Stable and Fair Power Control in Vehicle Safety Networks

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Abstract—Cooperative vehicle safety (CVS) systems operate based on frequent broadcast of vehicle state (e.g., position, heading and speed) and safety information to other vehicles over a wireless local area network. Such networks are formed using Dedicated Short Range Communications (DSRC) technology. In crowded networks the shared wireless channel may become congested and fail to deliver the broadcast information, potentially affecting the reliability of CVS applications. This issue, known as scalability issue, has been identified as one of the challenges to be addressed with high scale deployment of vehicle safety networks. Several scalability algorithms have recently been introduced and studied in literature; currently a select number of algorithms are under study by industry for possible adoption and standardization. One method used by some algorithms is congestion control through power or range adaptation. In this paper, we analyze two existing range control algorithms in terms of time stability and spatial fairness. We derive stability conditions for the main range control scheme that is currently under study. An improved algorithm that relaxes these conditions is then proposed and its convergence properties are studied. Finally, fairness properties of several power control schemes are studied and an enhancement method is presented that ensures spatial fairness is achieved by the presented power control schemes.

Keywords—VANET; Channel Busy Ratio; Congestion Control; Fairness; Cooperative Vehicular Safety Systems; Range Control Power Control, Scalability.

I. INTRODUCTION

The emerging active safety systems rely on vehicle-to-vehicle (V2V) communication to achieve real-time situational awareness. One such system, currently under initial field tests, is called Cooperative Vehicle Safety (CVS[1]) system and is expected to become operational and widely deployed before the end of the decade. CVS will also serve as a component of the eventual autonomous driving systems, forming a major part of its situational awareness subsystem. A major technical obstacle in large scale deployment of CVS is the scalability of the V2V communication network (called V2V safety network). V2V safety networks use a 10MHz DSRC (Dedicated Short Range Communication [2][3][4]) channel, which may be shared amongst up to thousands of nodes in dense traffic scenarios. The problem is compounded by the fact that vehicles sharing the same channel are required to frequently send messages indicating their current state (position, speed, heading, path history, etc.). Such state information is needed in CVS in order for each vehicle to build a real-time situational awareness map of its surrounding vehicles; the map is continuously analyzed for detection of possible collision hazards. The scalability issue can degrade the performance of CVS to levels that the real-time situational awareness map may not be trustworthy, rendering CVS applications unusable. In addition, emergency safety messages such as crash reports, sudden stopping (emergency electronic brake light) will also suffer in congested wireless channels and may not be delivered to desired destinations.

To address the issue of scalability, several solutions have recently been proposed [6]-[10],[13],[16],[21]-[25]. While solutions presented in these papers do offer valid mechanisms of reducing congestion in V2V safety networks, practical and implementation considerations had so far limited industry’s attention to the two methods presented in[7][23]. These two methods are currently under test and may possibly be modified by the vehicle safety communication consortium (VSCC). This paper seeks to provide an analysis of the stability and fairness properties of the solution in [7] (the only scheme under consideration which has a power control component) and presents an enhanced power control method which can significantly improve the solution in [7]. The work in [23] does not use power control and is not studied here. A comparison of the methods in [7] and [23] is currently being done by VSCC, which authors are also contributing to, and is outside the scope of this paper. However it is worth mentioning that congestion mitigation through rate control has been the subject of significant work in other areas (in particular related to TCP flows in the Internet). Rate control in TCP is based on observing end-to-end delay or packet loss and then adjusting the source rate using methods such as additive increase multiplicative decrease (AIMD) to avoid possible congestion failure in the Internet. The methods used in [7] and [23] control the rate by observing channel load (rather than implicit measures such as packet loss) and directly adjust the rate to control the load. The use of possibly disruptive methods such as AIMD is not necessary in one-hop network of CVS. In contrast to these methods, the solution in [7] takes a completely different approach by correlating message rate to vehicle dynamics and estimation error and...
leaves congestion control to power control schemes such as those described in this paper.

This paper makes several distinct contributions; first we present a stability analysis of the power control scheme of [7], by extending our previous work in [14], to set the stage for possible improvements. We then present a new power control algorithm that relieves many of the restrictions of the algorithm presented in [7]. A detailed analysis of the stability and convergence properties of this new algorithm is also presented. Following stability analysis, we consider the issue of fairness and analyze the properties of the original [7] and the new enhanced algorithm. We also compare the fairness findings with those of a similar algorithm, presented in [8], in order to provide an insight into the issues of distributed power control in Vehicular Ad-Hoc Networks.

In the next section, we first provide a brief background on V2V safety networks and the scalability issue. In section III, an overview of several congestion control algorithms are presented and their use of network feedback in the form of Channel Busy Ratio (CBR) is examined. In section IV, we present a detailed stability analysis of the main power control scheme that is currently under study for standardization. Section V introduces the enhanced power control scheme, and presents its stability conditions. Finally, fairness enhancement for power control schemes is presented in section VI.

II. BACKGROUND AND RELATED PREVIOUS WORK
A. Vehicular Safety Networks and the Issue of Communications Scalability

Active safety applications for crash avoidance highly depend on having accurate estimate of the states (position, speed, etc.) of neighboring vehicles in order to correctly analyze and detect hazardous crash situations. This will allow timely actions by the driver or autonomous driving control. To maintain accurate estimate of other vehicles’ states, a vehicle expects to receive enough information on a continuous basis from all its neighbors; therefore, each vehicle in a CVS system frequently broadcasts its state in a neighborhood of up to a few hundred meters (Fig. 1). This is generally done by reading GPS and on-board sensors and broadcasting the information in Basic Safety Messages (BSM) over a single-hop DSRC broadcast channel. BSMs form the bulk of messages delivered over DSRC channels and are responsible for congestion and the scalability issue. The occasional safety messages such as Emergency Electronic Break Light (EEBL) are relatively rare and while they suffer the effects of congestion, they do not contribute to the problem.

The initial design of CVS was based on broadcasting BSMs at 10Hz, with power set to levels enough to reach at least 250meters (generally higher, up to 1Km) [1]. This design was shown to lead to communications scalability issues (also called congestion failure) in congested highway or intersection scenarios [10][13][7][23].

The main reason for the scalability problem was the fact that when the number of vehicles sharing the same wireless broadcast channel increases beyond a certain limit, the throughput of the DSRC network which uses a CSMA (Carrier Sense Multiple) based MAC (as in 802.11 standard) quickly diminishes. The result is high loss rates which will make accurate tracking of the position and state of neighboring vehicles impossible. To overcome this issue the main two control levers of the network, which are rate and power for BSM transmission, can be used to control the network and relive the congestion issue. The next subsection briefly overviews few of the methods described in literature.

B. Existing Scalability Solutions

V2V communication in CVS prototypes used the basic design of broadcasting BSMs at 10Hz, to a range of at least 250meters (power set accordingly) [1]. The scalability issue of this design was demonstrated through simulation studies in several works including [10][13][7] [23]. An analytical model of the V2V network in [11] also confirmed this issue. To address the issue of scalability, several different algorithms have been proposed so far [6]-[10],[13],[16],[21]-[25]. These algorithms can be categorized into three groups of rate-control, power (range)-control, and joint rate and power control algorithms.

Methods such as [10][8] can be considered range or power control algorithms, as they assume that the rate is fixed or set by some other method, and they only focus on adapting the power or range of communication such that network congestion is avoided. The work in [10] achieves this through nodes advertising their last power setting and load (number of received packets) and then adjusting the power of each node to achieve a max-min fair distribution of the power assignments in a way that the load is kept around a maximum. The work in [8] provides a feedback linearized adaptive power control method that ensures an optimal level of Channel Busy Ratio (CBR) is achieved in the network, maximizing the broadcast throughput for any given rate and density of the nodes in network. The optimal CBR level, found from simulations and analytical models [12], is believed to be around 0.65. CBR is also called utilization or channel occupancy in other texts [11][6].

Methods such as [23][22][13] assume that transmission power is fixed and focus on adapting the rate of message generation. In [23], the rate is adaptively controlled to ensure that channel busy ratio is maintained around 0.6. Rezaei et.al. in [13] propose to use an error-dependent message generation policy to reduce the amount of communication needed for accurate tracking of vehicles. Though this method alleviates congestion to some degree, it is not adaptive to network
situations. In our previous work in [22] an adaptive mechanism is applied on top of the error dependent policy of [13]. The method responds to changes in CBR and packet loss ratio in the network to ensure accurate tracking.

While above methods focus mostly on either rate or range of communication, our previous work in [7] proposes a joint rate-power control scheme that combines the rate control concepts from [13] and [22] with an adaptive power control mechanism. The power control algorithm in [7] assumes that the message rate is controlled such that tracking error for neighboring vehicles is maintained below a threshold, if it then controls the range of the neighborhood by adaptively adjusting the power so that CBR is maintained in a near-optimal range (between 0.4 – 0.8). The power control scheme of [8] could also be used instead of power control of [7]; however these algorithms possess different robustness, convergence and fairness properties. One of the contributions of this paper is to analyze such properties in detail.

In section III we briefly describe the original power control algorithms from [7] and [8] to set the stage for analyzing their convergence and fairness features. As it was mentioned, the schemes proposed in [7] are currently under study for deployment in CVS. Therefore, the paper will focus more on [7] and also presents a new power control algorithm based on the concepts in [7]; the new algorithm is shown to be more robust and can be made to operate for a much wider range of system parameters.

Since we present simulation evaluations of some of the concepts in the coming sections, a detailed description of the simulation setup is provided in the next subsection.

C. Simulation Setup

For network simulations, NS-3 has been used which is an event-based open source network simulator widely used for research purposes. For physical layer, an OFDM PHY for the 5 GHz band with 10 MHz channel bandwidth has been used, which is the dedicated bandwidth for control channel and safety applications. We have also fixed some issues related to backoff process of the 802.11 which was not properly supported in the current version of the simulator (the fixes have been verified by other researchers and will be published in a separate paper). The BSM packet size is set to 500 bytes (close to current version of BSM), and the rate of beaconing is chosen to be fixed (2.5Hz, 5Hz, and 10Hz) to allow more focused study of range (power) control. The fixed values are based on [7] which studies error-based rate control mechanisms and the typical average rate was reported around 2Hz-3Hz and occasionally goes up to 5Hz, and in worst case scenarios it is set to 10Hz. In order to best represent the case with error-dependent rate control mechanisms, random time beaconing has been used with the specified average rate. In case of using an error-dependent rate control mechanism the vehicle will decide to send a packet or not based on its movement behavior; therefore, the random beaconing time will best represent this case.

To simulate the channel propagation behavior, we adopt the model derived in the study presented in [19]. The work in [19] presents models for large-scale path loss and fading of the channel. The models are derived from narrow-band measurements of the mobile V2V propagation channel under realistic driving conditions. Path loss has been modeled using a dual-slope log-distance model; whereas fading is modeled using Nakagami distribution. We adopt the parameters for these models for the data set which was collected in a suburban area.

For vehicle traffic simulation in our study, a 2000-meter two-way highway with 4 lanes in each direction is considered. A figure showing the road condition in SUMO GUI is shown in Fig. 2. We have used two sets of vehicles mobility on this road: 1) fixed vehicles trajectory 2) realistic scenario. In fixed vehicle scenario, vehicles are fixed at their position to allow for creating situations with precise densities ranging from very dense (0.1 vehicle per meter per lane) to very low density (0.01 v/m/l) covering values of : 0.01, 0.02, 0.03, 0.04, 0.05 and 0.1. The realistic scenarios are used to test the proposed algorithms in more realistic scenarios. In order to compile realistic scenarios we have used SUMO traffic simulator, which is an open source microscopic road traffic simulator. Given that we are interested in communication behavior and not vehicles’ physical interaction, the accuracy of SUMO in positioning vehicles on the road and their movement behavior is more than enough for our purpose. In this paper, we considered three realistic scenarios representing congested (17 mph), slow speed (30mph) and free flow (70 mph) conditions. The average speeds for these scenarios lead to different spacing of vehicles.

![Fig. 2. SUMO GUI showing realistic scenarios](image-url)
parameters of interest in CVS and generalizing this result may require certain conditions and is outside the scope of this paper. As it is seen in this figure, CBR values of 0.45 -0.9 generally yield IDR values near maximum (which is achieved by CBR ~ 0.65). Change of the DSRC parameters may slightly move the curve’s optimal point, but it generally stays in 0.55-0.75 for the range of CVS parameters.

CBR is calculated as the ratio of time that the channel has been sensed busy in a given time window T; for ease of presentation T is assumed to comprise of many mini-slots of T\text{slot} duration that can only be idle or busy. Therefore, CBR can be described as follows [8]:

\[ CBR = \frac{\sum_{i=1}^{T/T_{\text{slot}}} \Lambda_i}{T/T_{\text{slot}}} \] (1)

where \( \Lambda \) is 1 for busy slots and 0 otherwise. CBR is considered as a local feedback available for every node in the network at any time. It is computed in the MAC layer using clear channel assessment (CCA) reports from the PHY layer of 802.11. The two existing power control algorithms, which are the subject of study here, use CBR and are summarized below to allow readers easier understanding of the presented analysis; further details are found in [7][8].

A. Linear Memoryless Range Control Algorithm

The power control part of the algorithm in [7], called Linear Memoryless Range Control (LMRC) in this paper, is designed to maintain CBR in a range of ~0.4-0.8. The idea is to provide a robust mechanism of power adjustment that does not require knowing the optimal operation point exactly, but operates near that point (e.g., CBR of 0.65 if maximizing IDR is the objective). The algorithm can be described using the following update equation for the transmission range \( D \) (similarly power may be used) as a function of \( U \) (i.e., CBR) that is run every \( T \) seconds (in the range of ~0.1 to 1sec) [7]:

\[ D_{k+1} = f(U_k) = \begin{cases} D_{\text{max}} & U_k < U_{\text{min}} \\ D_{\text{min}} + \frac{U_{\text{max}} - U_k}{U_{\text{max}} - U_{\text{min}}} (D_{\text{max}} - D_{\text{min}}) & U_k < U_{\text{max}} \\ D_{\text{min}} & U_k \geq U_{\text{max}} \end{cases} \] (2)

Where \( U_k \) is the last measured CBR (at discrete time \( k \)), \( U_{\text{min}} \) is the least desired CBR, and \( U_{\text{max}} \) is the maximum desired CBR. The values for \( (D_{\text{min}}, D_{\text{max}}) \) are chosen based on safety requirements, such as those described in [12], and may take values from 50-100m for minimum and 250-400m for the maximum range. The update equation can be visualized against network characteristic curves (CBR vs. range \( D \) curves for different \( R \) and \( \rho \) values) in Fig. 4, for parameters of \( U_{\text{min}} = 0.3, U_{\text{max}} = 0.9 \) and \( D_{\text{min}} = 100, D_{\text{max}} = 250 \). The characteristic curves have been obtained through extensive simulation runs in NS3. The LMRC update function intersects all characteristic curves that are of practical interest, in the specified range of CBR which will keep IDR in the desired range. It must be noted that the practical implementation of the algorithm includes a guard message, which is a max power/range (\( D_c \)) message sent at a low rate (3.3 Hz to 1Hz or so), to ensure a coarse estimate is available even to far nodes in congested situations. The low rate ensures little effect on congestion, and its effect is further offset by reducing the output of the associated rate controller (e.g. [7]) by the same rate.

Fig. 3. IDR vs. channel occupancy for different values of rate \( R \) (5-115 msg/sec), range \( D \) (20-400m), and road density \( \rho \) (0.1-0.2 vehicle/m). Points belonging to the same simulation with different values of range \( d \) are connected by dotted line and different colors; although due to overlap they are indistinguishable.[8]

B. Gradient descent Range Control Algorithm

While LMRC tries to maintain CBR in a desired range, another alternative is to design an iterative descent based algorithm that tries to maintain CBR at a specific optimal point (e.g. at 0.7). The update function of the algorithm can be introduced as follows:

\[ D_{k+1} = \min(D_{\text{max}}, \max(D_{\text{min}}, D_k + \eta(U^* - U_k))) \] (3)

Where optimal CBR is denoted as \( U^* = 0.7 \). The value of gain (\( \eta \)) in this equation has been derived in [8] for the feedback linearized version of the algorithm that uses the following update function:

\[ D_{k+1} = D_k + \eta \ln\left(\frac{1-U_k}{1-U^*}\right) \] (4)

IV. STABILITY ANALYSIS OF RANGE (POWER) CONTROL BASED ON CBR

Stability of CBR based range control algorithms have been briefly analyzed in [14] for LMRC and [8] for Gradient-based Range Control (GRC). Stability is studied from the standpoint of an algorithm converging, in time, to a new valid power or range level in response to change in the network conditions, due to change of rate or road density. The change in network behavior, from algorithms' standpoint can be presented as a change in the relationship of Power (Range) and CBR. We show such relations as network characteristics curves as seen in Fig. 4.

Both GRC and LMRC algorithms demonstrate non-linear behavior which prevents straightforward analysis. However, a feedback-linearized version of GRC has been introduced in [8] that allows simple analysis through classic stability checks of feedback control systems. Nevertheless, our focus is on LMRC due to its importance and currently being under large scale field test study. Here, we present a slight modification of a lemma that we presented in [14] as follows, which applies to LMRC as well:

Lemma 1: Assuming that network density and average transmission rate experience no considerable change, so that
the network can be characterized by \( U_k = h(D_k) \), we can assert that any range control algorithm which uses a monotonically increasing function of CBR \( (D_{k+1} = f(U_k)) \) will converge to a single value of range (is stable in time) if the following condition is satisfied in each adaptation step:

\[
|h(D_{k+1}) - h(D_k)| < |f^{-1}(D_{k+1}) - f^{-1}(D_k)|
\]

(5)

This condition is intuitively obtained from the fact that to achieve convergence the subsequent difference in observed CBR will get smaller in every step:

\[
|U_{k+1} - U_k| < |U_k - U_{k-1}|
\]

(6)

This convergence condition can be interpreted as the need for inverse of control function \((f^{-1})\) to be always steeper than network characteristic curve for all ranges of \( D \).

When the above condition is applied to LMRC, for all choices of possible range values, the following limiting parameters are found for the algorithm:

\[
\begin{align*}
\{D_{min}, D_{max}\} &= (100,250) \\
\{U_{min}, U_{max}\} &= (0.3,0.9)
\end{align*}
\]

(7)

A tighter set of CBR limits, or wider range limits, could make the LMRC algorithm to converge very slowly or even to diverge. A more generalized form of the LMRC algorithm, first presented in [14], allows a somewhat wider range of parameters. This generalized form of the LMRC algorithm can be described as follow:

\[
D_{min} = f(U) = \begin{cases} 
D_{max} & U < U_{min} \\
D_{min} + \{(U_{max} - U) \times (D_{max} - D_{min}) \} & U_{min} \leq U < U_{max} \\
D_{max} & U \geq U_{max}
\end{cases}
\]

(8)

An exponent of \( \alpha = 1.5 \) has been investigated and shown to result in faster convergence than the original algorithm. Fig. 5, from [14], shows the resulting convergence steps on a typical scenario network characteristic curve. The simulation result of LMRC on a realistic scenario, in which vehicles have maximum speed of 30Mph, can be observed in Fig. 6. Fast convergence over time can be observed in this figure.

Despite the increased range of parameters that the extended LMRC algorithm provides, complete flexibility in setting minimum and maximum range (power) of transmission is still desired. The lower and higher CBR thresholds are also limited in LMRC, and a tighter range may be desirable. Such flexibility is not possible through enhanced LMRC and using limits such as (9) may result in algorithm divergence for certain channels. The result can be observed in Fig. 12(b).

\[
\begin{align*}
\{D_{min}, D_{max}\} &= (100,300) \\
\{U_{min}, U_{max}\} &= (0.4,0.8)
\end{align*}
\]

(9)
motivated us to present a new algorithm that allows almost any set of parameters, but remains stable under all operational conditions. The algorithm, called Stateful Utilization-based PoweR Adaptation (SUPRA), is described in the next section.

V. STATEFUL UTILIZATION-BASED POWER ADAPTATION (SUPRA)

The SUPRA algorithm uses the LMRC concept of linearly mapping CBR to Power, but controls the amount of change in each iteration using a configurable gain and a one-step memory. The gain can be adjusted such that the algorithm always converges to a solution in the desirable range of CBR values. We also opt for direct power control, rather than range control followed by mapping of range to power. The algorithm can be explained using the following formula that sets the power following a measurement of the CBR ($U_k$). Note that the formula uses the last set value of power $P_k$, to calculate $P_{k+1}$, thus becoming stateful.

$$P_{k+1} = P_k + \eta \times (f(U_k) - P_k)$$

The desired range of CBR is (0.4,0.8), but can be adjusted to be wider or narrower. The power minimum and maximum ($P_{min}$,$P_{max}$) values may now be arbitrarily set; the gain will then be adjusted to ensure the algorithm stays stable.

The improvement over LMRC is seen through regulation of the difference between power settings in two consecutive steps. In LMRC, the new value for range (power) is selected as in (2), which is the control function value at the measured CBR ($U_k$). However in SUPRA the new value of power is calculated by finding the difference between the target power from (2) (known as $f(U_k)$) and the last value of power ($P_k$), and then scaling the difference by gain ($\eta$) and adding it to last power value $P_k$. An appropriate value for the gain in the update equation ensures that the difference of two steps remains small enough so the algorithm converges to a solution. In fact, the addition of a state to the algorithm allows for controlling the amount of change in the value power in different iterations of the algorithm using a gain. The next section studies how the gain affects the stability of the algorithm.

The adjustment of step sizes results in a descent-based control method. While this may resemble the descent based GRC method, the two algorithms of SUPRA and GRC are very different. SUPRA does not target a single CBR value as GRC does. GRC uses a descent based and feedback linearized control function to make sure a certain reference value of CBR is reached by the network. In SUPRA, the target is not a single value of CBR and the control function is not feedback linearized. The SUPRA algorithm uses LMRC mapping of power and CBR and ensures stable descent towards a CBR value in the accepted range. This resulting CBR can be different from what LMRC and GRC achieve.

A. Stability Analysis of SUPRA

To derive the stability conditions of SUPRA, and therefore finding the appropriate gain for the update equation, we study the dynamics of the algorithm in this section. The stability requirement should be studied for all possible and likely operation conditions. These conditions will characterize the set of network characteristic curves for which we study the SUPRA algorithm. Fig. 7 shows a set of CBR vs. power curves that characterize the network (plant) behavior for a range of typical network settings in terms of the average transmission rate and density of nodes. Some extreme cases are also covered (e.g. $\rho = 0.1$, rate = 10). In many cases, where the network load is not high, the increase in rate or density has almost the same effect.

The SUPRA algorithm is stateful and uses the previous value of power ($P_k$) to find the appropriate new value. The operational area of the algorithm is shown in Fig. 7 and is marked with dashed line. It can be observed from the same figure, that the network characteristic curves, for most typical scenarios in the operational area can be very well approximated by a linear function of sensed CBR. So $U_k = h(P_k)$, or $P_k = g(U_k)$ (where we define $g=h^{-1}$) can be approximated as a linear function. The curves are similar for different typical scenarios with only slight differences in their slope. To study the stability of the algorithm, we use these reasonable linear approximated network characteristic curves in this section. The gain is found for the worst case scenario, so the algorithm remains stable under all conditions.

To study the stability, we consider two consecutive steps of the algorithm operation as plotted in Fig. 8. The symbols showed in this figure are defined as follows:

- $l_1 = |P_k - f(U_k)|$
- $l_2 = |f(U_{k+1}) - P_{k+1}|$
- $f(U_k)$: Main control function
- $f_1(U_k)$: SUPRA control Function
- $g(U_k)$: Network characteristics curve approximation in operational area

![Fig. 7. Communication characteristic curves for eight different scenarios, Power Control function (solid red line), Operational Area of the control scheme (dashed lined rectangle in blue). Note that the curves are shown with power P on the X axis, as U = h(P) or U = g^{-1}(P).](image-url)

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So we can correlate the slope of \( f(U_k) \) presented as \( tg(\alpha) \) and slope of \( f_\alpha(U_k) \) shown by \( tg(\beta) \) using the following equation:

\[
tg(\alpha) = tg(\beta) = \frac{(1 - \eta)(l_1 + l_2)}{h} \quad (11)
\]

The second term can be rewritten, by substituting \( l_1, l_2 \) values, as follow:

\[
\frac{(1 - \eta)(l_1 + l_2)}{h} = (1 - \eta)\left[|P_k - f(u_k)| + |f(u_{k+1}) - P_{k+1}|\right] \quad (12)
\]

We know that \( l_1, l_2 \) values are length (from the power axis) so we have the absolute values in (12). In order to eliminate absolute values we should look at different states of two consecutive steps of the control. This is illustrated in Fig. 9(a). Two possible cases for each two consecutive steps of the algorithm can be considered as follows:

\[
\begin{cases}
 f(u_{k+1}) > P_{k+1}, & f(u_k) < P_k \\
 f(u_{k+1}) < P_{k+1}, & f(u_k) > P_k
\end{cases}
\]

Theoretically we can then rearrange the absolute values in (12) and with a bit of tweaking, following equation will be obtained:

\[
\frac{(1 - \eta)(l_1 + l_2)}{h} = (1 - \eta)[m_{g^{-1}} + |m_f|] \quad (14)
\]

In which \( m \) is the slope of the function. Finally we can rewrite (11) as:

\[
tg(\alpha) = t.g(\beta) - (1 - \eta)[m_{g^{-1}} + |m_f|] \quad (15)
\]

On the other hand the convergence condition in (6) can be rewritten in terms of power assignments as follows:

\[
|P_{k+2} - P_{k+1}| < |P_{k+1} - P_k| \quad (16)
\]

This equation can be rewritten in terms of stabilized control function values, considering Fig. 8 as below:

\[
|f_\alpha(u_{k+1}) - P_{k+1}| < |f_\alpha(u_k) - P_k| \quad (17)
\]

Having the definition of tangent value in a right triangle which is the length of opposite by adjacent edge and looking at Fig. 8 we can write (17) as:

\[
tg(\alpha) < t.g\left(\frac{x}{2} - \theta\right) \quad (18)
\]

It can be observed from the same figure that:

\[
tg\left(\frac{x}{2} - \theta\right) = |m_{g^{-1}}|, \quad tg(\beta) = |m_f|, \quad t.g(\alpha) = |m_{f_s}|
\]

So by plugging in (15) into inequality (18) we will get:

\[
tg(\beta) - (1 - \eta)[|m_{g^{-1}}| + |m_f|] < |m_{g^{-1}}| \quad (19)
\]

Solving (19) can give us a limit for \( \eta \) as follows:

\[
\eta < \frac{2m_{g^{-1}}}{m_{g^{-1}} + m_f} \quad (20)
\]

Consequently the optimal value for gain \( \eta^* \), can be obtained if \( \alpha = 0 \) so the algorithm will converge in one step. Substituting the value of \( \alpha \) as zero in (15) will give the optimal value for gain(\(\eta\)):

\[
\eta^* = \frac{m_{g^{-1}}}{m_{g^{-1}} + m_f} \quad (21)
\]

We have calculated the gain values, \(\eta\) and \(\eta^*\) to guarantee convergence for all typical scenarios depicted in
Fig. 7 which covers the least to most dense possible scenarios in VANET and from low rates to 10Hz. These values can be observed in TABLE 1. As it is observed, the cases with $\eta < 1$ are the ones that could not be stabilized with LMRC. Moreover, all these scenarios, even the ones that are stable with LMRC are still very slow in convergence and a gain $\eta$ close to 0.5 results in much faster convergence for all of them.

TABLE 1 Gain($\eta$) values for typical scenarios, with $P_{\text{max}}=25\text{dBm}$ and $P_{\text{min}}=5\text{dBm}, U_{\text{max}}=0.8, U_{\text{min}}=0.4$

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\eta$ upper bound</th>
<th>$\eta$ best value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho=0.1$, rate=2.5,10</td>
<td>arbitrary</td>
<td>arbitrary</td>
</tr>
<tr>
<td>$\rho=0.05$, rate=5</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho=0.04$, rate=5</td>
<td>0.94</td>
<td>0.47</td>
</tr>
<tr>
<td>$\rho=0.03$, rate=5</td>
<td>0.94</td>
<td>0.47</td>
</tr>
<tr>
<td>$\rho=0.05$, rate=2.5</td>
<td>1.07</td>
<td>0.53</td>
</tr>
<tr>
<td>$\rho=0.04$, rate=2.5</td>
<td>1.1</td>
<td>0.55</td>
</tr>
<tr>
<td>$\rho=0.02$, rate=5</td>
<td>0.97</td>
<td>0.48</td>
</tr>
<tr>
<td>$\rho=0.03$, rate=2.5</td>
<td>1.1</td>
<td>0.55</td>
</tr>
</tbody>
</table>

To see how the gain affects the convergence speed of the algorithm, we used MATLAB simulations as seen in Fig. 10. The result for one scenario with $\rho = 0.04$ and rate = 5 can be observed in Fig. 10. As it can be observed in that figure for assuring convergence the upper bound for $\eta$ should be satisfied and as we choose a value closer to the optimal $\eta$ the convergence will be faster. We later present NS3 based simulations for evaluation of the findings in this section. The MATLAB tests were done to study the effects of different possible values for the gain ($\eta$). In CVS, fast convergence matters because road density is subject to change and it can be seen in TABLE 1 that the best values for $\eta$ are close for different scenarios since the behavior of typical scenarios were observed to be very similar. This helped us to choose $\eta=0.5$ for our NS3 simulations; robustness and fast convergence can be observed in results presented in Fig. 11. The NS3 simulations are done using typical DSRC settings with bitrate of 6Mbps and default vehicular network channel model (3-step Nakagami fading and 2 step log normal for path loss). Nodes are moving based on a low speed realistic

![Fig. 10. MATLAB simulation for $\rho = 0.04$ and rate=5 left) SUPRA divergence with $\eta = 0.96$ middle) SUPRA convergence with $\eta = 0.9$ right) SUPRA fast convergence with $\eta = 0.5$](image)

![Fig. 11. low speed scenario with average rate=5Hz (a) SUPRA algorithm result for $\eta=0.5$. Observing the change of CBR and Power along the time axis we can see that SUPRA converges in almost 3 steps (3sec) (b) LMRC algorithm converges in 10 iterations](image)
In Fig. 11 the speed of convergence can be compared for LMRC and SUPRA algorithm. In this case LMRC is using limits defined in (7) which are less tight than SUPRA limits. The other advantage of SUPRA over LMRC is its guaranteed convergence for different scenarios. This will assure a reliable performance for CVS in different conditions. One example of LMRC instability has been shown in Fig. 12. In the same scenario SUPRA succeeded to converge in 2 steps and stays stable in time and space.

This instability will affect the performance of CVS by increasing position tracking error (PTE). Position Tracking Error is a measure indicating how well vehicles in the CVS system are able to create real-time maps of their surroundings, and eventually react to traffic hazards. PTE, which was first presented in works such as [7] [12] is affected by amount of packet loss in the network. However quantifying this effect requires knowledge of vehicle movements, specific method of generating messages, and the rate of packet loss (or Packet Error Rate(PER)). PER is defined as the ratio of packets from a single sender to a single receiver that are lost during a time window. In this study, we have measured PER values for different distance bins for free flow scenario with 10Hz beaconing, running LMRC and SUPRA algorithms in different simulation runs. Since packets are generated using an algorithm that relies on vehicle dynamics, the smooth movements in SUMO are not useful for calculating PTE. Therefore, we have to rely on models that produce PTE as a function of PER (rate of arrived messages). Such a model is available from the work in [12] where a “rate-distortion model” is derived for empirical data for different rate control algorithms. Error in PTE is calculated as the 95th percentile, which indicates that 95% of the time tracking error is below this value. The model is shown in Fig. 13 and is used as a numerical function in our simulations.

The resulting PTE for the simulation scenario of Fig. 12 is shown in Fig. 14. For this test, we chose 100 random vehicles out of 571 vehicles in the free flow scenario with 10Hz beaconing rate, to calculate their PTE. A time window of 10sec-60sec is being considered for PER measurement, to avoid biases in PER calculations for start and end of the simulation run. 60sec has been chosen as the end time since some vehicles leave the road after that and there will not be enough sample of PER for different distance bins. As it can be observed in Fig. 14 the PTE improvement achieved by SUPRA over LMRC is considerable, especially at higher distances. The value of tracking error is not very different for the first distance bin (50m-100m) and is not shown, however the oscillation of PTE for vehicles using LMRC is observed. The difference in PTE can increase up to 0.6 meter for distances of 200m-250m which is considerable in CVS safety measures. This result shows the importance of stability of the...
algorithm over time in CVS systems. It is observed that instability in time in any probable scenario could threaten the performance of the whole system by unnecessarily increasing the congestion in the channel and leading to packet loss.

VI. FAIRNESS ENHANCEMENTS

Fairness in accessing channel is another issue that needs to be studied for any congestion control scheme. The idea is to fairly distribute the burden of congestion control. We define fairness as a spatio-temporal property as follows: nodes which experience the same road density (number of vehicles in a certain space span along the road, e.g., \( D_g \) meter radius) and have the same average transmission rate should use the same communication range or power. In other words, all nodes should have the same share of the channel. In more crowded situations all nodes should reduce their use of the channel, and not only some select group of nodes. Fairness can also be expressed as: nodes with the same density should put, on average, the same load on the network; meaning that the product of rate and range, \( R \times D \), should be the same for all nodes that are in an area with the same road density. Of course for a safety application such as CVS, nodes with special needs will be excluded from observing congestion control and fairness rules (for example, emergency messages or vehicles moving in the HoV lane at high speed).

When power or rate control algorithms based on local measurement of CBR are used, certain unfair situations may be possible. We had done a preliminary study of the LMRC and GRC algorithms in our previous works [14][26], and had particularly noted the issue with the GRC algorithm. The LMRC algorithm was shown to be more robust to unfairness. In this section we provide a more comprehensive study of the LMRC, GRC and the SUPRA algorithm proposed in this paper, and propose an enhancement to how CBR measurement is used in congestion control.

The unfair situation can be described as a situation in which on a road with the same road density and rate of transmission, a group of cars experience a low CBR while another group at a farther distance measures a high CBR. The group with low CBR will increase its power, further increasing the CBR of the second group. The higher CBR will drive the second group to use a lower power, further decreasing the CBR of the first group. The positive feedback situation causes very different power assignments while road density and transmission rates are the same for all nodes. Fortunately, this unfair situation is not easy to generate and cannot be observed in many typical situations. Nevertheless, one scenario where we observe the unfairness is when there is a very sharp change in road density, for example as in Fig. 15, where cars do not exist on part of the road and then a high density of cars appears, creating edge effects. The edge nodes naturally sense lower value of CBR since they have a lower density of less than half of the road density (\( D_g / 2 \)) for the first \( D_g \) meters (if a radius of \( D_g \) is considered for density measurement); this can be observed in Fig. 15. Therefore, based on the dynamics of the control schemes used, the edge nodes set their range of transmission to a higher value than the nodes in the middle of the road. This situation is fair for the first \( D_g \) meters or so because of different densities; however, as time goes on, with some algorithms such as GRC an unfair situation gradually develops farther from the edges, and a low-high rippled effect can be seen (Fig. 16) along the road where nodes with the same density and rate observe high and low CBRs and set their power accordingly, which is unfair.

This situation is more severe for GRC algorithm because of its dynamics. Every node in GRC aims at a single specified CBR (e.g., 0.65) rather than a range of CBR values as in LMRC or SUPRA. Consequently, edge nodes try to achieve higher range (power) due to their lower observed density, while nodes just as far as one maximum hop from the edge will choose a lower power. Additionally, GRC works based on previously adapted power with an addition of \( \eta (U^* - U_k) \). Therefore, nodes’ adapted range (power) will only change if there is a substantial deviation from the targeted CBR \( (U^*) \). For example, if all nodes start at the same low power, the edge nodes will accelerate their convergence to the...
specified CBR by using much higher power than their neighbors, forcing them to also see the same higher CBR without them using a higher value of power. This will ensure nodes up to one hop after the edge nodes stay in low power, in turn forcing nodes at two hops to use higher power.

Fig. 16 shows the results from this situation, where nodes up to 4000 meter are still hearing from edge nodes ranging from (4600-5000); therefore they all have lowered their initial power and adapted minimum power. The nodes at 4000meters are not hearing any of the edge nodes while their neighbors setting their power to min distance (power), causing them to sense lower CBR than 0.65 and adapting higher powers. This condition is to a large extent avoided in LMRC and SUPRA since nodes using these algorithms aim to achieve a range of CBR values and a sharp contrast in power assignment between neighbors does not exist at the edges. LMRC and the SUPRA work with a range of acceptable CBR values, thus assign a gradually decreasing power to nodes around the edge and avoid the unfair situation above, except for minor unfair situation like in Fig. 6 (for LMRC) and Fig. 11 (for SUPRA). In this section we examine and propose a method for improving this slight unfairness issue for LMRC and SUPRA algorithms. Since GRC is not currently considered for adoption by the industry we will not study it further.

We determined the main cause of unfairness to be the use of local measurements of CBR in certain scenarios where one group of nodes can interfere with a second group by sending messages at high power, but is not hearing that group due to the low power used by the second group. CBR is intrinsically a feedback measure that includes effects of nodes up to one hop far from the sensing node. Nevertheless, when nodes from one group use lower power than another, the effect on CBR will be unbalanced and the distributed nature of the local CBR measurement will be incomplete. To address this issue, we extend the perspective of nodes in sensing CBR using a simple distributed method as explained next.

**Using Maximum CBR of a Region**

The improved method of computing a feedback measure based on CBR is to have the sensed local CBR piggybacked on CVS messages to the neighboring nodes. The broadcast is done up to one adjacent hop (up to $D_g$ reached by guard messages). The overhead is negligible as the value of CBR can be reported in only a few bits. At each node the maximum value from the set of all received CBRs and the locally sensed CBR will be selected and used for control purposes in either LMRC or SUPRA algorithm. We refer to this enhancement as MaxCBR algorithm; a summary proof of why such a measure will provide fairness is provided next.

**A. Analysis of MaxCBR Fairnes Enhancement Algorithm**

The LMRC and proposed power control algorithm, SUPRA, work with a measurement of CBR. So for any two nodes $n_i$ and $n_j$, which have the same road density and rate (thus observing the same network characteristic curve), if $CBR_i = CBR_j$ then both LMRC and SUPRA will result in the same power for both nodes ($P_k+1$) in a few iterations (as was shown in the stability section). Now we will show that even if $CBR_i \neq CBR_j$, by using the enhancement proposed in this section, the situation will be resolved in few iterations of the enhancement algorithm.

Suppose we have the situation as depicted in Fig. 17 and all nodes experience the same density. Nodes u, v and w are sensing a low CBR therefore setting their power to a high value causing nodes q, and z to sense higher CBRs and setting their power to low values. This case is seen in simulation runs. In this scenario nodes u,v and w contribute to the CBR at q and z, while q and z use very low power due to the higher CBR (contributed by other nodes) and will not add to the CBR in u,v,w. The result is the situation in which nodes u,v,w end up with high power and low CBR, while nodes q and z use low power due to high CBR. Assuming this situation, if we introduce the enhancement algorithm using maximum reported CBR, the unfairness will resolve as explained next.

Fig. 17. Nodes q and z use lower power due to higher local CBR (without the MaxCBR method)

![Fig. 17. Nodes q and z use lower power due to higher local CBR (without the MaxCBR method)](image)

Fig. 18. In an unfair situation, false network characteristic curves $Hg$ and $Lg$ are observed by nodes seeing unbalanced high and low CBRs, resulting in false and unfair stability point (square); the real network characteristic curve $g$ and its stability point (circle) will result when nodes are synchronized using the MaxCBR algorithm.

![Fig. 18. In an unfair situation, false network characteristic curves $Hg$ and $Lg$ are observed by nodes seeing unbalanced high and low CBRs, resulting in false and unfair stability point (square); the real network characteristic curve $g$ and its stability point (circle) will result when nodes are synchronized using the MaxCBR algorithm.](image)
The MaxCBR algorithm will allow all nodes in $D_G$ range of each other to share the sensed CBR. The use of this mechanism will ensure that nodes are hearing the same set of CBR values will pick a single CBR value (the maximum) and become synchronized in their next power setting decision. For the example of Fig. 17, if $D_G$ is such that only 2 neighbors are covered, then the nodes u,v and w that have low CBR will follow one of the nodes q or z and will lower their power, which will cause lower CBR for q and z, and in few iterations this trend will cause equal CBR for all these nodes. In a memory-less algorithm like LMRC, this means that the next value of power (or range) would be the same for all nodes sharing the same CBR set. For stateful algorithms like SUPRA, the synchronization takes few steps since the initial choices of power will dictate the new value. To see why the nodes eventually converge to the same power setting, consider the fact that the algorithm for SUPRA can be written as:

$$P_{k+1} = P_k + \eta \times (f(U_{max}^k) - P_k)$$
$$= P_k (1 - \eta) + \eta (f(U_{max}^k))$$
$$= P_{k-j} (1 - \eta)^{j+1} + (1 - \eta) \eta f(U_{max}^{k-j}) + H(j, U_{max}, U_{max}^{k-j}, \ldots, U_{max})$$

(22)

where $H$ abstract the summation of previous fractions of $U_{max}$ values since iteration $k-j$ and is not directly correlated with the nodes choice of power at iteration $k$. As it can be seen the effect of $P_{k-j}$ and $f(U_{max}^{k-j})$ is faded over iterations with factors $(1-\eta)^{j+1}$ and $\eta(1-\eta)^j$. With increasing iteration $j$, the effect of the initial values fade and the nodes behave similarly. With nodes synchronized in their power/range setting, the algorithm progresses and converges under the stability conditions described in previous sections.

An illustration of the unfair situation can be visualized in Fig. 18, which shows that without the MaxCBR enhancement and in the unfair situation nodes with high and low CBR will observe false network characteristic curves that are due to the unbalance in power assignments. Notice that network characteristic curves are derived assuming similar behavior from all nodes, which is not the case in unfair situations. The sharing of CBR and choosing a single value will cause the nodes to behave similarly and therefore the observed characteristic curve will be the actual one and will guide the convergence in time.

An alternative enhancement algorithm that achieves faster synchronization of nodes for SUPRA is for each node to send out its $\{CBR, \ P, \ CBR_{max}, \ P_{max}\}$ set in every message which indicates what is the current CBRmax and its resulting Pmax for the next adaptation time. The receiving nodes will then use the Pmax of the node that had reported the maximum CBR and do not calculate Pmax based on their previous power level. This way, the adaptation function is bypassed until all nodes synchronize their CBR values. While this method may provide a faster synchronization we do not consider it in this paper and will study it in our upcoming publications. To get more insight into how each iteration of the algorithm with shared CBR works, the following table shows different iterations following the hypothetical unfair situation of Fig. 17.

<table>
<thead>
<tr>
<th>Node</th>
<th>U</th>
<th>q</th>
<th>v</th>
<th>z</th>
<th>w</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_k$</td>
<td>CBR$_u$</td>
<td>CBR$_q$</td>
<td>CBR$_v$</td>
<td>CBR$_z$</td>
<td>CBR$_w$</td>
</tr>
<tr>
<td>$P_k$</td>
<td>$P_u$</td>
<td>$P_q$</td>
<td>$P_v$</td>
<td>$P_z$</td>
<td>$P_w$</td>
</tr>
<tr>
<td>$U_{k+1}$</td>
<td>CBR$_v$</td>
<td>CBR$_v$</td>
<td>CBR$_v$</td>
<td>CBR$_v$</td>
<td>CBR$_v$</td>
</tr>
<tr>
<td>$P_{k+1}$</td>
<td>$P_u$</td>
<td>$P_q$</td>
<td>$P_v$</td>
<td>$P_z$</td>
<td>$P_w$</td>
</tr>
<tr>
<td>$U_{k+2}$</td>
<td>CBR$_u$</td>
<td>CBR$_q$</td>
<td>CBR$_v$</td>
<td>CBR$_z$</td>
<td>CBR$_w$</td>
</tr>
<tr>
<td>$U_{k+2}$</td>
<td>CBR$_v$</td>
<td>CBR$_v$</td>
<td>CBR$_v$</td>
<td>CBR$_v$</td>
<td>CBR$_v$</td>
</tr>
</tbody>
</table>

To verify the effect of the enhancement, we have run the simulation and compared the result with the original method. The enhancement improves many of earlier presented results in terms of providing a fairer channel to all the vehicles. For example results in Fig. 19 show the same scenario as in Fig. 11, but with the MaxCBR enhancement. A considerable improvement can be observed and the slight ripple in the result is faded away. Fig. 20 shows a similar scenario when LMRC is used with enhanced use of CBR.

![Fig. 19. SUPRA algorithm, $\eta=0.5$ and MaxCBR Enhancement applied, realistic scenario (low speed) rate=5Hz](image)

To better see how the algorithm resolves unfairness, in another simulation we manually set unfair power assignments to nodes to create a situation similar to Fig. 17. The results for SUPRA without the MaxCBR enhancement is seen in Fig. 21.
An unfairness situation develops and remains present as time passes. By applying the MaxCBR enhancement, the unfairness is completely resolved as is seen in Fig. 22.

Fig. 20. LMRC with Enhancement applied, scenario $\rho = 0.03$ and rate=5Hz

Fig. 21. SUPRA algorithm, $\eta=0.5$ and No Enhancement, scenario $\rho=0.04$ and rate=5Hz with manually inserted unfairness

VII. CONCLUSIONS

Congestion control in vehicle safety networks is one of the major issues currently under investigation. We presented a detailed analysis of LMRC, which is the only power control scheme that is currently under consideration by industry for standardization. The convergence conditions of this algorithm are analyzed, leading to derivation of the restrictions that need to be in place to ensure algorithm convergence. We then introduced SUPRA, an enhanced version of the algorithm, which is a major contribution of this paper and lifts the restrictions on the original algorithm, allowing it to operate almost with any set of safety parameters. A study of the convergence properties and conditions of SUPRA has been provided, showing that a wide range of gain values are possible and allow stable power control with flexible choices for power and CBR limits. A mechanism to ensure robustness to unfair spatial distribution of power assignments has also been presented and analyzed. Simulation results further verified the stability and fairness properties of SUPRA, and the improvement in vehicle position tracking error that is possible when power control can be ensured to remain stable using SUPRA.

The presented enhancements are all simple to implement and incur minimal implementation or operation overhead. More complicated enhancements are also possible, including an alternative method of sharing CBR and power assignment that is expected to result in faster convergence to fair operation. We will present a study of such enhancements in our upcoming publications.
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