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## Rectangular waveguide radiator miniaturization using electromagnetic infinity-shaped metamaterial resonator

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**Abstract:** A miniaturized open-ended rectangular waveguide antenna radiating below the cut-off frequency of the waveguide is proposed. Waveguide miniaturization is achieved by periodically loading the antenna with electromagnetic metamaterial (MTM) consisting of infinity shaped resonators. The metamaterial gives the waveguide the ability to support propagation of the backward wave below the cut-off frequency. The proposed open waveguide radiator was designed, optimized and simulated using High Frequency Structure Simulator HFSS<sup>TM</sup> commercial software. Comparing previous work of miniaturization of waveguides, a higher miniaturization ratio with a bandwidth about 327 MHz and good matching was obtained.

**Keywords:** Waveguide miniaturization, left-handed media, metamaterials, Infinity-shaped resonator.

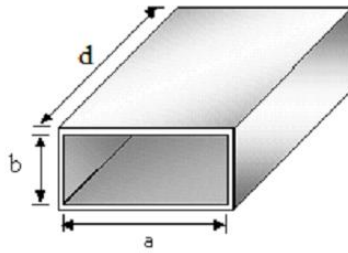
### تصغير حجم الموجه الموجي باستخدام مرنان ميتاماتيريال على شكل رمز اللانهاية

**ملخص:** يدرس هذا البحث تصغير أبعاد الموجه الموجي مفتوح الطرفين بجعله يمرر ترددات أقل من التردد الأدنى له. تصغير الموجه الموجي يتم من خلال وضع مصفوفة من الوحدات المتماثلة من الميتاماتيريال المكونة من المرنانات الحلقية المجزأة على شكل رمز اللانهاية داخل الموجه الموجي، حيث أن الميتاماتيريال تعطي الموجه الموجي الإمكانية لتوفير خاصية الموجات الراجعة وبترددات أقل من التردد الأدنى. البرنامج المستخدم في تصميم ومحاكاة الموجه الموجي هو برنامج HFSS<sup>TM</sup>. مقارنة مع دراسات سابقة على تصغير الموجه الموجي حصلنا على قيمة أكبر لنسبة تصغير الترددات، وعرض النطاق الترددي حوالي 327 ميجاهيرتز، وكفاءة عالية في التوافق الموجي.

## 1. INTRODUCTION

High frequency and high power technologies such as microwave devices, radars and antenna employ waveguides as a guiding structure for more than six decades. In spite of the increasing popularity of planar structure application, the

usage of waveguides is inevitable in many applications such as a feeding network for open-ended radiators and a large antenna array. Furthermore, waveguides are employed to fit multi-frequency interlaced antenna arrays and open-ended radiators operating at different frequencies into restricted space. The transverse length of the rectangular waveguide is at least half wavelength long [1], and there is an increased need for its miniaturization since it is considered to be the biggest disadvantage of waveguides.



**Figure 1:** Rectangular waveguide.

There are several ways that can be used for the miniaturization of waveguides. According to [1], the waveguide cut-off frequency  $f_c$  can be decreased by the square root of the relative permittivity when the waveguide is filled by dielectric materials leading to waveguide miniaturization by the same factor. The cut-off frequency  $f_c$  can be calculated as follows:

$$f_c = \frac{c}{2\pi\sqrt{\epsilon_r\mu_r}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}, \quad b = a/2 \quad (1)$$

Where  $a$  and  $b$  are the inside width and height of the rectangular waveguide, respectively,  $m$  and  $n$  are the number of half - wavelength variations of the field components along  $a$  and  $b$ , respectively,  $\epsilon_r$  is the relative permittivity of filling dielectric material in the waveguide,  $\mu_r$  is the relative permeability and  $c$  is the speed of light in free space.

Complex artificial perfect magnetic conductor surfaces can be used for the waveguide walls to reduce its width, [2]. Dielectric loaded hard-walled waveguide that supports TEM propagation was studied theoretically and reported in [3]. Furthermore, metamaterials can be used to load the waveguide in order to reduce its size. Metamaterials is composed of artificial periodic structures consisting of normal metals and dielectric materials and possess

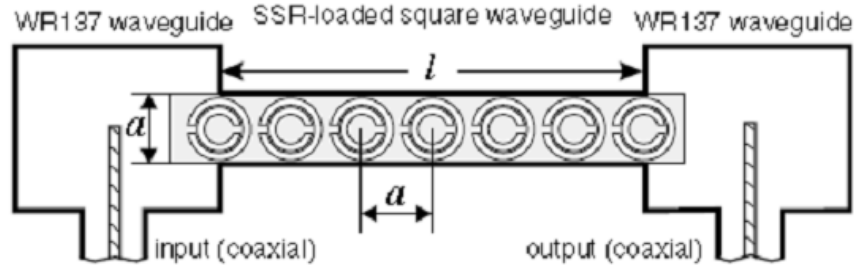
## Rectangular waveguide radiator miniaturization using

exotic characteristics such as negative permeability, permittivity and index of refraction, [4]. The miniaturization of guiding structures by loading a rectangular metallic waveguide structure with periodic split ring resonators (SRRs) were investigated in [5] and further investigated in [6]. Miniaturization depends on the resonator's geometry, which leads to different resonant and different cut-off frequencies. The split rings are important in constructing new types of metamaterial. Various types of ring and ring-like structures such as circular, square, U-shaped, S-shaped,  $\Omega$ -shaped, and others are used to create new metamaterials, [7]. Hedge studied the use of circular SRR in loading waveguide in order to reduce its size, [8].

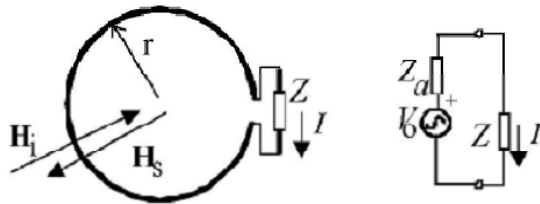
In this paper, we use a new metamaterial called infinity shaped resonator as proposed in [9]. This structure is used in an open ended waveguide antenna and is simulated in the frequency range 4.3-10 GHz.

## 2. THEORETICAL INVESTIGATIONS

The circular SRR – loaded waveguide can be analysed based on effective homogeneous medium method where the unit cell size ‘ $a$ ’ of the medium is much smaller than the guided wavelength  $\lambda_g$  ( $a \ll \lambda_g$ ), Fig. 2. The single SRR can be thought of as a capacitive loaded loop antenna, Fig. 3, and the narrow split in the ring as a loading capacitor of impedance ‘ $Z$ ’.



**Figure 2:** Experimental waveguide setup showing unit cell size “ $a$ ”.



**Figure 3:** Capacitive loaded loop antenna and equivalent circuit diagram.

When the structure is illuminated by a plane wave,  $H_i$ , the current flowing in the loop produces a new magnetic field which causes scattering in the opposite direction of the original plane wave. The resultant magnetic field is sum of scattered and incident fields,

$$H = H_i + H_s = H_i \left[ 1 - j \frac{K\omega\mu_0 A}{Z_a + Z} \right] \quad (2)$$

Where  $Z_a$  is the impedance of antenna loop,  $Z$  is the load capacitance,  $A$  is the loop area,  $\omega$  is the angular frequency of incident field,  $\mu_0$  is the free space permeability,  $K$  is a constant depending on the geometry of the system.

The inductive nature of the loop and the loading capacitance give rise to resonant behavior. The resultant field will be:

$$H = H_i \left[ 1 - j \frac{K\omega\mu_0 A}{R + j\omega L - \frac{1}{j\omega C}} \right] \quad (3)$$

Where  $R$ ,  $L$  and  $C$  are resistance, inductance, and capacitance of the loop circuit respectively.

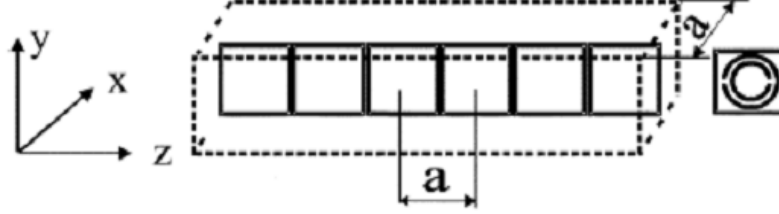
Slightly above the resonance of the structure, the scattered field  $H_s$  will be almost out of phase with the incident field  $H_i$ , yielding lower local resultant field. This will result in negative polarization and negative effective permeability given by:

$$\mu_{eff} = \mu'_{eff} - j\mu''_{eff} = 1 - \frac{f_{mp}^2 - f_0^2}{f^2 - f_0^2 - j\gamma f} \quad (4)$$

Where  $f$  is the frequency of the incident wave,  $f_{mp}$  is the magnetic plasma frequency (when  $\mu_{eff} = 0$ ),  $f_0$  is the resonant frequency of SRR (when  $\mu_{eff}$  diverges) and  $\gamma$  is the loss factor.  $f_{mp}$  and  $f_0$  are dependent on lattice constant and geometry of the SRR.

When the magnetic field vector of incident plane wave is perpendicular to the SRR (x direction), it will induce a current in the rings yielding negative permeability, Fig. 4. On the other hand, the parallel components of the magnetic vector cannot induce any current in the rings.

### Rectangular waveguide radiator miniaturization using



**Figure 4:** Square waveguide filled with SRR.

So components in both parallel directions (y and z directions) of the rings cannot affect the permeability leaving it equal to that of vacuum ( $\mu_{pr} = 1$ ). So, the filling shown in Fig. 4 gives rise to uniaxial metamaterial with a permeability given by:

$$\bar{\mu} = \mu_0 \begin{bmatrix} \mu_{tr} & 0 & 0 \\ 0 & \mu_{pr} & 0 \\ 0 & 0 & \mu_{pr} \end{bmatrix} = \mu_0 \bar{\mu}_r \quad (5)$$

Where  $\mu_0$  the permeability of free space,  $\mu_{tr}$  is relative permeability in x direction (transverse) and  $\mu_{pr}$  is relative permeability in y and z directions (parallel).

Since  $\mu_{pr}$  appears in both y and z directions,  $\mu_{pr}$  can be written as  $\mu_{lr}$  (longitudinal) and the tensor can be reduced to 2 x 2 matrix,

$$\bar{\mu} = \mu_0 \begin{bmatrix} \mu_{tr} & 0 \\ 0 & \mu_{lr} \end{bmatrix} = \mu_0 (\bar{\mu}_r' - j\bar{\mu}_r'') = \mu_0 \bar{\mu}_r \quad (6)$$

The wave equation for the waveguide in Fig. 4 is given by:

$$\nabla \times \bar{\mu}_r^{-1} (\nabla \times \bar{E}) = k_0^2 \epsilon_r \bar{E}, \quad k_0^2 = \omega^2 \mu_0 \epsilon_0 \quad (7)$$

Where  $\bar{E}$  is electric field,  $k_0$  is free space propagation constant,  $\epsilon_0$  is free space permittivity and  $\epsilon_r$  is relative permittivity of the medium.

Assuming that the waveguide supports the TE mode, the dispersion equation can be written as:

$$\frac{k_x^2}{\mu_{lr}} + \frac{k_z^2}{\mu_{tr}} = \epsilon_r k_0^2 \quad (8)$$

From equation 8, the longitudinal propagation constant is given by:

$$k_z = \pm \sqrt{\epsilon_r \mu_{tr} (k_0^2 - \frac{k_x^2}{\epsilon_r \mu_{lr}}} = \beta_z - j\alpha_z, \quad k_x = \frac{m\pi}{a}, \quad m = 1, 2, 3, \dots \quad (9)$$

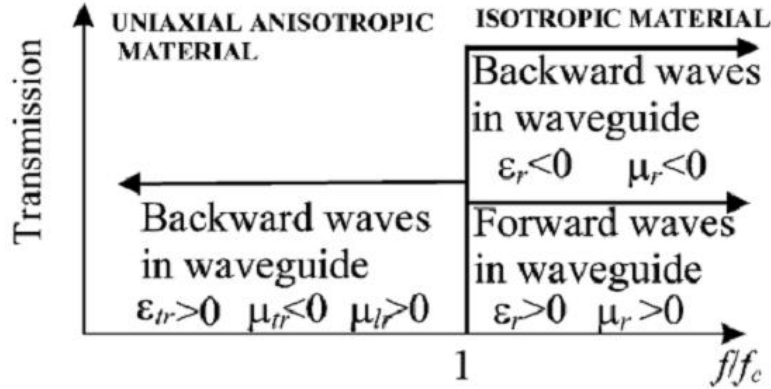
Where  $a$  is the width of the waveguide,  $k_x$  is the propagation factor in transverse direction,  $k_z$  is the propagation factor in longitudinal direction,  $\beta_z$  is the phase constant and  $\alpha_z$  is the attenuation constant.

Depending on the solution of the root term, there can be two possible values for the propagation constant  $k_z$ . Since the flow of energy must decay from the source, the meaningful solution occurs when  $\alpha_z > 0$ . Considering a lossless material for the rings ( $\epsilon_r'' = \mu_r'' = 0$ ), the longitudinal propagation constant in equation 9 can be written as:

$$k_z = \pm k_0 \sqrt{\epsilon_r \mu_{tr} \left[ 1 - \left( \frac{f_c}{f} \right)^2 \right]} = \beta_z, \quad f_c = \frac{f_{c0}}{\sqrt{\epsilon_r \mu_{lr}}}, \quad f_{c0} = \frac{mc}{2a} \quad (10)$$

Where  $f$  is the frequency of the incident wave,  $f_{c0}$  is the cut-off frequency of the empty waveguide,  $f_c$  is the cut-off frequency of the waveguide with filling.

The effect of filling material in the waveguide is shown in the Fig. 5.



**Figure 5:** Effect of filling material in the waveguide.

One can clearly notice, filling of the uniaxial anisotropic metamaterial with negative transverse permeability, waveguide can support backward wave below

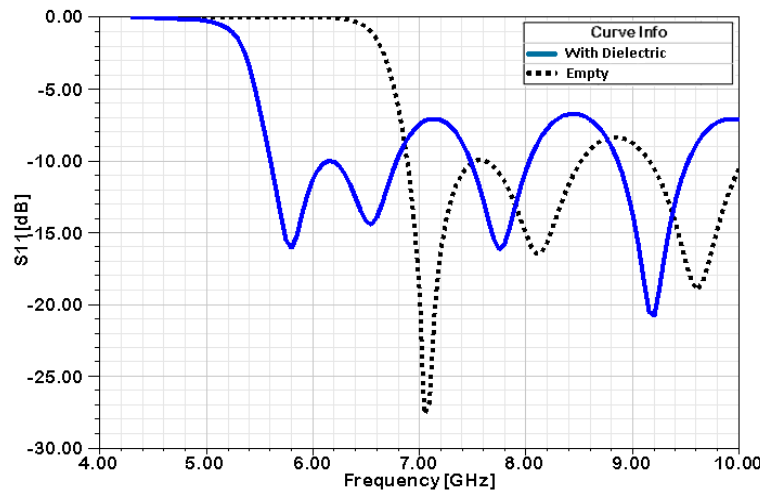
## Rectangular waveguide radiator miniaturization using

cut-off frequency of the waveguide. This kind of waveguide exhibits lowpass behavior, and it can be seen as the dual of ordinary waveguide with highpass behavior. The bandwidth of the backward wave propagation is governed by the dispersion property of the material filling. In the discussion above, we have considered material with no dispersion. In nature, every material has dispersion. Therefore, the material filling can support backward wave propagation only within the limited frequency band.

### 3. SIMULATIONS AND RESULTS

The Ansoft's finite-element method based High Frequency Structure Simulator commercial software (HFSS) is used for the simulation. The dimensions of the waveguide structures is taken to be the same as given in [8] for comparison purposes. The simulated wave guide antenna is operating at X-band (cut-off frequency 6.6 GHz) with dimensions  $22.86 \text{ mm} \times 60 \text{ mm} \times 10.16 \text{ mm}$ . The waveguide was excited by C-band waveguide-to-coaxial transition (cut-off frequency 4.3 GHz) with dimensions  $35 \text{ mm} \times 25 \text{ mm} \times 15 \text{ mm}$ . The model was simulated from 4.3 GHz to 10 GHz.

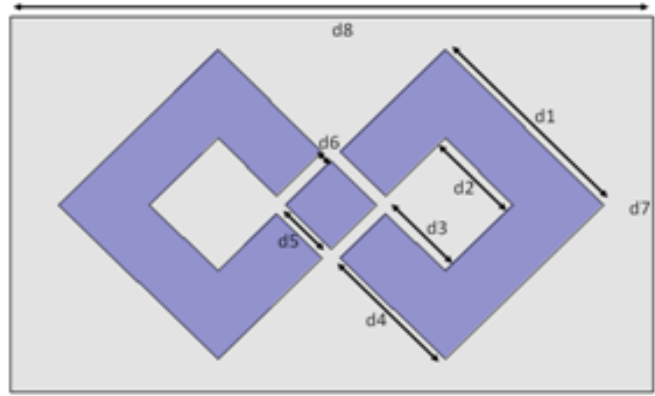
The return loss of the empty waveguide open radiator compared to that of loaded waveguide with dielectric slab (FR4 sheet of  $10.16 \text{ mm} \times 84 \text{ mm}$  and  $1.524 \text{ mm}$  thickness with relative permittivity 4.4) is shown in Fig. 6. It can be seen that the open waveguide radiator starts transmitting at 6.6 GHz while the loaded waveguide radiator starts transmitting at 5.4 GHz.



**Figure 6:** Return loss for empty and dielectric slab loaded waveguide.

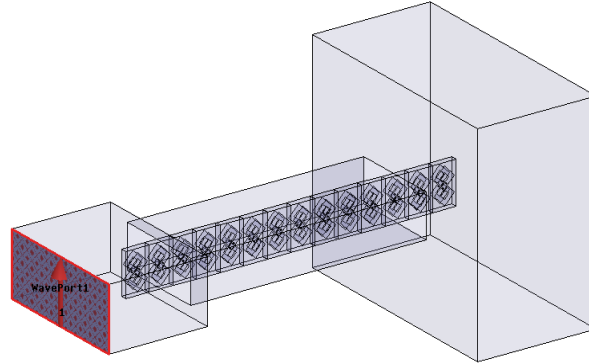


Furthermore, the X-band waveguide was loaded by a regular array of 14 infinity shaped metamaterial cells, every unit cell contains a FR4 substrate with dimensions  $1.524 \text{ mm} \times 6 \text{ mm} \times 10.16 \text{ mm}$  and a copper infinity shape with conductivity of  $5.8 \times 10^7 \text{ S/m}$ , 0.01 mm thickness, and dimensions as shown in Fig. 7.



**Figure 7:** Top view of infinity shaped resonator dimensions:  $d1=3.5\text{mm}$ ,  $d2=1.5\text{mm}$ ,  $d3=1.3\text{mm}$ ,  $d4=2.3\text{mm}$ ,  $d5=1\text{mm}$ ,  $d6=0.2\text{mm}$ ,  $d7=6\text{mm}$  and  $d8=10.16\text{mm}$ .

The loaded waveguide model built using HFSS software is shown in Fig. 8. The first cell and half of the second were placed out from the X-band waveguide to ensure the excitation of the first cell in the array. The last two cells and half of the twelfth cell were placed out from the open end of the waveguide in order to improve the radiation.

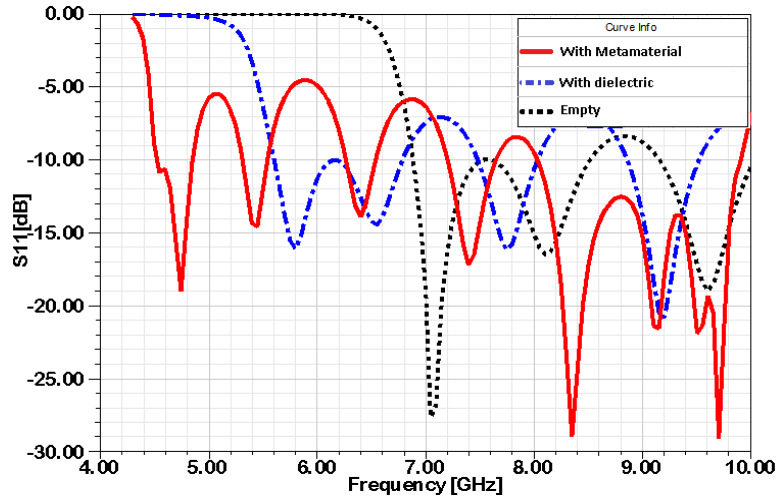


**Figure 8:** Loaded waveguide model.

The return loss  $S_{11}$  at the input port of waveguide antenna that is loaded with

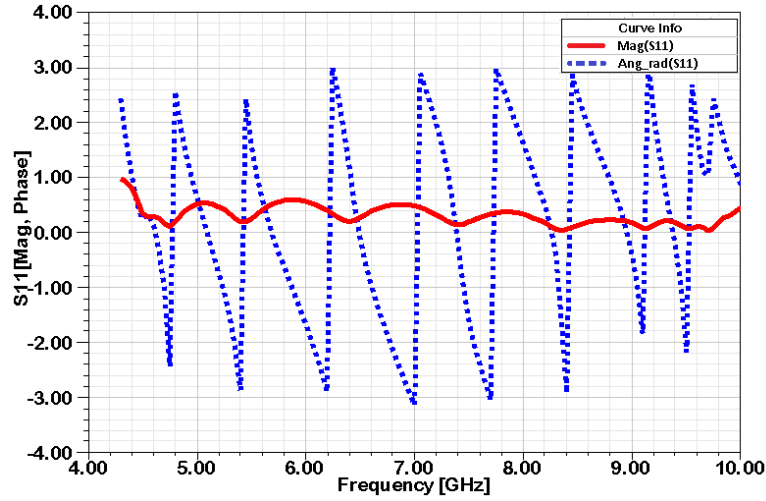
## Rectangular waveguide radiator miniaturization using

14 infinity shape cells was plotted and shown in Fig. 9. The results are compared with that of the empty waveguide, and that of a waveguide with only the dielectric slab inside it. From the plot, it is clear that the waveguide propagates below its cut-off frequency of 4.43 GHz and the passband bandwidth is 327 MHz which represents a bandwidth reduction of about 2.17 GHz. A minimum return loss of -19dB at 4.75 GHz was achieved.



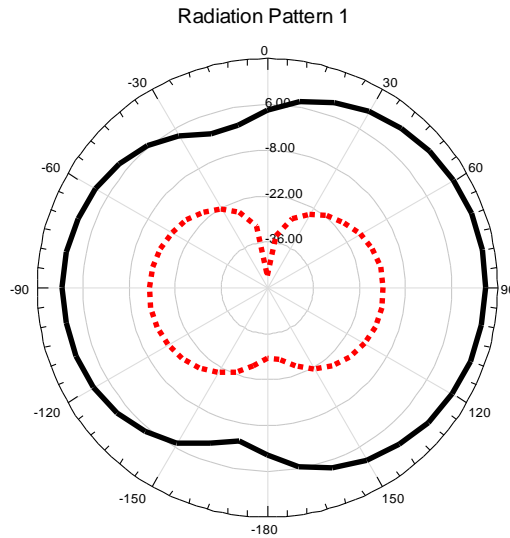
**Figure 9:** Return loss for empty, dielectric slab loaded, and metamaterial loaded waveguide.

Existence of the passband well below the cut-off frequency of the waveguide alone is not a proof of a backward wave. Fig. 10 shows the phase and magnitude of the guided wave. It is obviously seen that the phase of the wave in the passband increases unlike as in an ordinary waveguides where phase decreases. Thus, physically longer waveguides exhibit larger phase of  $S_{11}$  because the energy flow direction opposes the phase velocity. This proves that there is a phase advance in such waveguides unlike phase delay in an ordinary waveguide. With this, one can conclude that physically longer backward wave waveguide appears electrically shorter with phase advance, [10].



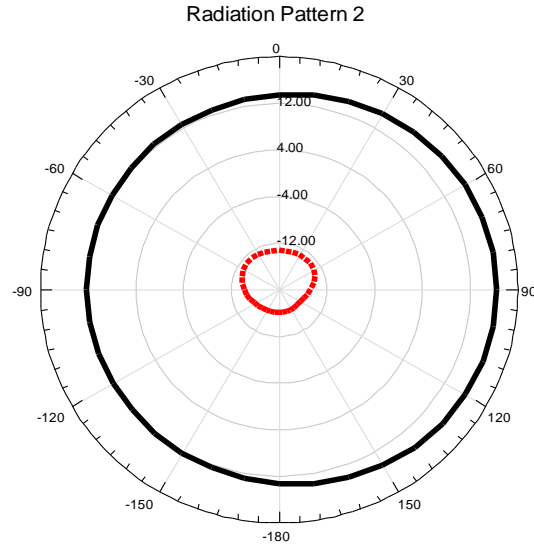
**Figure 10:** Phase and magnitude of S11 for the infinity shaped loaded waveguide antenna.

The E-plane and H-plane patterns of the waveguide antenna at 4.75 GHz are shown in Fig. 11 and 12, respectively. It is clear that it radiates at the resonant frequency like an omni-directional antennas.



**Figure 11:** E-plane radiation pattern for metamaterial loaded open waveguide radiator (solid: co-polarized, dotted: cross-polarized).

## Rectangular waveguide radiator miniaturization using



**Figure 12:** H-plane radiation pattern for metamaterial loaded open waveguide radiator (solid: co-polarized, dotted: cross-polarized)

## 4. PARAMETRIC ANALYSIS

Waveguide antenna characteristics such as cut-off frequency and return loss are affected by many parameters such as the number of metamaterial cells, and the typed of used dielectric. The effect of changing the number of unit cells from 11 through up to 14 unit cells is shown in Fig. 13. We can see that the widest bandwidth, best reduction and minimum return loss were obtained with 14 unit cell. A narrower bandwidth and smaller bandwidth reduction would be obtained if the number of unit cells is less than 11, and a lower performance will be obtained with more than 14 unit cells. The effect of changing the substrate material is shown Fig. 14. Two different materials have been used, the first one is FR4\_epoxy with relative permittivity of 4.4 and dielectric loss tangent of 0.02, and the second is Rogers RT/duroid 5870 with relative permittivity of 2.33 and dielectric loss tangent of 0.0012. It is noticeable that there is more reduction with FR4\_epoxy material which is in agreement with the classical method of miniaturization of guiding structures.

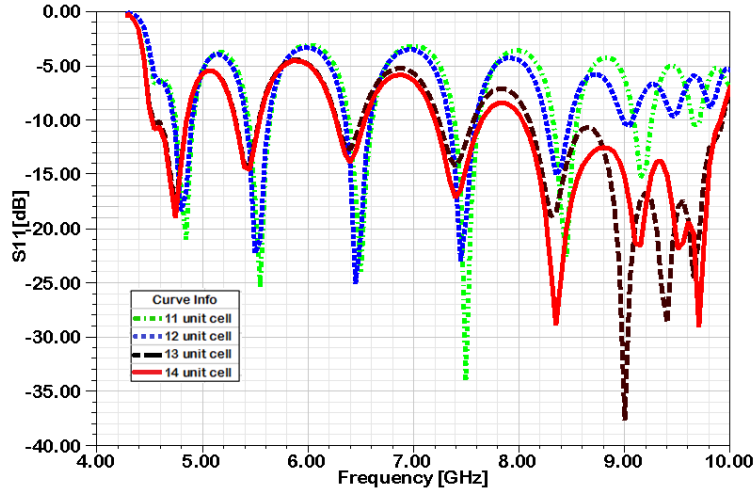


Figure 13: Return loss for the antenna with different number of unit cell.

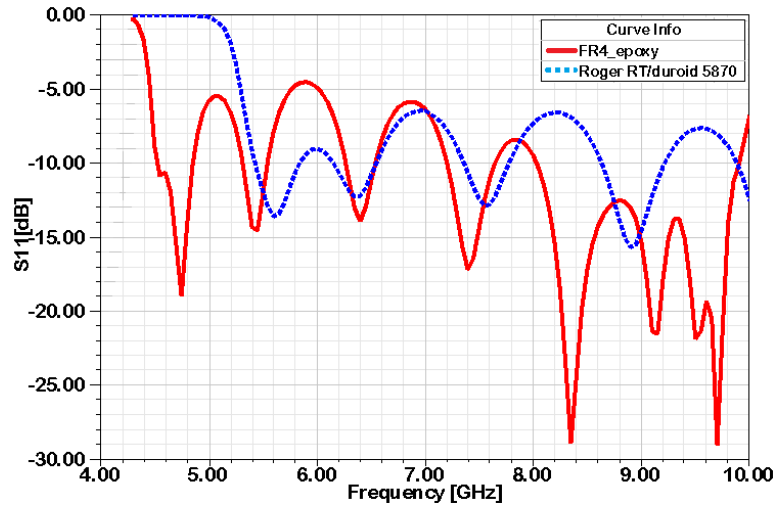


Figure 14: Return loss for the waveguide antenna with different substrate materials.

## 5. CONCLUSION

The infinity shape metamaterial cells were used to miniaturize the open-ended waveguide radiator. The antenna is able to radiate below the cut-off frequency of the waveguide by supporting backward waves. Simulation of

## **Rectangular waveguide radiator miniaturization using**

infinity shape loaded X-band waveguide antenna radiating below the cut-off frequency was successfully carried out. According to simulations, the cut-off frequency is 4.43 GHz, 2.17 GHz below the cut-off frequency of the unloaded waveguide, a minimum return loss of -19dB at 4.75 GHz and bandwidth of 327 MHz. Comparing previous work of miniaturization of waveguides, a higher miniaturization ratio with a bandwidth about 327 MHz and good matching is obtained.

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***Mohamed K. Ouda and Nidal A. Abutahoun***

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