Abstract— In this paper, an optimal design of linear phase digital high pass (HP) finite impulse response (FIR) filter has been proposed using Particle Swarm Optimization with Constriction Factor and Inertia Weight Approach and Wavelet Mutation (PSOCFIWA-WM). PSOCFIWA-WM incorporates a new definition of swarm updating in PSOCFIWA with the help of wavelet based mutation. Wavelet mutation enhances the PSOCFIWA to explore the solution space more effectively. In the design process, filter length, pass band and stop band edge frequencies, feasible pass band and stop band ripple sizes are specified. Other evolutionary algorithms like real coded genetic algorithm (RGA), particle swarm optimization (PSO), PSO with constriction factor and inertia weight approach (PSOCFIWA), and the proposed PSOCFIWA-WM have been used for the comparative design of linear phase FIR HP filter. A comparison of simulation results reveals the optimization superiority of the proposed technique over the other optimization techniques for the solution of FIR HP filter design.

Keywords- FIR Filter, Wavelet Mutation, Evolutionary Optimization, High Pass Filter.

I. INTRODUCTION

A digital filter is simply a discrete-time, discrete-amplitude convolver. Digital filter can be classified in two categories: finite impulse response (FIR) and infinite impulse response (IIR) [1]. By designing the filter taps to be symmetrical about the centre tap position, a FIR filter can be guaranteed to have linear phase. The most frequently used method for the design of exact linear phase weighted Chebyshev FIR digital filter is the one based on the Remez-exchange algorithm proposed by Parks and McClellan [2]. Further improvements in their results have been reported in [3].

The objective function for the design of optimal digital filters involves accurate control of various parameters of frequency spectrum and is thus highly non-uniform, non-linear, non-differentiable and multimodal in nature, which classical gradient based techniques cannot handle efficiently. So, evolutionary optimization methods have been implemented for the design of optimal digital filters with better control of parameters and these try to achieve the highest stop band attenuation.

Different evolutionary optimization methods are reported in the literatures. GA [4] and their variations like orthogonal genetic algorithm (OGA) [5], hybrid Taguchi GA (TGA) [6], simulated annealing [7], Tabu Search [8], an artificial bee colony optimization [9], Differential Evolution (DE) [10], particle swarm optimization (PSO) [11-12], GA proves itself to be more efficient but they are not very successful in determining the global minima in terms of convergence speed and solution quality. In this paper, the benefits of designing the FIR filter using an improved Particle Swarm Optimization called PSOCFIWA-WM have been explored. PSO is an evolutionary optimization technique developed by Eberhart et al. [13]. Several modifications of the conventional PSO technique have been made towards the optimization of the FIR filter. The merits of PSO lie in its simplicity in implementation as well as its convergence can be controlled by few parameters. More recently, craziness based PSO (CRPSO) [14-15] and the PSO with constriction factor and inertia weight approach [16] have been applied for FIR filter design problem.

Most of the above algorithms show the problem of premature convergence, stagnation and revisiting of the same solution over and again. In order to overcome these deficiencies, in this paper, PSOCFIWA-WM and a novel fitness function are employed to find the best coefficients those result in closely matched ideal frequency response for optimal FIR HP filter design. The PSO’s mutating space is varying dynamically based on the properties of the wavelet function that has been discussed in [17].

The rest of the paper is arranged as follows. In section II, the FIR HP filter design problem is formulated. Section III briefly discusses the proposed PSOCFIWA-WM algorithm. Section IV describes the simulation results obtained for FIR HP filter using PM algorithm, RGA, PSO, PSOCFIWA and the proposed PSOCFIWA-WM. Finally, section V concludes the paper.

II. FIR HIGH PASS FILTER DESIGN

The main advantage of the FIR filter structure is that it can achieve exactly linear-phase frequency responses.

A digital FIR filter is characterized by,

$$H(z) = \sum_{n=0}^{N} h(n)z^{-n}, \quad n=0, 1\ldots N$$  \hspace{1cm} (1)$$

where N is the order of the filter which has (N+1) number of coefficients; h(n) is the filter’s impulse response. The values of h(n) will determine the type of the filter, e.g., low pass, high pass, band pass etc. The values of h(n) are to be determined in the design process. This paper presents the design of FIR HP filter with h(n) as positive even symmetric and the filter order (N) is even. In any optimization algorithm,
the individual to be optimized represents (N/2 + 1) number of h(n). In each iteration cycle, these individuals are updated. Fitness values of individuals are calculated using the new coefficients and these fitness values are used to improve the search and the individual obtained after a certain number of iterations or after the error is below a certain limit is considered to be the optimal result. Because the h(n) coefficients are symmetrical, the dimension of the problem reduces by a factor of 2. The optimized (N/2+1) coefficients are then flipped and concatenated to find the required (N+1) filter coefficients.

Various filter parameters which are responsible for the optimal filter design are stop band and pass band normalized frequencies \((\omega_s, \omega_p)\), pass band and stop band ripples \(\delta_s\) and \(\delta_p\), stop band attenuation and transition width. Now for (1), coefficient vector \([h_0, h_1 \ldots h_n]\) is represented in D dimensions, where D = (N/2 +1) for the case of Nth order linear phase FIR filter.

In this paper, a novel error fitness function has been adopted in order to achieve higher stop band attenuation and to have an accurate control on the transition width. The fitness function used in this paper is given in (2).

\[
J = \sum_{k} \left| \left| H_{i}(\omega) - H_{i}(\omega)\right|\right| + \sum_{k} \left| \left| H_{i}(\omega) - \delta_{i}\right|\right|.
\]

For the first term of (2), \(\omega \in \) pass band including a portion of the transition band and for the second term of (2), \(\omega \in \) stop band including the rest portion of the transition band. The portions of the transition band chosen depend on pass band edge and stop band edge frequencies. The word ‘abs’ or ‘| |’ stands for absolute value. The error function given in (2) represents the generalized error fitness function to be minimized using the evolutionary algorithms RGA, PSO, PSOCFIWA and PSOCFIWA-WM individually. Each algorithm tries to minimize this error fitness \(J\) and thus improves the filter performance. Unlike other error fitness functions as given in [2, 5, 8, 10, 11] which consider only the maximum errors, \(J\) involves summation of all absolute errors for the whole frequency band. Minimization of \(J\) yields higher stop band attenuation and lesser pass band ripples. Since the coefficients of the linear phase filter are matched, the dimension of the problem is thus halved. By determining only half of the coefficients (N/2+1), the filter can be designed. This greatly reduces the computational burdens of the algorithms, applied to the design of linear phase FIR filters.

III. OPTIMIZATION TECHNIQUES EMPLOYED

A. Particle Swarm Optimization with Constriction Factor and Inertia Weight Approach (PSOCFIWA)

Standard PSO [13] is developed through simulation of bird flocking in multidimensional space. Bird flocking optimizes a certain objective function. Each agent knows its best value so far (pbest). This information corresponds to personal experiences of each agent. Moreover, each agent knows the best value so far in the group (gbest) among pbests. Namely, each agent tries to modify its position using the following information:

- The distance between the current position and pbest.
- The distance between the current position and gbest.

Mathematically, velocities of the particles are modified according to the following equation:

\[
v_{i}^{k+1} = w \cdot v_{i}^{k} + C_1 \cdot \text{rand} \cdot \left( \text{pbest}_i - s_i^k \right) + C_2 \cdot \text{rand} \cdot \left( \text{gbest} - s_i^k \right)
\]

where \(v_{i}^{k}\) is the velocity of agent \(i\) at iteration \(k\); \(w\) is the inertia weighting factor; \(C_1\) is the weighting factor; \(\text{rand}\) is the random number between 0 and 1; \(s_i^k\) is the current position of agent \(i\) at iteration \(k\); \(\text{pbest}_i\) and \(\text{gbest}\) is the pbest of agent \(i\) and gbest is the gbest of the group. The first term of (3) is the previous velocity of the agent. The second and third terms are used to change the velocity of the agent. Without the second and third terms, the agent will keep on “flying” in the same direction until it hits the boundary. Namely, it corresponds to a kind of inertia and tries to explore new areas. For PSOCFIWA [16], the velocity is manipulated in accordance with (4).

\[
v_{i}^{k+1} = CF_a \cdot \left( w \cdot v_{i}^{k} \right) + C_1 \cdot \text{rand} \cdot \left( \text{pbest}_i - s_i^k \right) + C_2 \cdot \text{rand} \cdot \left( \text{gbest} - s_i^k \right)
\]

Normally, \(C_1 = C_2 = 1.5 - 2.05\) and constriction factor \(CF_a\) varies from 0.6-0.73. The best values of \(C_1, C_2, \) and \(CF_a\) are found to vary with the design sets. In Inertia Weight Approach (IWA), inertia weight \(\left\{ w^k \right\}\) at (k+1)th cycle is as given in (5)

\[w^{k+1} = w_{\text{max}} - \frac{w_{\text{max}} - w_{\text{min}}}{k_{\text{max}}} \times (k + 1)
\]

where \(w_{\text{max}} = 1.0; w_{\text{min}} = 0.4; k_{\text{max}} = \text{Maximum number of iteration cycles. The solution updating is given by the following equation:}

\[s_i^{k+1} = s_i^k + v_{i}^{k+1}\]

B. PSOCFIWA-WM

I. Basic Wavelet theory: a concept

Certain seismic signals can be modelled by combining translations and dilations of an oscillatory function with a finite duration called a “wavelet”. Wavelet transform can be divided into two categories: continuous wavelet transform and discrete wavelet transform. The continuous wavelet transform \(W_a(x)\) of function \(f(x)\) with respect to a mother wavelet \(\psi(x) \in L^2(\mathbb{R})\) is given by the following equation [17-18].

\[W_{a,b}(x) = \frac{1}{\sqrt{C_p}} \int_{-\infty}^{\infty} f(x) \psi^{*}_{a,b}(x)dx\]

where \(\psi^{*}_{a,b}(x) = \frac{1}{\sqrt{a}} \psi \left( \frac{x-b}{a} \right); \ a \in \mathbb{R}, \ a, b \in \mathbb{R}, \ a > 0\)

In (7), \((*)\) denotes the complex conjugate, \(a\) is the dilation (scale) parameter, and \(b\) is the translation (shift) parameter. It is to be noted that \(a\) controls the spread of the wavelet and \(b\) determines its control position. A set of basis function \(\psi_{a,b}(x)\) is derived from scaling and shifting the
mother wavelet. The mother wavelet has to satisfy the following admissibility condition.

\[ C_v = 2\pi \int_{-\infty}^{\infty} |\hat{\psi}(\omega)|^2 d\omega < \infty \]  
(8)

where \( \hat{\psi}(\omega) \) is the Fourier transform of \( \psi(\omega) \) and given by the following equation.

\[ \hat{\psi}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x) e^{-j\omega x} dx \]  
(9)

Most of the energy of wavelet function \( \psi(x) \) is confined to a finite domain and is bounded.

2) Association of Wavelet based Mutation with PSOCFIWA (PSOCFIWA-WM)

It is proposed that every element of the particle of the population will mutate. In the population, a randomly selected \( i^{th} \) particle and its \( j^{th} \) element (within the limits \([S_{j,\text{min}}, S_{j,\text{max}}]\)) at the \( k^{th} \) iteration will undergo mutation as given in the following equation.

\[
S_{ij}^{(k)} = \begin{cases} 
S_{ij}^{(k-1)} + \sigma x (S_{j,\text{max}} - S_{j,\text{min}}), & \text{if } \sigma > 0 \\
S_{ij}^{(k-1)} + \sigma x (S_{j,\text{min}} - S_{j,\text{max}}), & \text{if } \sigma \leq 0
\end{cases}
\]  
(10)

where the mutation operator, \( \sigma = \sigma_{a,0}(x) = \frac{1}{\sqrt{a}} \psi \left( \frac{x}{a} \right) \); A

Morlet wavelet \( \psi(x) \) (mother wavelet) is defined in (11).

\[ \psi(x) = e^{-\frac{x^2}{2}} \cos(5x) \]  
(11)

Thus, the mutation operator is given by the dilated wavelet as (12).

\[ \sigma = \frac{1}{\sqrt{a}} e^{-\frac{x^2}{2}} \cos(\frac{5x}{a}) \]  
(12)

Different dilated Morlet Wavelets are shown in Figure 1. From Figure 1, it is clear that as the dilation parameter \( a \) increases, the amplitude of \( \psi_{a,0}(x) \) will be scaled down. In order to enhance the searching performance in the fine tuning stage, this property will be utilized in mutation operation. As over 99% of the total energy of the mother wavelet function is contained in the interval \([-2.5, 2.5]\), \( x \) can be randomly generated from \([-2.5\times a, 2.5\times a]\) [17]. The value of \( a \) is set to increase with the value of \( k/K \) governed by the monotonically increasing function (13) [17-18], so that the mutation operator \( \sigma \) decreases from a high value to a very low value, thus resulting in appreciable mutation during early search or exploration stage and fine tuning (i.e., lesser mutation) during local search or exploitation stage near the end of maximum iteration cycles. The significance of the mutation is thus reduced in the fine tuning stage. \( k \) is the current iteration cycle and \( K \) is the maximum number of iteration cycles.

\[ a = e^{-\ln(g_1) \times \left(1 - \frac{k}{K}\right) \xi_{\text{om}} + \ln(g_1)} \]  
(13)

where \( \xi_{\text{om}} \) is the shape parameter of the monotonically increasing function, and \( g_1 (=10000) \) is the upper limit of the parameter \( a \). The value of \( a \) is set to increase from unity to \( g_1 \) with increasing iteration cycle \( k \). A perfect balance between the exploration of new regions and the exploitation of the already sampled regions in the search space is expected in PSOCFIWA-WM by the right choices of the control parameters, e.g., the swarm size \( (n_p) \), the probability of mutation \( (p_m) \), and the shape parameter of WM \( (\xi_{\text{om}}) \).

Changing the parameter \( \xi_{\text{om}} \) will change the characteristics of the monotonically increasing function of WM. The dilation parameter \( a \) will take a higher value to perform fine tuning faster, i.e., in the early exploration stage as \( \xi_{\text{om}} \) increases. The best \( \xi_{\text{om}} \) chosen is 2.0 to avoid such fine tuning in the early exploration stage.

IV. SIMULATION RESULTS AND DISCUSSIONS

A. Analysis of Magnitude Response of FIR HP Filter

The simulation results discussed in this section clearly justify the superiority of the proposed PSOCFIWA-WM over the other methods like RGA and conventional PSO and PSOCFIWA. All simulations have been performed in MATLAB 7.5 version on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM. The order of the linear phase FIR HP filter has been taken as 20, so that the length of the coefficient vector is 21. The sampling frequency has been fixed to \( f_c \) = 1Hz. The number of sampling points taken is 128. In order to extract the best results, each algorithm is made to run for 40 times.

Table 1 shows the parameters chosen for PSOCFIWA-WM algorithm. The proper selection of these parameters plays an important role in the convergence profile of the algorithm. Table 2 shows the optimized coefficients of FIR HP filter. Table 3 shows the comparative results of performance in terms of maximum and minimum stop band ripples (normalized), transition width (normalized) using PM, RGA, PSO,
PSOCFIWA and the PSOCFIWA-WM, respectively. It is noticed that for minimum or almost same level of transition width, the PSOCFIWA-WM results in the minimum (0.02604) stop band ripple among all algorithms.

In designing the filter using all methods, the main focus has been kept on maximizing the stop band attenuation as far as possible. Table 4 shows the statistical results as maximum, mean, variance and standard deviation for stop band attenuations (dB) achieved by all four algorithms for the design of 20th order FIR HP filter.

### Table I. Parameters of PSOCFIWA

<table>
<thead>
<tr>
<th>Population Size</th>
<th>Iteration cycles</th>
<th>$C_1$ and $C_2$</th>
<th>$w_{\text{max}}$, $w_{\text{min}}$</th>
<th>$CFa$, $g_{\text{mult}}$</th>
<th>$Pm$</th>
<th>$g_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>200</td>
<td>2.05</td>
<td>1.0, 0.4</td>
<td>0.73</td>
<td>2.0</td>
<td>0.15</td>
</tr>
</tbody>
</table>

The maximum stop band attenuation achieved by PSOCFIWA-WM is 31.69 dB as compared to 30.15 dB of PSOCFIWA, 28.1 dB of conventional PSO, 25.25 dB of RGA and 23.55 dB of PM. Table 4 dictates that for maximum number of iterations the standard deviation of the proposed PSOCFIWA-WM algorithm is minimum (2.4292) as compared to 4.53818 (RGA), 4.01134 (PSO) and 2.9655 (PSOCFIWA) in terms of stop band (dB) attenuation. The simulation results of [19] show that for order 20, the maximum stop band attenuation (dB) is 22.83, maximum pass band ripple (normalized) is 0.126, maximum stop band ripple (normalized) is 0.0722, transition width is 0.0794. Thus the proposed method shows better results in terms of maximum stop band attenuation, stop band ripple for slightly greater or almost same level of transition width for the designed optimal FIR HP filter.

### Table II. Optimized Coefficients of the FIR HP Filter of Order 20

<table>
<thead>
<tr>
<th>h(N)</th>
<th>RGA</th>
<th>PSO</th>
<th>PSOCFIWA</th>
<th>PSOCFIWA-WM</th>
</tr>
</thead>
<tbody>
<tr>
<td>h(1)=h(21)</td>
<td>0.021731353326545</td>
<td>0.025559145974814</td>
<td>0.026662794210328</td>
<td>0.024287726290049</td>
</tr>
<tr>
<td>h(2)=h(20)</td>
<td>-0.048131602272058</td>
<td>-0.047413653181042</td>
<td>-0.045462848237385</td>
<td>-0.033215947202983</td>
</tr>
<tr>
<td>h(3)=h(19)</td>
<td>0.006298189918824</td>
<td>0.005135430273491</td>
<td>0.003126203180894</td>
<td>-0.015592651936326</td>
</tr>
<tr>
<td>h(4)=h(18)</td>
<td>0.041895534595676</td>
<td>0.039988099089174</td>
<td>0.039099240233353</td>
<td>0.05034572812006</td>
</tr>
<tr>
<td>h(5)=h(17)</td>
<td>0.008879943669486</td>
<td>0.001405996354139</td>
<td>-0.000706589413391</td>
<td>-0.00291851202379</td>
</tr>
<tr>
<td>h(6)=h(16)</td>
<td>-0.059028766591514</td>
<td>-0.060283192968605</td>
<td>-0.057811895154533</td>
<td>-0.049872338753173</td>
</tr>
<tr>
<td>h(7)=h(15)</td>
<td>-0.00013559660394</td>
<td>0.000768613197325</td>
<td>-0.001609397095625</td>
<td>-0.015851362470313</td>
</tr>
<tr>
<td>h(8)=h(14)</td>
<td>0.104257677520726</td>
<td>0.105120739785348</td>
<td>0.104297654243400</td>
<td>0.116041736515584</td>
</tr>
<tr>
<td>h(9)=h(13)</td>
<td>0.00382934541217</td>
<td>0.0014719279711810</td>
<td>0.003692321019440</td>
<td>-0.002627298198950</td>
</tr>
<tr>
<td>h(10)=h(12)</td>
<td>-0.316631427228320</td>
<td>-0.31547190838371</td>
<td>-0.316440835851818</td>
<td>-0.312990388248335</td>
</tr>
<tr>
<td>h(11)</td>
<td>0.499468012052621</td>
<td>0.499981461098444</td>
<td>0.499981461098444</td>
<td>0.499981461098444</td>
</tr>
</tbody>
</table>

### Table III. Other Comparative Results of Performance Parameters of All Algorithms for the FIR HP Filter

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Stop Band Ripples (normalized)</th>
<th>Transition Width (normalized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM</td>
<td>0.06645</td>
<td>0.06622</td>
</tr>
<tr>
<td>RGA</td>
<td>0.05461</td>
<td>0.09693</td>
</tr>
<tr>
<td>PSO</td>
<td>0.03935</td>
<td>0.01125</td>
</tr>
<tr>
<td>PSOCFIWA</td>
<td>0.03109</td>
<td>0.01249</td>
</tr>
<tr>
<td>PSOCFIWA-WM</td>
<td>0.02604</td>
<td>0.01137</td>
</tr>
</tbody>
</table>

### Table IV. Statistical Results for Stop Band Attenuations (dB) Obtained by Different Algorithms for the FIR HP Filter

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Maximum</th>
<th>Mean</th>
<th>Variance</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM</td>
<td>23.55</td>
<td>23.56</td>
<td>0.0002</td>
<td>0.014142</td>
</tr>
<tr>
<td>RGA</td>
<td>25.25</td>
<td>25.27</td>
<td>20.5905</td>
<td>4.53818</td>
</tr>
<tr>
<td>PSO</td>
<td>28.10</td>
<td>35.396</td>
<td>16.0908</td>
<td>4.011337</td>
</tr>
<tr>
<td>PSOCFIWA</td>
<td>30.15</td>
<td>35.79</td>
<td>8.79396</td>
<td>2.9654612</td>
</tr>
<tr>
<td>PSOCFIWA-WM</td>
<td>31.69</td>
<td>35.136</td>
<td>5.900984</td>
<td>2.429194</td>
</tr>
</tbody>
</table>

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V. CONCLUSIONS

This paper presents the best optimal method for designing linear phase digital FIR high pass filter by using proposed evolutionary optimization based on PSO with Constriction Factor and Inertia Weight Approach and wavelet mutation (PSO-CFIWAWM). Comparison of the results of PM, RGA, conventional PSO, PSOCFIWA and the PSOCFIWA-WM algorithm has been made. It is revealed that the PSOCFIWA-WM has the ability to converge to the best quality near optimal solution and possesses the best convergence characteristics among other algorithms considered in this
work. Extensive simulation results justify that the proposed algorithm PSOCFIWA-WM outperforms Park McClellan, RGA, conventional PSO and PSOCFIWA in achieving the best magnitude response of the filter in terms of stop band attenuation, stop band and pass band ripples, with a very little deterioration in the transition width. Thus, the PSOCFIWA-WM may be used as a good optimizer for the design of optimal linear phase FIR filter of digital signal processing systems.

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