—the top 5% requested information items (80% of the requests), that are restored in a direct accessed database. Class B—the 15% less requested information items (19% of the requests), that are restored in an archive database. Class C—the least 80% requested information items (1% of the requests), that are restored in an external archive device (e.g. External Cassettes or CD’s). Upon requesting an information item, the system has to inform the user if it is available and where. The user has to decide where to locate the requested item. If a requested item will not be found the search will be continued automatically in the other classes (from the faster accessed level to the slower one). Table 5 shows the possible states of nature and the signals received from the information system.

Suppose $Q_{1,0}$ represents an information system dealing with all the information items in both warehouses, accessible at a probability $p$ and producing a no-information signal when it is not accessible. $Q_{2,0}$ represents an information system dealing with the top 5% information items, it is completely accessible and producing a no-information signal in all other states of nature. The utility of the information system is measured by the expected mean response time per a transaction of information item request. In order to ensure equality with the returns of the system that deals with only part of the states of nature (break-even or equilibrium point), we can examine the minimal level of accessibility $p$ required of the system that deals with all the states of nature. This is demonstrated in Table 6.

According to this example it is worthwhile to invest in developing an information system that deals with all the states of nature only if one can ensure a level of accessibility of 90.90%. Otherwise, it is preferable to build an information system for managing the location of the information that is stored in the Direct Accessed D.B. rather than to invest in a whole information system for managing the location of every information item.

Thus, we have witnessed another aspect of the productivity paradox: sometimes it is not worthwhile to invest in a comprehensive information system (e.g. ERP). Computerizing the information

<table>
<thead>
<tr>
<th>State of nature</th>
<th>Signals (mutually exclusive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$—Top 5% requested information item</td>
<td>$Y_1$—Top 5%</td>
</tr>
<tr>
<td>$S_2$—Less requested information item</td>
<td>$Y_2$—Less requested—15%</td>
</tr>
<tr>
<td>$S_3$—Least requested information item</td>
<td>$Y_3$—Least requested—80%</td>
</tr>
<tr>
<td></td>
<td>$Y_2$—A no-information signal (inaccessibility)</td>
</tr>
</tbody>
</table>
### Table 6
Comparison of a central information system at a level of accessibility \( p \) with a local system at a level of absolute accessibility

<table>
<thead>
<tr>
<th>Information structure</th>
<th>System dealing with part of the states of nature</th>
<th>System dealing with all of the states of nature</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_{20} = \begin{pmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 \ 0 &amp; 0 &amp; 0 &amp; 1 \end{pmatrix} )</td>
<td>( Q_{10} = \begin{pmatrix} p &amp; 0 &amp; 0 &amp; 1-p \ 0 &amp; p &amp; 0 &amp; 1-p \ 0 &amp; 0 &amp; p &amp; 1-p \end{pmatrix} )</td>
<td></td>
</tr>
</tbody>
</table>

The matrix of a priori probabilities for the states of nature

\[ \Pi = \begin{pmatrix} 0.8 & 0 & 0 \\ 0 & 0.19 & 0 \\ 0 & 0 & 0.01 \end{pmatrix} \]

The time cost matrix (time units)

\[ U = \begin{pmatrix} -1 & -3 & -13 \\ -2 & -2 & -12 \\ -10 & -10 & -10 \end{pmatrix} \]

Optimal decision rule:

\[ D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

Max\(_0\)(trace(\( \Pi \ast Q_{2,0} \ast D \ast U \)) = -1.30

Max\(_0\)(trace(\( \Pi \ast Q_{1,0} \ast D \ast U \)) = p \ast 1.28 \ast (1-p) \ast 1.5 = -1.30

Level of accessibility \( p \)

100%

90.90%

---

**8. Summary and conclusions**

An attempt has been made in this study to explain various aspects of the productivity paradox. Three examples have been presented to demonstrate why investing in information systems is not always justified.

**8.1. Significance of the results**

Using the information structure model with various levels of accessibility enables to represent an information system with level of accessibility that was decreased over time, due to environmental changes (such as new information technologies and faster response times of competitors). This phenomenon might force the user of this information system to decrease its critical response time in order to respond faster. In such cases, the model illuminates situations when it is needed to improve a system just in order to survive, and not in order to improve performance.

Using the model of an information system that deals with only part of the states of nature makes it clear that sometimes a comprehensive information system (e.g., ERP, a broad Data-Warehouse) is not always preferable to a partial information system. Sometimes, due to system or data accessibility problems, it is not worthwhile to invest in developing a comprehensive information system that deals with all the states of nature. In such cases it is preferable to develop an information system that deals with only part of them, giving the decision-maker a random signal or a no-information signal under the rest of the states of nature. This contributes to clarifying another aspect of the productivity paradox: sometimes it is better to invest less in order to achieve more. An information system that can deal with only part of the states of nature, but with better accessibility can be no less beneficial than a comprehensive but more expensive or less accessible information system.

An information system that is generally more informative than another system at a level of complete accessibility may be not generally more
informative than this other system under incomplete level of accessibility. This can explain certain aspects of the productivity paradox. For example, introducing a new system that is designed to be generally more informative will not bring about the expected improvement if the accessibility is not improved and, in fact, the new system is no better in absolute terms. Indeed, there may even be a worsening of the expected payoff. This may be attributed to two factors:

(1) The system is not able to cope properly with situations of lack of information (characterization and planning problems).

(2) The system signals situations of lack of information but the user assumes that there is no need to exercise judgment or, not knowing how to make decisions without an accessible system (having developed a dependence on the information system), the user decisions are random and not based on rational behavior.

8.2. A practical interpretation

Though this study deals with theoretical models, it also has some practical implications. At the stage of examining technological alternatives for an information system, and particularly when making a data configuration decision (centralization or various forms of decentralization), the expected level of accessibility becomes an additional decision-making criterion, because of its direct effect on the value of the information.

At the stage of analyzing and planning the information system it is sometimes possible to compare the benefit of computerizing all states of nature and that of computerizing just part of them. Such comparison is suitable for information systems that meet the following conditions:

(1) The possible states of nature are given in advance and the a priori probabilities of their occurrences are known in advance.

(2) The returns arising from any given decision in each possible state of nature are known in advance and they yield benefits that can be quantified in monetary values.

In such cases, the following actions have to be carried out:

(1) Identification of infrequent states of nature,
(2) Presentation of alternatives for computerizing parts of the states of nature,
(3) Calculation of the expected value of the information for each action,
(4) Cost–benefit analysis for each alternative,
(5) Selection of an alternative: finding the optimal level of computerization.

For example, of the 800 or so formulae for calculating salaries in the Ministry of Education’s payroll system, 20 are relevant to the calculations of 98% of the salaries, the rest relating to an isolated number of individual teachers. It has to be decided whether it is worthwhile to computerize all the formulae for all the teachers on the payroll list.

In comparing the performance of several proposed information systems (for example, in examining alternatives for purchasing a software application or a new system), we have to take into account the level of accessibility and the way the systems operate in conditions of non-accessibility (such as whether the system produces a no-information signal). It must be remembered that the preferred information system under complete accessibility is not always the preferred choice under incomplete accessibility.

Empirical research into the impact of technological developments such as centralization and decentralization of information systems on organizational productivity will contribute to further understanding of these issues.

Appendix A

**Theorem 1.** Let $Q$ be an information structure. Let $S = \{S_1, \ldots, S_n\}$ be the set of states of nature of $Q$. Let $Y = \{Y_1, \ldots, Y_m\}$ be the set of signals of $Q$. Let $M(Q)$ be a structure representing lack of information. Let $S$ be its set of states of nature and $Y$ its set of signals.

\[
\forall i, 1 \leq i \leq n, \quad \forall k, 1 \leq k \leq n, \quad \forall j, 1 \leq j \leq m,
\]

\[
M(Q)_{ij} = M(Q)_{kj}
\]
(an information structure whose rows are all identical). Let $Q_1$ be a convex combination of $Q$ and $M(Q)$.

$$Q_1 = p_1 \ast Q + (1-p_1) \ast M(Q),$$

where $p_1$ is the level of accessibility (certainty) of $Q_1$.

Let $Q_2$ be a convex combination of $Q$ and $M(Q)$.

$$Q_2 = p_2 \ast Q + (1-p_2) \ast M(Q),$$

where $p_2$ is the level of accessibility (certainty) of $Q_2$. Suppose $0 \leq p_2 \leq p_1 \leq 1$. Then $Q_1$ is generally more informative than $Q_2$. □

We will first prove a lemma.

**Lemma**

(1) Let $Q_1$, $Q_2$, and $Q$ be information structures representing information systems operating on the same set of states of nature and having the same signals. Let $S = \{S_1, \ldots, S_n\}$ be the set of states of nature of $Q_1, \ldots, Q$. Let $Y = \{Y_1, \ldots, Y_m\}$ be the set of signals of $Q_1, \ldots, Q$. Let $Q_1$ be generally more informative than $Q_2, \ldots, Q_n$, that is, $Q = Q_1 \ast R$, $\forall i$, $i = 1, \ldots, r$.

(2) Let $Q$ be an information structure describing any convex combination whatsoever of the information structure $Q_1$ (Ahituv, 1981).

$$Q = \sum_{i=1}^{n} p_i \ast Q_1, \quad \sum_{i=1}^{n} p_i = 1.$$  
Hence, $Q_1$ is generally more informative than $Q$.

**Proof**

(1) $Q = \sum_{i=1}^{n} p_i \ast Q_1, \quad \sum_{i=1}^{n} p_i = 1.$

(2) Substituting $Q_i = Q_1 \ast R_i$, $\forall i$, $i = 1, \ldots, r$,

(3) We obtain $Q = \sum_{i=1}^{n} p_i \ast Q_1 \ast R_i, \quad \sum_{i=1}^{n} p_i = 1.$

(4) Hence, $Q = Q_1 \ast \sum_{i=1}^{n} p_i \ast R_i, \quad \sum_{i=1}^{n} p_i = 1.$

(5) Denote $R = \sum_{i=1}^{n} p_i \ast R_i$.

(6) As $R$ is a convex combination of stochastic matrices $R_i, \quad i = 1, \ldots, r$, $R = \sum_{i=1}^{r} p_i \ast R_i, \quad \sum_{i=1}^{r} p_i = 1$, as $R$ is stochastic of the order $m \times m$.

(7) (4), (6) $\Rightarrow Q = Q_1 \ast R$.

(8) According to the second condition of the Blackwell Theorem (McGuire and Radner, 1986, pp. 105–109), $Q_1$ is generally more informative than $Q$. □

**Proof of the theorem**

(1) Assume that $p_1 > 0$, since if $p_1 = 0$ then $p_2 = 0$ and clearly the theorem will hold.

(2) Thus $\frac{p_2}{p_1} \leq 1$.

(3) As $Q_1$ is generally more informative than $M(Q)$, then according to the lemma $Q_1$ is generally more informative than

$$\frac{p_2}{p_1} Q_1 + \left(1 - \frac{p_2}{p_1}\right) M(Q),$$

a convex combination.

We will now calculate the value of the convex combination $\frac{p_2}{p_1} Q_1 + \left(1 - \frac{p_2}{p_1}\right) M(Q)$, substituting for $Q_1$ according to: $Q_1 = p_1 \ast Q + (1-p_1) \ast M(Q)$.

(4) $\frac{p_2}{p_1} Q_1 + \left(1 - \frac{p_2}{p_1}\right) M(Q) = \frac{p_2}{p_1} \left(p_1 \ast Q + \left(1 - p_1\right) \ast M(Q)\right) + \left(1 - \frac{p_2}{p_1}\right) M(Q)$

(5) $= \left(p_2 \ast Q + \left(\frac{p_2}{p_1} - p_2\right) \ast M(Q)\right) + \left(1 - \frac{p_2}{p_1}\right) M(Q)$

(6) $= p_2 \ast Q + (1 - p_2) \ast M(Q)$

Thus (3) and (6) $\Rightarrow Q_1$ is generally more informative than $Q_2$. □

**Appendix B**

**Theorem 2.** Let $Q_1$ and $Q_2$ be two information structures operating on the same set of states of nature. Let $S = \{S_1, \ldots, S_n\}$ be the set of states of nature of $Q_1$ and $Q_2$. Let $Y_1 = \{Y_{1-1}, \ldots, Y_{1-m}\}$ be the set of signals of $Q_1$ and let $Y_2 = \{Y_{2-1}, \ldots, Y_{2-m}\}$ be the set of signals of $Q_2$. $Q_1$ is generally more informative than $Q_2$, that is,

$$Q_2 = Q_1 \ast R,$$

where $R$ is a stochastic matrix.

Let $N(Q_1), N(Q_2)$ be an information structures at absolute lack of information, suitable for the states of nature and the signals of $Q_1$ and $Q_2$, respectively. Suppose $N(Q_1) = N(Q_2)$.

Let $p$ be the level of accessibility of the systems $Q_1$ and $Q_2$. 


Let us now examine the \( j \) columns in which \( \sum_{i=1}^{m} R_{i,j} > 1 \) and in which we denote the minimal element in the \( j \) column as \( \text{Min}_j R_{i,j} \). A sufficient condition for \( Q_3 = p \ast Q_1 + (1 - p) \ast N(Q_1) \) being generally more informative than \( Q_4 = p \ast Q_2 + (1 - p) \ast N(Q_2) \) is that for each such column \( j \) there exists
\[
\text{Min}_j R_{i,j} - (1 - p) \left( \frac{1}{m} \sum_{i=1}^{m} R_{i,j} - \frac{1}{m} \right) \geq 0,
\]
where \( \text{Min}_j R_{i,j} \) is the minimal element of \( R \) in the \( j \) column and \( \sum_{i=1}^{m} R_{i,j} \) is the sum of the \( j \) column elements.

First we will prove two lemmas.

**Lemma 2.1.** Let \( A \) be a stochastic matrix of the order \( n \times m \). Let \( M(A) \) be a stochastic matrix of the order \( n \times m \), the rows of which are identical.

\[
\forall i, 1 \leq i \leq n, \quad \forall k, 1 \leq k \leq n, \quad \forall j, 1 \leq j \leq m, \\
M(A)_{i,j} = M(A)_{k,j}.
\]

Let \( M(m) \) to be a stochastic matrix of the order \( m \times m \) with rows identical to those of \( M(A) \).

\[
\forall i, 1 \leq i \leq m, \quad \forall k, 1 \leq k \leq m, \quad \forall j, 1 \leq j \leq m, \\
M(m)_{i,j} = M(A)_{i,j}.
\]

Hence, \( A \ast M(m) = M(A) \).

**Proof**

(1) \( A \ast M(m) \) is of the order \( n \times m \). Let us look at its \( i, j \) term:
\[
\sum_{k=1}^{m} A_{i,k} \ast M(m)_{k,j} = 1 \leq i \leq n, \quad 1 \leq j \leq m.
\]

(2) We utilize the stochasticity of \( A \) and the fact that all the rows of \( M(m) \) are identical
\[
= \sum_{k=1}^{m} A_{i,k} \ast M(m)_{1,j} = 1 \ast M(m)_{1,j} = M(m)_{1,j}, \\
1 \leq i \leq n, \quad 1 \leq j \leq m.
\]

(3) However, it is thus clear that
\[
M(A)_{i,j}, \quad 1 \leq i \leq n, \quad 1 \leq j \leq m. \quad \square
\]

**Lemma 2.2.** Let \( R \) be a stochastic matrix of the order \( m \times m \).

\[
R = p \ast I + (1 - p) \ast M(m),
\]

where \( M(m) \) is a stochastic matrix such that for every stochastic matrix \( A \) of the order \( m \times m \)
\[
A \ast M(m) = M(m),
\]

\( I \) — the identity matrix of the order \( m \times m \),
\( 0 < p \leq 1 \).

Hence \( R \) is a reciprocal and its terms are
\[
R^{-1} = \left( \frac{1}{p} \ast I - \frac{1 - p}{p} \ast M(m) \right).
\]

**Proof**

(1) \( R = p \ast I + (1 - p) \ast M(m) \).

Let us now look at the product.

(2) \( R \ast \left( \frac{1}{p} \ast I - \frac{1 - p}{p} \ast M(m) \right) = (p \ast I + (1 - p) \ast M(m)) \ast \left( \frac{1}{p} \ast I - \frac{1 - p}{p} \ast M(m) \right) \).

(3) \( = \left( \frac{1}{p} \ast I \ast p \ast I \right) + \left( \frac{1 - p}{p} \ast M(m) \ast I \right) - (1 - p) \ast I \ast M(m) + \left( \frac{1 - p}{p} \ast M(m) \ast M(m) \right) \).

Using
\[
M(m) \ast I = I \ast M(m) = M(m) \ast M(m) = M(m),
\]

(4) \( = I + \frac{1 - (1 - p) - (1 - p)^2}{p} \ast M(m) \).

(5) \( = I + \frac{1 - 2p + p^2 - 1 + 2p - p^2}{p} \ast M(m) = I. \quad \square
\]

**Proof of the theorem**

(1) Let us assume a general informativeness ratio between \( Q_3 \) and \( Q_4 \), that is, there exists a stochastic matrix \( R_2 \) such that
\[
(p \ast Q_1 + (1 - p) \ast N(Q_1)) \ast R_2 = \left( p \ast Q_2 + (1 - p) \ast N(Q_2) \right).
\]

(2) Or, substituting \( Q_2 = Q_1 \ast R, N(Q_1) = N(Q_2) \).

(3) \( (p \ast Q_1 + (1 - p) \ast N(Q_1)) \ast R_2 = (p \ast Q_1 \ast R + (1 - p) \ast N(Q_1)) \).

Utilizing the fact that \( Q_1 \ast N(m) = N(Q_1) \), according to Lemma 2.1, we obtain
\[
Q_1 \ast (p \ast I + (1 - p) \ast N(m)) \ast R_2 = Q_1 \ast \left( p \ast R + (1 - p) \ast N(m) \right) \ast R_2.
\]

\( (I \) is the unit matrix of the order \( m \times m \).\)
A sufficient condition for the existence of a general informativeness ratio will be

\[ (p \ast I + (1 - p) \ast N(m)) \ast R_2 = p \ast R + (1 - p) \ast N(m) \]

and if we utilize Lemma 2.2, which we have proved,

\[ (p \ast I + (1 - p) \ast N(m))^{-1} = \left( \frac{1}{p} \ast I - \frac{1 - p}{p} \ast N(m) \right) \]

\[ R_2 = \left( \frac{1}{p} \ast I - \frac{1 - p}{p} \ast N(m) \right) \ast (p \ast R + (1 - p) \ast N(m)) \text{ is a stochastic matrix.} \]

(Note: The condition is sufficient and not necessary, because it is possible that \( Q_1 \) is a structure that gives rise to the equality given in (4). For example, \( Q_1, N(Q_1) \) will often give rise to an equality. On the other hand, in many cases it is necessary and if it does not exist the following equality will not hold, as we demonstrated in Section 6.)

\[ I \ast R - (1 - p) \ast N(m) \ast R + \frac{1 - p}{p} \ast I \ast N(m) - (1 - p) \ast I \ast N(m) \ast N(m) = 0 \]

\[ R_2 = (1 - (1 - p) \ast N(m) \ast R + (1 - p) \ast N(m), \]

\( R_2 \) is a stochastic matrix.

(a) We will now examine the meaning of the stochasticity.

\[ R_2 \] is a convex combination of stochastic matrices, the sum of the terms of their rows being 1 and the sum of their coefficients being 1 \((1 - (1 - p) + (1 - p) = 1)\). Thus the sum of the terms in a row of \( R_2 \) is always 1.

Explanation: The sum of the terms in row \( i \) is

\[ \sum_{j=1}^{m} R_{i,j} - (1 - p) \ast \left( \sum_{k=1}^{m} N(m), k \ast R_{i,j} - \frac{1}{m} \right) \]

\[ = \sum_{j=1}^{m} R_{i,j} - (1 - p) \ast \left( 1 - \frac{1}{m} \ast \sum_{k=1}^{m} \sum_{k=1}^{m} R_{i,k} \right). \]

We utilize the stochasticity of \( R, \sum_{j=1}^{m} R_{i,j} = 1 \)

(13) that is, the sum of the terms of row \( i, \)

\[ 1 - (1 - p) + (1 - p) = 1, i = 1, \ldots, m. \]

(b) We will examine when the terms of \( R_2 \) will be non-negative (because of the negative coefficient!!)

(14) Let us look at the \( j \) columns in which \( \sum_{i=1}^{m} R_{i,j} > 1 \) in relation to the minimal term in the \( j \) column, \( \text{Min}_i R_{i,j} \). We do this because then there is a chance of a negative term, as the product of a row of \( N(m) \) in the column that fulfills \( \sum_{i=1}^{m} R_{i,j} > 1 \) is greater than \( \frac{1}{m} = N(m), j \).

(15) If for every such column \( j \) it holds that

\[ \min_{j} R_{i,j} - (1 - p) \ast \left( \frac{1}{m} \sum_{i=1}^{m} R_{i,j} - \frac{1}{m} \right) \geq 0, \]

then clearly all the terms are non-negative and the structure is stochastic. \( \Box \)

Appendix C

Theorem 3

1. Let \( Q \) be an information structure. Let \( S = \{S_1, \ldots, S_k\} \) be the set of states of nature of \( Q \). Let \( Y = \{Y_1, \ldots, Y_n\} \) be the set of signals of \( Q \).

2. Let \( Y_{m+1} \) be a no-information signal that is not part of the set of signals of \( Q \). Let \( Q_0 \) be an information structure with the addition of a no-information signal \( (Y_{m+1}) \). Let \( S = \{S_1, \ldots, S_n\} \) be the set of the states of nature of \( Q_0 \). Let \( Y_0 = \{Y_1, \ldots, Y_m, Y_{m+1}\} \) be the set of signals of \( Q_0 \).

3. Let \( N_0(Q) \) be a structure that represents receipt of a no-information signal. Let \( S \) be the set of its states of nature and \( Y_0 \) its set of signals.

\[ N_0(Q)_{i,j} = 0 \text{ when } 0 \leq j \leq m, \]

\[ N_0(Q)_{i,j} = 1 \text{ when } j = m + 1. \]

4. Let \( Q_1 \) be a convex combination of \( Q_0 \), and \( N_0(Q) \)

\[ Q_1 = p \ast Q_0 + (1 - p) \ast N_0(Q), \quad 0 \leq p < 1. \]

5. Let \( Q_2 \) be an information structure representing an information system that deals with part of the states of nature dealt with by \( Q_0 \), that is, the set of its states of nature is \( \{S_1, \ldots, S_k\} \), \( n > k \), and the system is completely available (at a probability of \( 1 \)).

Let \( Y_0 = \{Y_1, \ldots, Y_m, Y_{m+1}\} \) be the set of signals of \( Q_2 \).

\[ \forall i, i = 1, \ldots, k, \forall j, j = 1, \ldots, m, \quad Q_{2i,j} = Q_{i,j}; \]

\[ \forall i, i = k + 1, \ldots, n, \forall j, j = 1, \ldots, m, \quad Q_{2i,j} = 0; \]

\[ \forall i, i = k + 1, \ldots, n, j = m + 1 \quad Q_{2i,j} = 1. \]

Then \( Q_1 \) is not generally more informative than \( Q_2 \).
Proof

(1) For the sake of argument, let us suppose that $Q_1$ is generally more informative than $Q_2$. This means that there is a stochastic matrix $R$ of the order $m + 1 \times m + 1$:

$$Q_1 \ast R = Q_2.$$

(2) In particular, for every one of the last $n - k$ rows

$$\left( \sum_{h=1}^{m} (p \ast Q_{1,h}) \ast R_{h,j} \right) + (1 - p) \ast R_{m+1,j} = Q_{2,j} = 0.$$

(3) Hence, $(1 - p) \ast R_{m+1,j} = 0$, $1 \leq j \leq m$. This is true for all the first $m$ columns of $R$.

(4) That is, $\sum_{j=1}^{m} R_{m+1,j} = 0$.

(5) Hence, because $R$ is a stochastic Matrix, $R_{m+1,m+1} = 1$.

(6) Looking at one of the first $k$ rows of $Q_2$,

$$Q_{2, m+1} = 0$$

(from the definition of the structure)

$1 \leq i \leq k$.

(7) Seemingly,

$$\left( \sum_{h=1}^{m} (p \ast Q_{1,h}) \ast R_{h,m+1} \right) + (1 - p) \ast R_{m+1,m+1} = Q_{2,m+1} = 0.$$

(8) On the other hand, according to (5) $(1 - p) \ast R_{m+1,m+1} > 0$.

This is a contradiction: $Q_1$ is not generally more informative than $Q_2$. $\square$

References


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