Macro and Micro diversity improvement with Patched Dynamic Decode and Forward Relaying

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Abstract—In this paper, we consider the improvement of the macro and micro diversity by the help of relays using a dynamic decode and forward (DDF) protocol. We first derive bounds on the macro and micro diversity for DDF and Patched-DDF protocols using distributed Mono-stream and Alamouti schemes. We then derive an auto-organized behavior of the relay that maximizes the macro and micro diversity of the broadcast application.

I. INTRODUCTION

In cellular networks, multimedia broadcast/multicast has gained interest for outdoor and home communications. A broadcast transmission is generally designed for providing a given quality of service to the terminal experiencing the worst link quality. Open-loop transmissions are by essence diversity limited and the micro diversity order improvement by the help of relays has already been investigated. The improvement of the SNR, also called macro diversity gain, by transmitting the same signal from non co-localized transmitters is also well known [1][2]. However, to our knowledge, it is not known how to define a macro diversity order or how the macro diversity is exploited by error correction coding. Indeed, when we address the problem of broadcasting to several destinations, the long-term SNR may be modeled by a random variable that varies with the destination location and can fade close to zero. One aim of this paper is to consider the theoretical bounds on error correction coding for the exploitation of SNR-fading or slow-fading along with the fast-fading. Among the wide variety of relaying protocols, the dynamic decode and forward (DDF) protocol [3] has the appealing property of taking the best benefit from the relay use [4][5][6][7][8][9]. As soon as the relay correctly decodes the message sent by the source, which occurs at a random time, a joint transmission scheme between the source and the relay is received by the destination. The transmission ends as soon as the destination correctly decodes the message, the DDF protocol applies more naturally to closed-loop transmissions.

In this paper, we consider the broadcast services improvement as an example of application of DDF and define more diversity efficient Patched-DDF protocols. We consider a multi-cell system, where each base station is one source of the broadcast service, and a deployment of several relays in each cell in order to improve the coverage of the broadcast service. Broadcasting in a multi-user and multi-relay environment give some interesting constraints to the choice of the relaying protocol:

• When inter-cell interference is experienced in the downlink, the deployment of relays close to the cell edge can drastically increase the interference level of one cell on another. Thus, we consider DDF relaying protocols as the relay is in a transmission mode only a fraction of time and transmits error-free signals.
• When using the DDF relaying protocol, each relay activation time is random. The source cannot adapt its transmission according to the activation time of each relay and behaves independently of the relays states. Each relay is assumed to work in an auto-organized way without feedback or control signaling specific to relaying between the source and the relay. We call this situation relay unaware source.
• The destination detects the presence of the relay by the help of dedicated pilot symbols, and we focus on low-complexity receivers.

We propose to design DDF protocols for improving the diversity order of open-loop transmissions from a relay-unaware source to as many destinations as possible, without any feedback or control signaling specific to relaying.

In section II, we present the system model and parameters. In section III, we derive a criterion for the macro diversity order. In section IV, we recall some results on the diversity exploitation by error correction coding for block-fading and Matryoshka channels. We then analyze the micro and macro diversity behavior of DDF and Patched-DDF relaying protocols with a Mono-stream or Alamouti distributed scheme. Finally, in V, we present simulations results enlightening the theoretical results.

II. SYSTEM MODEL AND PARAMETERS

We consider a communication between a source $S$ and a destination $D$, helped by a relay $R$; each one having $N_r$ reception antennas. The relay $R$ can either be in a phase 1 where it receives the signal from the source $S$, or in a phase 2 where it transmits a signal to the destination $D$. A complex Gaussian noise plus interference signal of variance $2N_0$ is added at each receive antenna. We consider an OFDM-based transmission so that each sub-channel associated to each sub-carrier is flat. We furthermore consider that the so-called quasi-static channel remains constant during at least the transmission of one codeword and is independent from one transmission to another. The $N_r$-length channel coefficients vectors between the source $S$ and the destination $D$ are
denoted $h_{SD}$ and are complex Gaussian distributed with zero mean and variance unity. As for the source-destination link, $h_{SR}$ and $h_{RD}$ respectively denote the source-relay and relay-destination channel coefficients.

We note $\rho_{SD}$ the long term signal to noise plus interference ratio (SINR) of the source-destination link. During phase 1, the long term SINR $\rho_{SR}$ is observed on the source-relay link and during phase 2, the long term SINR $\rho_{RD}$ is observed on the relay-destination link. When considering a transmission between the source and large number of destinations, or equivalently when considering the whole positions in the coverage area of the source, the variables $\rho_{SD}$, $\rho_{RD}$ and $\rho_{SR}$ can be considered as random variables. Indeed, the attenuation between one device to the other encompasses the loss due to the reception antenna diagrams and the path loss which are dependent on the positions of the source and destination. Furthermore, the path gain also contains a shadowing random variable that characterizes the neighborhood of the two devices and which is often log-normal distributed.

The transmission of one information word follows a Bit-Interleaved Coded Modulation (BICM) structure [10], i.e., an information word of length $K$ bits is encoded into an interleaved codeword of a binary code $C$, and modulated using a QPSK discrete modulation carrying $m_S = 2$ coded bits. Vectors of modulation symbols are given to the input of the OFDM modulation that produces OFDM symbols.

Finally, the codeword is sent during one frame, which is segmented into $N_f$ sub-frames comprising several OFDM symbols, the length of the $i$-th sub-frame being $B_i$ coded bits, with $B_i \geq K$. We assume that the sub-frames are sent in a dis-continuous fashion, there are for example several broadcast or unicast services scheduled through frequency and time. A Cyclic Redundancy Check (CRC) code is embedded in the information word, allowing for the destination $D$ to try to decode the concatenation of the sub-frames, and to stop listening at the source $S$ as soon as the CRC check is correct. By this mean, the battery of the high quality link user can be saved till the next decoding attempt.

The relay $R$ receives data sent by the source during a phase 1 of the DDF protocol, during which the relay keeps listening to the source as long as decoding failures occur. After correct decoding, the relay $R$ switches into a phase 2 of the codeword transmission. It transmits additional redundancy with a $2^{m_R}$-QAM modulation to $D$ on the same frequency and time resource as the source $S$. Consequently, the addition of the source and relay signals is received at the destination during phase 2. The relay $R$ transmits $B_i m_R/m_S$ coded bits during the $i$-th sub-frame. We note $L_1$ the number of coded bits transmitted by the source during phase 1, and $L_2$ and $L_R = L_2 m_R/m_S$ respectively the number of bits transmitted by the source and by the relay during phase 2. We call $M$ the first sub-frame index after which the relay could correctly decode the information word. The frame and sub-frame structure, the phases of reception and transmission of the relay, and the decoding failures at the relay are illustrated in Fig. 1.

The codewords of $C$ and the additional redundancy sent by the relay are for example generated from a rate matching algorithm associated to a Rate-1/3 turbo-code, as in the 3GPP-LTE standard.

### III. Macro and Micro Diversity Order

In this section, we introduce the material needed for characterizing the robustness of a coded scheme in a macro and micro-diversity context. Let us consider a $N_r \times N_t$ matrix $H$ which entries are independent complex Gaussian distributed with variance unity and zero mean. We assume that the $N_t$ transmit antennas are not co-localized, and the signal transmitted from the $i$-th antenna is received with a SINR $\rho_i$ at each of the $N_r$ destination receive antenna. The channel model is written as

$$Y = H \rho X + \eta$$

where $\rho = \text{diag} \left( \sqrt{\rho_1}, \ldots, \sqrt{\rho_{N_t}} \right)$ and $\eta_i \sim \mathcal{N}_c(0,1)$. After averaging on all possible realizations of the fast fading channel $H$, the pairwise error probability between $X$ and $X'$ is equal to

$$P(X, X'|\rho) \simeq \frac{1}{\det(I + \rho(X - X')(X - X')^\dagger\rho^\dagger/4)^{N_r}}$$

leading to the well-known rank and determinant criteria [11] for the design of space-time codes in terms of micro-diversity and coding gain.

Then, we consider the variation of the random variables $\rho_i$, varying according at least to the position of the devices. We assume that the distance between the transmit antennas (or Source and Relay in the relaying case) is sufficiently high to consider an independence of the $\rho_i$ random variables. Thus, the pairwise error probability averaged on multiple positions of the destination can be rewritten as

$$P(X, X') \simeq E_{\rho_i} \left[ \frac{1}{1 + \sum_i \rho_i^{N_r} \|X_i - X_i'\|^{2N_r}/4} \right]$$

where $X_i$ is the $i$-th row of $X$.

![Fig. 1. The frame and sub-frame decomposition of one codeword.](image)
In [12], the computation of $P(X | X')$ is done for log-normal correlated shadowing. Unfortunately, the distributions of $\rho_i$ depend on all variables defining the path gain from each transmitter and for each possible destination position and are particular to each deployment type. The macro-diversity order can be defined as the number of sources playing a role in the SNR observed at the destination. In our case, it may vary from one pairwise error probability to the other, and can be defined as the minimal number of non-null coefficients multiplying the $\rho_i$ variables in (2). Thus, one can derive the following definition of the macro diversity, which does not require the knowledge of the $\rho_i$ distributions:

$$D_{macro} \triangleq \min_{X, X'} \left\{ \text{rank} \left( I \odot (X - X')(X - X')^\dagger \right) \right\}$$  \hspace{1cm} (3)

where $\odot$ denotes the term-by-term multiplication operator. If the $\rho_i$ random variables are correlated, the criterion includes the hermitian part of the covariance matrix, this is out of the scope of this paper. The SINR $\rho_i$ contributes to the system performance as soon as $X_i - X'_i$ is non-null for any pairs of codewords. We can show that the macro-diversity order is always larger than the micro diversity order, which induces that if the full micro-diversity order is achieved, so does the full-macro-diversity order.

The same definition of the macro diversity order is obtained by computing the expectation of the pairwise error probabilities when $H = I$, which means that the macro diversity can be analyzed independently from the micro diversity. Furthermore, from (2), we observe that after averaging over the micro SINR values, $P(X | X')$ does not exhibit the transmit micro-diversity order. This suggests that the micro-diversity improvement might not be helpful for the global system performance. This assertion will be clarified in the simulation results.

IV. ANALYSIS OF THE MACRO DIVERSITY AND MICRO DIVERSITY ORDER OF DDF RELAYING

In this section, we derive the micro diversity and macro diversity analysis and control of DDF relaying protocols for two distributed MIMO schemes with low complexity receivers: the Mono-stream DDF relaying and the distributed Alamouti scheme.

A. Results on the diversity exploitation with error correcting codes

1) Euclidean code definition: We consider the definition of the Euclidean code $\mathcal{E}$, as in [13]: for a given codeword transmission, the information is converted in a point $x$ belonging to a multi-dimensional space. The conversion encompasses the transmission scheme and the channel realization. With a global ML decoding and Gaussian noise assumption, the pairwise error probability is a function of the squared Euclidean distance $\|x - x'\|^2$, which can be factorized as follows:

$$\|x - x'\|^2 = \sum_i \lambda_i d_i^2$$  \hspace{1cm} (4)

where $\lambda_i$ are independent squared fading variable corresponding to one so-called channel state, and $d_i^2$ is the amount of squared Euclidean distance $\|x - x'\|^2$ transmitted through the $i$-th channel state. The distances $d_i$ mainly depend on the Hamming weight between the two codewords of $\mathcal{C}$ associated to $x$ and $x'$, on the interleaver design, on the binary labeling of QAM modulation symbols, and on other processing on symbols, such as algebraic precoding [13]. Thus, the decoder input sees a block channel; the $i$-th channel being of length $L_i$ coded bits and seeing a random attenuations $\lambda_i$.

2) Block fading channels: The bound on the diversity order of a coded modulation transmitted on a block-fading channel of equal-length blocks and independent fading realizations has been derived in [14]. For variable-length blocks, the full diversity order is achieved as soon as $K \leq \min (L_i)$ [15].

3) Matryoshka channels: In [16], the Matryoshka channel $\mathcal{M}(D, \mathcal{L})$ is defined by $N$ blocks, where $\mathcal{L} = (L_1, \ldots, L_N)$ is the vector of block lengths and $D = (D_1, \ldots, D_N)$ is the vector of diversity order intrinsic to each block. The Matryoshka channel is characterized in that the fading random variable of the $i$-th block is a component of the fading random variable of the $i-1$-th block. As a result, the full diversity order is achieved as soon as $K \leq L_1$, where $L_1$ is the block of highest diversity order.

B. Mono-stream DDF relaying

In this section, we assume that after having correctly decoded the information word after the reception of $L_1$ bits during the $M$ first sub-frames, the relay regenerates the $L_2$ remaining coded bits and transmits the same constellation symbols on the same resource during the $N_0 - M$ last sub-frames. The signals received at the destination for each channel use during the two phases of the DDF protocol are

$$\begin{align*}
y_1 &= z_1 \sqrt{\rho_{SD} h_{SD}} + \eta_1 \\
y_2 &= z_2 (\sqrt{\rho_{SD} h_{SD}} + \sqrt{\rho_{RD} h_{RD}}) + \eta_2
\end{align*}$$

We consider the distance between two codewords $x$ and $x'$ of the euclidean code $\mathcal{E}$

$$\|x - x'\|^2 = \|h_{SD} x + h_{RD} x'\|^2$$

where $\rho = \text{diag} (\sqrt{\rho_{SD}}, \sqrt{\rho_{RD}})$.

$$X = X' = \begin{bmatrix} d_1 & d_2 \\ 0 & d_2 \end{bmatrix}$$

and $d_2^1$ and $d_2^2$ denote the factorized squared euclidean distances as in (4) corresponding to non-null bits of the codeword associated to $x - x'$ and occurring respectively in phase 1 and phase 2.

From (1), we obtain

$$P(x, x' | \rho) \sim \left( 1 + \frac{d_1^1 \rho_{PSD}}{4} \right) \left( 1 + \frac{d_2^1 \rho_{RD}}{4} \right) + \frac{d_2^2 \rho_{PSD}}{4}$$

Asymptotically (when $N_0 \to 0$), the right hand side term of the denominator becomes negligible. Thus, the system behaves as a transmission on a block-fading channel with two independent blocks of length $L_1$ and $L_2$, the full diversity order is constrained to the Singleton bound on block-fading.
channels, which gives $\min(L_2, L_1) \geq K$. If $L_1 = 0$, i.e., if the whole transmission is done from the two transmitters at the same time, the diversity order is limited to one, as expected.

For the macro diversity, we compute from (3)

$$(1 \otimes (X - X') (X - X')^\dagger) = \text{diag}(d_1^2 + d_2^2, d_2^2)$$

If $d_2 = 0$ (no error within the $L_2$ coded bits of phase 2), the macro diversity order is $D_{\text{macro}} = 1$, while if $d_1 = 0$ (no error within the $L_1$ coded bits of phase 1), the macro diversity order is $D_{\text{macro}} = 2$. We define the macro diversity equivalent binary-input binary-output channel as a block-SNR channel, following a Matryoshka $\mathcal{M}([1,2], \{L_1, L_2\})$ structure. As a result, a macro diversity order of two is observed after decoding only if $K \leq L_2$.

It has to be noted that in this case the error correcting code plays a different role on the micro diversity and macro diversity recovery. For example, when $L_2 \geq L_1$ the maximal macro diversity order might be achieved when the maximal micro diversity order cannot. Such a mismatch between the macro diversity and the micro diversity comes from the following observation: $\sqrt{\rho_{SD} h_{SD}} + \sqrt{\rho_{RD} h_{RD}}$ carries a micro diversity order one when $\rho_{SD}$ and $\rho_{RD}$ are set constant, while $E(\|\sqrt{\rho_{SD} h_{SD}} + \sqrt{\rho_{RD} h_{RD}}\|^2) = \rho_{SD} + \rho_{RD}$ carries a macro diversity order of two.

In the particular case of DDF, we always have $L_1 \geq K$, otherwise the relay cannot decode the information without errors: the micro and macro diversity orders are always achieved altogether as soon as $L_2 \geq K$.

C. Patched-DDF relaying with Mono-stream scheme

The main constraint of DDF-protocols is the random activation time of the relay, which makes $L_1$ and $L_2$ vary. When the relay activation is late, then $L_2 < K$ and the full macro and micro diversity orders are not achieved. The main idea of Patched-DDF, initially proposed under the form of Patched-DSTBC in [17] for micro diversity enhancement, is to virtually move some coded bits sent during phase 1 to the block associated to phase 2. In order to do so, the relay transmits a function $f(z_1, z_2)$ of vectors of symbols $z_1$ sent during phase 1 and $z_2$ sent during phase 2, as illustrated in Fig. 2. The destination performs a combination, for example linear, of the signals received from the source only during phase 1 and the superimposed signals received from the source and the relay during phase 2. We focus on the cases where the combination lead to an equivalent MIMO scheme allowing for a low complexity decoding.

For the Mono-stream case, the relay transmits during phase 2 a symbol $x_2$ belonging to a $2^{m_R}$-QAM modulation, and derived from the combination of the QPSK symbol $z_{m_R/2}$ transmitted during phase 2 and of $m_R/2 - 1$ QPSK symbols $z_1, \ldots, z_{m_R/2 - 1}$ transmitted during phase 1 by the source. The relay knows $z_{m_R/2}$ by the re-encoding of the information word it correctly decoded at the end of phase 1. The signals received by the destination during phase 1 and phase 2 are as follows:

$$\begin{align*}
y_{1,i} & = z_{1,i} \sqrt{\rho_{SD} h_{SD}} + \eta_{1,i}, \quad 1 \leq i \leq m_R/2 - 1 \\
y_{2} & = z_2 \sqrt{\rho_{SD} h_{SD}} + x_2 \sqrt{\rho_{RD} h_{RD}} + \eta_2
\end{align*}$$

The combination for building the $2^{m_R}$-QAM sent by the relay is defined by:

$$x_2 = \sum_{i=1}^{m_R/2-1} a_i z_{1,i} + a_{m_R/2} z_2, \quad a_i = \sqrt{\frac{3}{2^{m_R} - 1}} 2^{i-1}$$

The destination combines the signal $y_2$ received during phase 2 with the $m_R/2 - 1$ signals $y_{1,i}$ received during phase 1 according to the definition of the symbol sent by the relay, i.e.:

$$\begin{align*}
y & = \sum_{i=1}^{m_R/2-1} a_i y_{1,i} + a_{m_R/2} y_2 \\
& = x_2 \left( \sqrt{\rho_{SD} h_{SD}} + a_{m_R/2} \sqrt{\rho_{RD} h_{RD}} \right) + \eta
\end{align*}$$

where the noise $\eta$ is complex Gaussian distributed of variance unity. We consider here $2^{m_R}$-QAM modulations re-building, or patching, in order to keep the receivers complexity as low as possible.

At the end of phase 2, $\min((m_R/m_S - 1) L_2, L_1)$ bits have been virtually moved from block 1 to block 2. This is true for both the macro diversity block-SNR channel and micro diversity block-fading channel. If $(m_R/m_S - 1) L_2 > L_1$, the relay transmits the same symbols as the source for the last bits of phase 2, as for the classical Mono-stream DDF protocol.

When considering the micro diversity, the new equivalent block-fading channel has two blocks of length $L_1' = \max(L_1 - L_2(m_R/m_S - 1), 0)$ and $L_2' = \min(L_2 m_R/m_S, L_1 + L_2)$ coded bits. Thus, the full diversity is only obtained if $\min(L_1', L_2') \geq K$. In the case of Patched Mono-stream DDF, the micro diversity order of the first block might be lost for early activation of the relay, which was not the case with classical Mono-stream DDF.

When considering the macro diversity, the new equivalent block SNR channel is a Matryoshka $\mathcal{M}([L_1', L_2'], \{2, 1\})$, and the full macro diversity order is achieved as soon as $L_2' \geq K$. As $L_2' \geq L_2$, we understand that the Patched-DDF can recover the macro diversity in more cases than with the classical Mono-stream DDF. Furthermore, some configurations illustrate the fact that full macro diversity can be observed when full micro diversity is not. As a remark, since $a_{m_R/2} \leq 1$, the SNR is degraded for the relay to destination link. The SNR is left constant on the source to destination link but a larger modulation scheme is used, which results in a loss of coding.
gain. The SNR degradation is a price to pay for recovering the diversity with a low complexity. More efficient schemes have been presented in [17] but need more complex receivers.

We propose that the relay chooses its spectral efficiency $m_B$ in the aim of maximizing the macro diversity, then the micro diversity, and finally minimizing the coding gain degradation. Thus, the relay might choose the minimal $m_B$ from the set of modulations that satisfy $\min(L_2 m_B/m_S, L_1 + L_2) \geq K$. This behavior of the relay magnifies the dynamic property of the DDF protocol. Furthermore, the source transmission is independent of the relay state or constellation, which gives us one practical example of auto-organized relay-unaware source systems.

One can use more efficient distributed space-time codes between the source and the relay, that will lead to a joint maximization of the macro and micro diversity, even with Patched-DDF strategies.

### D. Patched-DDF relaying with distributed Alamouti schemes

In this section, we consider that during phase 2, the relay will transmit the modified symbols sent by the source according to a distributed Alamouti scheme. After proper processing at the destination using the Alamouti scheme orthogonality, the channel coefficients are combined coherently, i.e., the coded bits of phase 1 see an equivalent channel equal to $\sqrt{\rho_{SD}[h_{SD}]^2}$; and the coded bits of phase 2 see an equivalent channel equal to $\sqrt{\rho_{SD}[h_{SD}]^2 + \rho_{RD}[h_{SD}]^2}$.

For the micro diversity behavior of the DDF protocol, the block-fading channel is a Matryoshka $M(\{L_2, L_1\}, \{2,1\})$, and the full micro diversity order is achieved as soon as $L_2 \geq K$. If a Patched-Alamouti DDF protocol is used [17], the full micro diversity order can be achieved as soon as $L_2 m_B/m_S \geq K$. As the SNR coefficients $\rho_{SD}$ and $\rho_{RD}$ are tied to the channel coefficients $|h_{SD}|^2$ and $|h_{RD}|^2$, it is easy to understand that the micro diversity behavior is equal to the macro diversity behavior. Patched-DDF-Alamouti schemes also are candidate for relay-unaware source systems with low-complexity receivers. Table IV-D summarizes the conditions for obtaining macro and micro diversity with DDF and Patched-DDF Mono-stream and Alamouti schemes.

### V. COMPUTER SIMULATION RESULTS

It was shown in [18] that the discrete input outage probability follows the same diversity order as a coded system, and in [13] that very close to the outage performance can be obtained by a proper design of the channel interleaver. Thus, we compute the discrete input outage probability for varying values of the SNR between the source and the destination $SNR_{SD} = 10 \log_{10}(\rho_{SD})dB$ and between the relay and the destination $SNR_{RD} = 10 \log_{10}(\rho_{RD})dB$. We store the couple of SNRs leading to a target probability error of $10^{-3}$, the shown behaviors being similar for a target of $10^{-2}$.

First, in Fig. 3 and Fig. 4, we have $N_r = 2$, $N_b = 2$ sub-frames, $B_1 = K$ and $B_1/B_2 = 2$ which leads to a final coding rate $R_c = 2/3$. Fig. 3 illustrates the discrete input outage probability for $M = 1$ when $SNR_{SD} = SNR_{RD}$.

We observe that for the DDF-Mono-stream scheme $\min(L_1 = K, L_2 = K/2) < K$, for the DDF-Alamouti scheme $L_2 = K/2 < K$ and for the Patched-DDF-Mono-stream scheme $\min(L_1 = K/2, L_2 = K) < K$, the full micro diversity order cannot be achieved. For the Patched-DDF-Alamouti, $L_2 = K$ and the full diversity order 4 is achieved as expected. We observe as well the degradation in coding gain due to the patched strategy.

In Fig. 4, we compare the DDF Mono-stream and DDF-Alamouti schemes for varying $SNR_{SD}$ and $SNR_{RD}$ and a target error rate of $10^{-3}$. The curve labeled with No Relaying corresponds to $M = 2$, and the target error rate is achieved.
for $SNR_{SD} = 15dB$. When $M = 1$, the two blocks of the block-SNR channel have lengths $K$ and $K/2$, leading to a macro diversity order one. This is illustrated by the vertical asymptote at $SNR_{SD} = 12.5dB$, corresponding to the SNR needed for achieving the target error rate with the source to destination link only and a coding rate $R_c = 1/2$. Indeed, when $SNR_{RD}$ tends to infinity, the $K/2$ coded bits associated to the second block can be decoded with no error, which leaves $K/2$ bits to be decoded with $K$ bits transmitted only on the source to destination link. We then consider Patched-DDF schemes for $M = 1$ with 16-QAM patching ($m_R = 4$). There is an horizontal asymptote on the left which illustrates that the target error rate can be obtained even for null $\rho_{SD}$ (contrary to macro diversity one cases). The horizontal asymptote is the needed SNR for achieving the target error rate with the relay to destination link only and a coding rate $R_c = 1$ and illustrates that the Patched-DDF allows for recovering the macro diversity when the DDF cannot. However, full macro diversity is achieved at the price of a performance degradation for low $SNR_{RD}$ values, which is due to the use of a higher spectral efficiency modulation at the same SNR as with no patching. Fortunately, this difference appears for high $SNR_{SD}$ and high $M$ indexes, for which the destination will often decode the information before the relay. We observe that the Patched-DDF-Alamouti scheme highly outperforms the Patched-DDF-Mono-stream scheme for $SNR_{SD} \approx SNR_{RD}$. Indeed, the Patched-DDF-Mono-stream does not have the full micro diversity order. For low values of $SNR_{SD}$, the two schemes perform equally and the micro diversity does not bring performance improvement.

In Fig. 5 and Fig. 6, we have $N_r = 2$, $N_b = 7$ subframes, $B_1 = K$ and $\forall i < 7$, $B_i/B_i = 3$ which leads to a final coding rate $R_c = 1/3$. In Fig. 5, we compare the DDF and Patched-DDF Mono-stream and Alamouti schemes. The target error rate is achieved for $SNR_{SD} = 12dB$ with no relay activation ($M = 7$). For the DDF-Mono-stream, as $L_1 \geq B_1 = K$, the macro diversity and micro diversity are obtained only for $L_2 \geq K$, i.e. for $M \leq 4$. This is confirmed by the vertical asymptotes for $M = 5$, $M = 6$ and $M = 7$ and corresponding to a source-to-destination-only transmission with coding rates respectively equal to $R_c = 1/7$, $R_c = 1/4$ and $R_c = 1/3$. Furthermore, the horizontal asymptotes correspond to the full macro diversity cases $M = 1, M = 2, M = 3$ and $M = 4$, and are given by the performance of a relay-to-destination-only transmission, with rates respectively equal to $R_c = 1/2$, $R_c = 3/5$, $R_c = 3/4$ and $R_c = 1$. For cases $1 \leq M \leq 4$ when the full macro diversity is achieved, the DDF-Alamouti scheme outperforms the DDF-Mono-stream scheme, which is explained by the coherent/non-coherent sum of the $h_{1SD}$ and $h_{1RD}$ coefficients. For cases $5 \leq M \leq 7$ when the full micro diversity is not achieved, the performance are dominated by non-full diversity error events.

**TABLE I**

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Micro Diversity</th>
<th>Macro Diversity</th>
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<tbody>
<tr>
<td>DDF Mono-stream</td>
<td>$K \leq \min(L_1, L_2)$</td>
<td>$K \leq L_2$</td>
</tr>
<tr>
<td>DDF Alamouti</td>
<td>$K \leq L_2$</td>
<td>$K \leq L_2$</td>
</tr>
<tr>
<td>Patched-DDF Mono-stream</td>
<td>$K \leq \min((m_R / m_S - 1) L_2, m_R / m_S L_2)$</td>
<td>$K \leq m_R / m_S L_2$</td>
</tr>
<tr>
<td>Patched-DDF Alamouti</td>
<td>$K \leq m_R / m_S L_2$</td>
<td>$K \leq m_R / m_S L_2$</td>
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*Conditions on full macro and micro diversity orders.*
which have the same performance for the DDF-Mono-stream and DDF-Alamouti scheme. By using Patched-DDF schemes for $M = 5$ with 16-QAM patching and $M = 6$ with 64-QAM patching, we observe that full macro diversity is achieved at the price of a performance degradation for low $SNR_{BD}$ values.

In Fig. 6, we compute the probability of activation of the relay from the discrete input outage probability of the source-to-relay link with $SNR_{SR} = 2dB$. As the probability of having a non-full diversity configuration is non-null (i.e., the probability of non activation of the relay), the averaged performance among all possible relay activation times is not theoretically full diversity. However, we observe that for Patched-DDF protocols all cases $1 \leq M \leq 6$ have a full macro diversity order. As the non-relaying case probability is negligible for $SNR_{SR} = 2dB$, the system behaves as a full macro diversity one. The classical DDF protocols exhibits a final macro diversity one in that case. Furthermore, we observe that the degradation in coding gain due to the patching strategy is negligible, as foreseen in section IV-C. Finally, we see that the Alamouti scheme always outperforms the Mono-stream scheme.

In Fig. 7, we compute the averaged Patched-DDF-Alamouti performance for $SNR_{SR} = \{-6, -2, -1, 0, 1, 2, 3, 4\}dB$ corresponding to a set of curves read from right to left, with a flat fading channel and the classical typical urban channel with six taps $TU6$ over 5 MHz. Recovering the macro diversity by Patched-DDF drastically improve the coverage area of a broadcast service, even for high micro diversity systems. The macro diversity gain would be observed as well on AWGN channels, which allows us to argue that the Patched-DDF strategy is also efficient in closed-loop system including H-ARQ. Finally, we can observe that the Patched-DDF schemes are particularly useful even for low $SNR_{SR}$ values, which could address the case of cooperative broadcast when destinations help each other [19].

VI. CONCLUSIONS

In this paper, we have illustrated the importance of controlling the macro diversity order in DDF relaying system, and have proposed a Patched version of the protocol for improving it. We highlight that the presented macro and micro diversity efficient Patched-DDF protocols are relay unaware, and can be used in signaling-free broadcast system with multiple auto-organized relays.

REFERENCES