CONTROL OF CHAOS IN MECHANICAL ENGINEERING

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Abstract:

Each and every system in existence falls under chaotic behaviour in a certain stage of its life span. This paper deals with a problem of controlling such a chaotic behaviour, suppressing and increasing the behaviour by OGY method. The goal is to control the chaos in a smaller expense of power.

Keywords: Poincare map, OGY method, Euler’s method, orthogonal matrices.

Introduction:

The problem of control of chaos is studied usually in three historically earliest and most actively developing directions of research. The study is based on periodic system excitation, the method of Poincare’ map linearization (OGY method), and the method of time-delayed feedback (Pyragas method) \[1\]. This paper deals with a chaotic problem in mechanical engineering. The problem is analysis in the light of OGY method.

Methodology:

The problem is defined based upon the method put forward by Ott, Grebogi and Yorke \[2\]. This method proposes that the chaotic behavior can be controlled by generating a similar one of the infinite unstable periodic orbits embedded in the attractor of the chaos.

Let us consider a smooth family of nonlinear autonomous systems of ordinary differential equations

\[ \dot{x} = F(x, \mu), x \in M \subset \mathbb{R}^m, \mu \in L \subset \mathbb{R}^k, F \in C^\infty \]  \(1\)

given in phase space \(M\) by smooth vector fields \(F\), depending on coordinates of vectors of system parameters \(\mu\), lying in the region \(L\) of the space \(\mathbb{R}^k\).

Let the unstable limit cycle \(x^*(t, \mu^*)\) be the required solution of family of systems \((1)\) which has in addition a regular or singular attractor at the same parameter value \(\mu = \mu^*\) at a moment of time \(t\).

Let us consider the Poincare section \(S\) passing through a point \(x_0 = x^*(0, \mu^*)\). Also consider a Poincare map \(x \rightarrow G(x, \mu)\) where \(G: (\mathbb{R}^N \times \mathbb{R}) \rightarrow \mathbb{R}^N\). This map linearizes the unstable states of the Poincare section.
**Tracking Chaos:**

In mechanics, any dynamism can be characterized as a system of vibrations. These vibrations, in the case of discrete dynamical system or continuous dynamical system, can be constructed as a transversal section [3] of trajectories (Poincare’ map).

Consider the discrete dynamical system [4],

\[ x_{n+1} = F(x_n, \mu) \]  \hspace{1cm} (2)

with initial condition \( x = x_0 \) when \( n = 0 \).

There exists a curve of fixed points \( x = H(\mu) \) satisfying

\[ x_f = F(x_f, \mu). \]  \hspace{1cm} (3)

The curve is then linearized iteratively by Euler’s predictor and corrector to track the zeros of \( F \). The chaotic system is traced by Euler’s predictor method

\[ x_{n+1} = x_n - h \left( \frac{d}{dx} F(x_n, \mu_n) \right)^{-1} \cdot \frac{d}{d\mu} F(x_n, \mu_n) \]  \hspace{1cm} (4)

from a fixed point say \((x_0, \mu_0)\) with a step size \( h \).

By making such a prediction, an approximation error of order \( h \) is made. This error is then corrected by solving

\[ F(x, \mu_0 + h) = 0. \]  \hspace{1cm} (5)

iteratively, taking the final point \( x_n \) as the initial value for iteration.

Thus the curve of steady state of fixed points of the discrete system is traced.

**Controlling Chaos:**

Let us consider, the solution of (1) is

\[ x_f^*(t) = x^*(t, \mu_f^*) \]  \hspace{1cm} (6)

at \( \mu = \mu_f \). Allowing some noise or disturbance \( \nu(t) \) about time \( t \) from outside the system, we get

\[ x(t) = x^*(t, \mu_f^*) + \nu(t). \]  \hspace{1cm} (7)

Let us assume such a linear variational system as

\[ x_{n+1} = x_n A + \nu B \]  \hspace{1cm} (8)
where \( A = \frac{d}{dx} F(x, \mu) \) and \( B = \frac{d}{d\mu} F(x, \mu) \) are matrices.

The matrix \( A \) is assumed to be diagonalizable such that,

\[
A = S\Lambda S^{-1}
\]

where the columns of \( S \) are the right-eigen vectors \( e_i \), the rows of \( S^{-1} \) are the left eigen vectors \( f_i \) and \( \Lambda \) contains the eigen values \( \lambda_i \).

And \( f_i e_j = \delta_{ij} \) where \( \delta_{ij} \) is Kroniker-\( \delta \).

Now the value of \( \nu \) is obtained by substituting the matrices in (8).

**Conclusion:**

The controlling chaos, which is created externally, is created by supplying suitable values to the eigen vectors and eigen values of the matrix \( A \) in (8). This method is effective when the external noise is minimum.

A number of applications in mechanical engineering gained, including control of pendulum systems, beams, plates, control of stick–slip friction motion, control of vibro-formers and control of micro-cantilevers.

**References:**


