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Elimination of Oscillations in a Central Heating System using Pump Control

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Abstract

In central heating systems with thermostatic valve temperature control it is a well known fact that room temperature oscillations may occur when the heat demand becomes low due to the non-linear behavior of the control loop. This is not only discomforting but it also increases the energy cost of heating the room. Using the pump speed as an active part in control is it shown that the room temperature may be stabilized in a wider interval of heat demand. The idea is to control the pump speed in a way that keeps the thermostatic valve within a suitable operating area using an estimate of the valve position. The position is estimated from the pump terminals, using the pump flow and the pump differential pressure. The concept is tested on a small central heating test bench. The results show that it is possible to stabilize the room temperature even at part load conditions.

Introduction

Controlling room temperature using a thermostatic valve saves energy and gives good comfort under normal operating conditions [2]. A minor drawback using a thermostatic valve is that a low heat demand in a central heating system using constant boiler temperature and a fixed pump speed results in a low water flow and a high gain in the control loop. This may cause oscillations in the temperatures in seasons with low heat demand especially if the pump is over-dimensioned [8, 9].

Figure 1 shows a block diagram of the closed loop control of the room temperature using a thermostatic valve. T_{set} is the temperature set-point value, T_g is the valve head gas temperature, Δp_v is the valve differential pressure, q is the radiator flow Φ is the radiator heat flux, and T_r is the room temperature. Φ_{loss} is the heat loss from the room to the environment. Due to the nonlinear characteristics of the valve and the nonlinear dynamics of the radiator, oscillations may occur at low heat demands and flows, where the small signal gain is high.

To overcome this problem, the thermostatic valve must be given working conditions which keep the small signal gain sufficiently low to avoid instability. There are two different ways to maintain a satisfactory low gain, one is to decrease the boiler temperature (radiator inlet temperature) and another is to reduce the pump pressure by reducing the pump speed. In modern pumps speed control is an integrated feature, making it possible to control the pressure in a central heating system. In this work the possibility of using pump speed control to eliminate oscillations is investigated. A small central heating system test bench is used to verify the results.

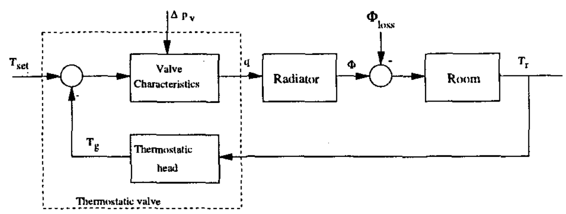

Figure 1: *Thermostatic valve closed loop*

Central Heating Test System

Grundfos A/S [4] has constructed a test system capable of emulating a small central heating system. The test bench consists of two closed boxes with radiator and thermostatic valves inside for simulating rooms and a heating unit that supply the two boxes with hot water. On the side of each room Peltier elements are mounted to simulate various heat demands. Flow, temperature and pressure are measured in several places of the test bench. For a full description of the test bench see [5, 7].

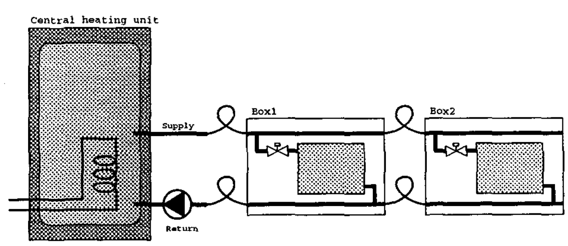

Figure 2: *Central heating test system*

The central heating unit contains an 80 l water tank with a 2.5 kW electric heat element. The UPE25-40 Grundfos pump is controlled with a 0-5 V signal.

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Each room contains a small radiator with a gas based thermostatic valve at the inlet. Two 150 W Peltier cooling elements are placed in the top and front plates of the rooms.

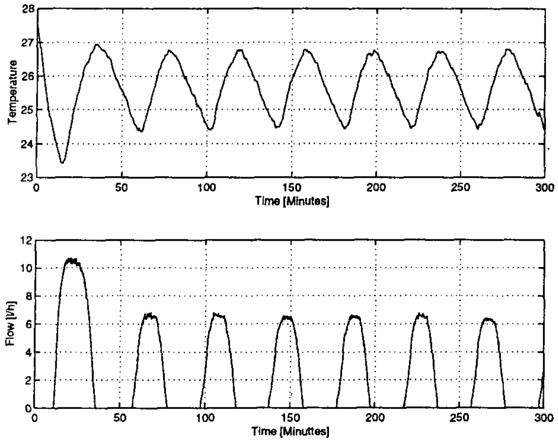

Figure 3: *Room air temperature and flow through the radiator*

Figure 3 shows an experiment where instability occurs. In the experiment the pump speed is at its maximum, the inlet water temperature is 50 $°C$ and the Peltier elements draw approximately 40 % of their maximum cooling power.

Central Heating System Models

In order to design the controller, a simple non-linear model [5, 6] as well as a linear model have been developed. The model consists of a hydraulic and a thermal part. The components in the hydraulic part are the thermostatic valve, the pump, pipes and flow-transducers. The model for the hydraulic part is static using (1)—see [5, 3]—for the thermostatic valve, (2) for the pump. The radiator and pipes are modeled by static pressure-flow relations, (3).

$$\begin{aligned} q &= a \cdot \tanh(b(T_{\text{set}} - T_g + 2)) \cdot \sqrt{\Delta p_v} \\ &= a \cdot \tanh(b T_{\text{error}}) \cdot \sqrt{\Delta p_v} \\ &= K_v(T_{\text{error}})\sqrt{\Delta p_v} \qquad (1) \\ \Delta p_p &= p_{\max}\left(\left(\frac{n_p}{n_{\max}}\right)^2 - \left(\frac{q}{q_{\max}}\right)^2\right) \qquad (2) \\ \Delta p_r &= R_p q^2 \qquad (3) \end{aligned}$$

q is the radiator flow, Δp_v is the thermostatic valve differential pressure, Δp_p is the pump differential pressure and Δp_r is pressure drop across the radiator and pipes. a and b are constants describing the thermostatic valve. n_p denotes the actual pump speed determined by the external speed control signal and $p_{\max}$, $q_{\max}$ and $n_{\max}$ are constants characteristic for the pump. R_p is the hydraulic resistance of the radiator and pipes. Furthermore the pressure generated by the pump will equal the sum of the pressure drops across the valve, radiator, pipes and flow-meter, giving a single equation for the flow:

$$q = \frac{n_p}{n_{\max}}\sqrt{\frac{p_{\max}}{\frac{1}{K_v^2} + R_p + \frac{p_{\max}}{q_{\max}^2}}} \qquad (4)$$

The thermal part consists of models of the radiators, the thermostatic valves and the rooms.

The radiator is a distributed system. It is modeled by lumping it into 11 sections. Using a one exponent model for the radiator power, the j'th section is given by [11]:

$$\begin{aligned} \frac{C_{rw}}{N}\frac{dT_j}{dt} &= c_{pw}\rho_w q(T_{j-1} - T_j) \\ &\quad - \frac{\Phi_o}{N}\left(\frac{T_j - T_r}{\Delta T_0}\right)^n \end{aligned} \qquad (5)$$

where C_{rw} denotes the heat capacity of water and radiator material lumped into one control volume, T_j is the temperature of the j'th section, $N = 11$ is the number of sections, c_{pw} specific heat capacity of water, ρ_w density of water and Φ_o is the nominal radiator power, i.e. the radiator power at the nominal mean temperature difference $\Delta T_0 = 60°C$.

In the room model, the heat balance equation gives:

$$\begin{aligned} C_a\frac{dT_r}{dt} &= \frac{\Phi_0}{N\Delta T_0^n}\sum_{j=1}^{N}(T_j - T_r)^n \\ &\quad + \frac{1}{R_r}(T_a - T_r) - \Phi_p \end{aligned} \qquad (6)$$

where C_a is air heat capacity, R_r thermal resistance of room walls, T_a ambient temperature and Φ_p Peltier element power.

The thermostatic valve gas dynamics are modeled by assuming a first order heat transfer relation from the room:

$$\frac{dT_g}{dt} = \frac{1}{\tau_g}(T_r - T_g) \qquad (7)$$

where T_g is the gas temperature in the thermostatic head and τ_g is a time constant.

The model is linearized giving the model structure shown in Figure 4:

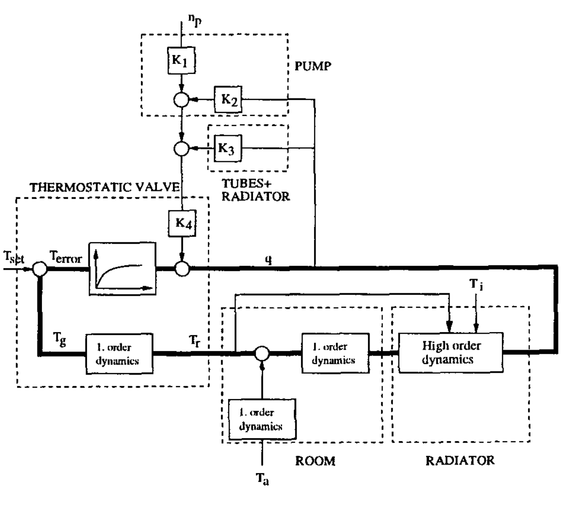

Figure 4: *Linear model of the central heating system*

The hydraulic equation (4) may be linearized to

$$\hat{q} = \frac{1}{n_{\max}}\sqrt{\frac{p_{\max}}{z}}\hat{n}_p + \frac{\bar{n}_p\sqrt{p_{\max}}}{n_{\max}} z^{-\frac{3}{2}} K_v^{-3} a\, b\, \mathrm{sech}^2(b\bar{T}_{\mathrm{error}})\hat{T}_{\mathrm{error}} \qquad (8)$$

with $z = \frac{1}{K_v^2} + R_r + \frac{p_{\max}}{q_{\max}^2}$. The linearized equation for the j'th radiator section is given by

$$\begin{aligned}\frac{d\hat{T}_j}{dt} = \; & \frac{c_{pw}\rho_w N\bar{q}}{C_{rw}}\hat{T}_{j-1} \\ & - \left(\frac{c_{pw}\rho_w N\bar{q}}{C_{rw}} + \frac{\Phi_0 n(\bar{T}_j - \bar{T}_r)^{n-1}}{\Delta T_0^n C_{rw}}\right)\hat{T}_j \\ & + \frac{c_{pw}\rho_w N(\bar{T}_{j-1} - \bar{T}_j)}{C_{rw}}\hat{q} + \frac{\Phi_0 n(\bar{T}_j - \bar{T}_r)^{n-1}}{\Delta T_0^n C_{rw}}\hat{T}_r\end{aligned} \qquad (9)$$

Where $\hat{}$ denotes a deviation from operating point and $\bar{}$ denotes the value at the operating point.

The linearized room equation is

$$\begin{aligned}\frac{d\hat{T}_r}{dt} = & \sum_{j=1}^{N}\frac{\Phi_0 n(\bar{T}_j - \bar{T}_r)^{n-1}}{\Delta T_0^n C_a N}\hat{T}_j \\ & + \left(-\frac{1}{R_r C_a} - \left(\sum_{j=1}^{N}\frac{\Phi_0 n(\bar{T}_j - \bar{T}_r)^{n-1}}{\Delta T_0^n C_a N}\right)\right)\hat{T}_r \\ & - \frac{1}{C_a}\hat{\Phi}_p + \frac{1}{R_r C_a}\hat{T}_a\end{aligned} \qquad (10)$$

The equation for the thermostatic head is already linear, meaning, that the small signal equation may be found by substituting the variables with the corresponding $\hat{}$ value.

In the linearized equations the operating point values of the variables are found as equilibrium points of the non-linear equations. The equilibria for $\bar{T}_1, \ldots, \bar{T}_{11}, \bar{T}_r, \bar{q}$ have been calculated for varying values of Φ_p and fixed values of the radiator inlet temperature T_i, T_a and $n_p = n_{\max}$.

The linearized equations may be arranged to a state space description of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\hat{q}' + \mathbf{E}\mathbf{d} \qquad (11)$$

$$\hat{q}'' = \mathbf{C}\mathbf{x} + D\hat{q}' + \mathbf{F}\mathbf{d} \qquad (12)$$

where $\hat{q}'$ and $\hat{q}''$ are input and output flows with the loop broken at the flow side of the thermostatic valve, and

$$\mathbf{x} = [\hat{T}_1, \cdots, \hat{T}_{11}, \hat{T}_r, \hat{T}_g]^T \qquad (13)$$

$$\mathbf{d} = [\hat{\Phi}_p, \hat{T}_a, \hat{T}_{\mathrm{set}}]^T \qquad (14)$$

The values of the matrix elements which may be extracted from equations (7)-(10) are dependent on the operating point flow.

To investigate the open loop gain it is divided into the radiator - room gain and the valve gain. The radiator - room gain is found using the linearized equations with C changed to $\mathbf{C}' = [0, \cdots, 0, 1, 0]$ and the thermostatic valve gain can be extracted from (8).

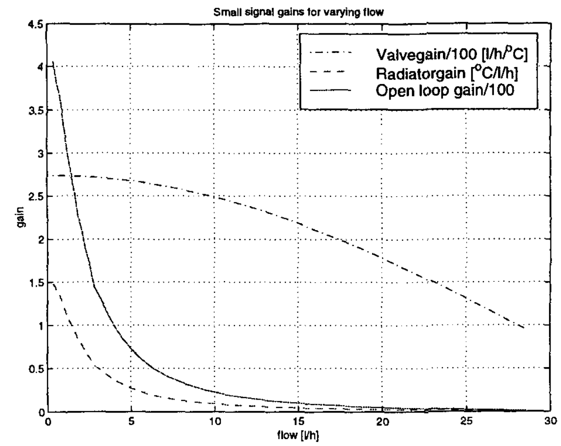

Figure 5: *Small signal gain from room temperature to valve flow, small signal gain from radiator flow to room temperature, and total open loop small signal gain for various values of the equilibrium flow*

Figure 5 shows that the small signal radiator - room gain is extremely dependent on the operating point value of the flow with high gains at low flows. The small signal gain of the thermostatic valve is near constant at low flows and goes to zero as the valve saturates at high flow.

Using the linearized equations it is possible to calculate the open loop poles as well as the closed loop poles as a function of the operating points. The model has 13 poles where 11 poles are associated to the radiator, one pole to the room and finally one pole to the thermostatic valve head. A calculation shows that below a certain value of $\bar{q}$ the closed system becomes unstable as shown in Figure 6, where the closed loop poles are shown for three different values of operating flows.

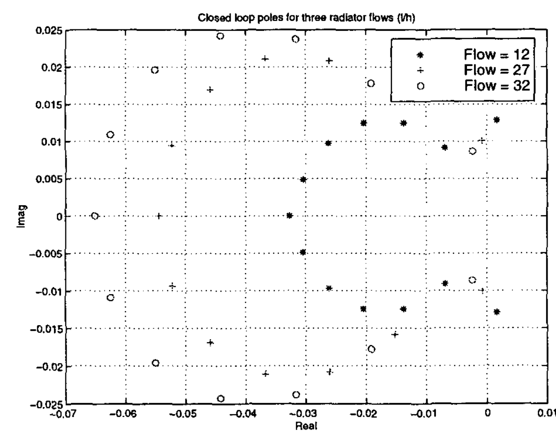

Figure 6: *Closed loop poles at 3 selected operating points showing instability at the lowest flow*

Control Structure

In order to avoid oscillations in the thermostatic valve loop at low heat demands the dynamics in the loop must be modified. The small signal gain in the valve is given by (8), where it is seen that the gain can be controlled by the pump speed. At full speed the valve gain is shown in Figure 5 as a function of the equilibrium flow. The stability problem in the loop may arise from the high open loop gain at low flows as shown in Figure 5. In order to achieve lower loop gains at low flows it is desirable to have valve characteristics with increasing gain as a function of the equilibrium flow. A constant loop gain arise if the valve characteristics is proportional to the inverse of the radiator gain shown in Figure 5.

To investigate the valve small signal gain, $\bar{n}_p$ is isolated in (4) and inserted in (8) giving:

$$\hat{q} = \bar{q}z^{-1}K_v(\bar{T}_{\text{error}})^{-3}ab\,\text{sech}^2(b\bar{T}_{\text{error}})\,\hat{T}_{\text{error}} \quad (15)$$

An idea is to keep the term $\bar{T}_{\text{error}}$ at a constant value at low flows [7]. This implies that K_v is kept constant. The valve gain is now a linear function of the equilibrium flow. K_v can be estimated from measured flow and pressure and controlled by the pump speed. This will have the effect that the loop gain will be less flow dependent because the valve gain increases with $\bar{q}$, see (15), and the radiator gain decreases with $\bar{q}$, see Figure 5. In Figure 7 the open loop gain for various flows is shown. While this certainly shows less variation than the gain with constant pump speed it still shows substantial variation with a significant peak.

With this control concept it is possible to choose a suitable value of K_v making the inner loop stable in every operating point. The curve in Figure 7 corresponds to a K_v which stabilizes the loop, as shown in Figure 8 showing Nyquist plots for varying flows. None of the curves encloses $(-1, 0, q)$, shown as a vertical line.

Using an outer loop which keeps K_v close to stabilizing set-points as calculated above ensures stability in fixed operating points. However this analysis does not guarantee stability during rapid transitions between operating points. Methods to analyze stability using Lyapunov theory are suggested in e.g. [10]. This analysis leads to limitations on the speed of variation for the operating point, in this case the flow. General experience suggests though that this analysis gives overly conservative limits, and to the authors knowledge there is no general method giving tight stability bounds.

In a practical setting an outer controller can use the pump speed to control the value of K_v. In the test system the actual value of K_v may be estimated from measurements of the pressure drop across the valve and the flow through it using (1). In a usual central heating system fewer measurements will be available, here a useful procedure could be to estimate K_v from measurements near the pump e.g. flow and pressure or even measurements on the electrical side of the pump.

Another idea is to use pump speed to influence the thermostatic valve in a way, that will keep the small signal gain in the radiator - room - thermostatic valve loop gain approximately constant independently of the heat demand. A given heat demand corresponds approximately

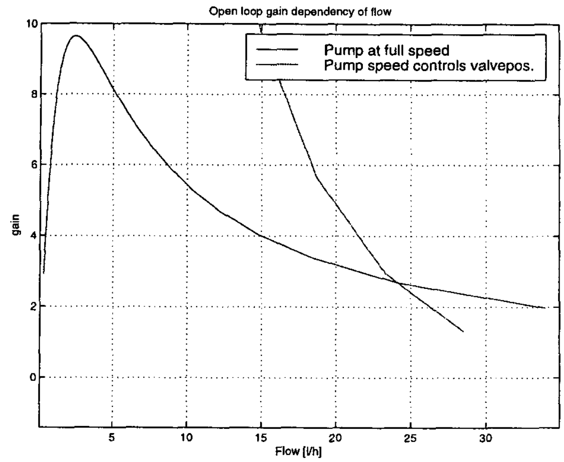

Figure 7: *Open loop gain with pump speed controlled to give fixed valve position*

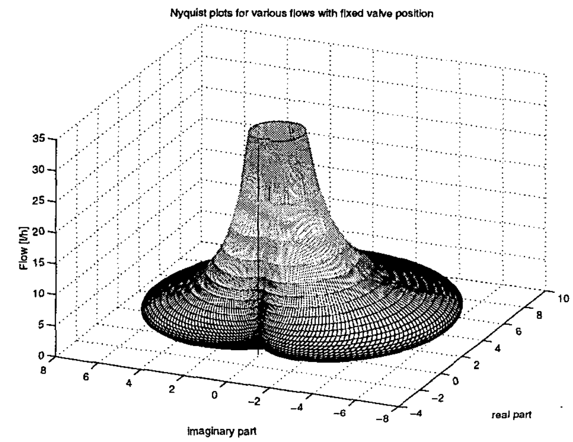

Figure 8: *Nyquist plots with pump speed controlled to give fixed valve position*

to a given flow. A constant loop gain implies that the small signal loop gain should equal the radiator - room small signal gain, G_{rad}, multiplied by the valve gain (15).

$$G_{\text{loop}} = G_{\text{rad}}(\bar{q})\bar{q}z^{-1}K_v(\bar{T}_{\text{error}})^{-3}ab\,\text{sech}^2(b\bar{T}_{\text{error}}) \quad (16)$$

For a given $\bar{q}$ this may be solved for T_{error} or equivalently K_v, giving a control scheme where the set-point for K_v is scheduled according to the equilibrium flow. This scheme would trade off a varying steady state error versus a more constant loop gain in the inner loop and a better stability margin.

The above mentioned control structures describe only the control of one radiator - thermostatic loop. Normally several radiators will be supplied by one pump. This implies that loops can not be optimized individually using the pump speed. In many cases though, the heat demands of individual rooms will follow similar patterns determined by climate et cetera. Even in cases with different room heat demands the climate disturbances will act in the same direction facilitating a larger interval without oscillations.

Controller Test

The control strategy with fixed K_v has been implemented on the central heating system using a configuration with one radiator [7]. The value of K_v was estimated from measurements of pressure drop across the valve and flow.

The outer loop is controlled by a PID-controller. The PID-controller is tuned using a relay experiment [1] followed by a hand tuning to give a good compromise between overshoot and rise-time resulting in $K = 127\frac{\%}{l/sec/bar^{0.5}}$, $T_i = 31\ min$, $T_d = 7.4\ min$. Figure 9 shows K_v, q and T_r in a situation with a low initial heat demand and some subsequent disturbances. At time zero the controller is put into operation and the loop stabilizes with a $K_v = 0.0065$. This is lower than the calculated value $K_v = 0.0130$ used in the Figures 7 and 8. This value should cause instability at the flow $2\ l/h$ according to calculations, but due to inaccuracies in the model parameters the test system allows lower values of K_v before oscillations appear than predicted by the calculations. A low set-point value of K_v is desirable because it results in a higher loop gain and a smaller stationary error.

At $time = 450\ min$ the cooling effect of the Peltier elements has been decreased by approximately 7 W. At $time = 1150\ min$ the ambient temperature begins to rise due to the rising sun. At $time = 1370\ min$ the morning cleaning staff at Aalborg University opened the window. The disturbances caused no oscillations, and the room temperature variations are smaller than 0.5 $°C$.

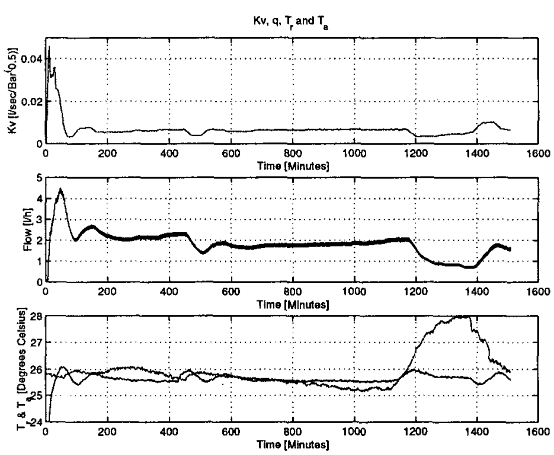

Figure 9: *Controller test with disturbances,* K_v = 0.0065. *Note the abscence of oscillations.*

Conclusion

Oscillations may appear in central heating systems with thermostatic valves in situations with low heat demands. In this paper it has been shown that this phenomenon may be analyzed using linearized operating point models. It turns out that the oscillations may be related to large gains in the radiator at low flows in the radiator. By changing the speed of pump it is possible to change the gain in the thermostatic valve in a way that ensures stability in the radiator - room - valve loop. A concept trying to keep the valve at a fixed position has been tested with a positive result on a test central heating system. Other similar concepts are suggested. The analysis using linear operating models is able to predict stability in fixed operating points. Attention should be taken to the fact that this type of analysis does not incorporate the effect of rapid changes in operating point for which Lyapunov-like methods should be used. Practical laboratory tests have shown stability also with changes in operating point.

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