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Citation: AIP Conference Proceedings **1605**, 452 (2014); doi: 10.1063/1.4887631
View online: http://dx.doi.org/10.1063/1.4887631
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1605?ver=pdfcov
Published by the AIP Publishing

Fourier Analysis of Conductive Heat Transfer for Glazed Roofing Materials

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Abstract. For low-rise buildings, roof is the most exposed surface to solar radiation. The main mode of heat transfer from outdoor via the roof is conduction. The rate of heat transfer and the thermal impact is dependent on the thermophysical properties of roofing materials. Thus, it is important to analyze the heat distribution for the various types of roofing materials. The objectives of this paper are to obtain the Fourier series for the conductive heat transfer for two types of glazed roofing materials, namely polycarbonate and polyfilled, and also to determine the relationship between the ambient temperature and the conductive heat transfer for these materials. Ambient and surface temperature data were collected from an empirical field investigation in the campus of Universiti Teknologi MARA Shah Alam. The roofing materials were installed on free-standing structures in natural ventilation. Since the temperature data are generally periodic, Fourier series and numerical harmonic analysis are applied. Based on the 24-point harmonic analysis, the eleventh order harmonics is found to generate an adequate Fourier series expansion for both glazed roofing materials. In addition, there exists a linear relationship between the ambient temperature and the conductive heat transfer for both glazed roofing materials. Based on the gradient of the graphs, lower heat transfer is indicated through polyfilled. Thus polyfilled would have a lower thermal impact compared to polycarbonate.

Keywords: Mathematical modeling, Heat transfer, Roofing materials, Periodic function.
PACS: 02.30.Nw, 07.20.Dt

INTRODUCTION

Malaysia is situated in tropical climate with high temperatures and humidity, and abundant solar radiation throughout the year. Daytime temperatures rise above 30°C (86°F) year-round and night-time temperatures rarely drop below 20°C (68°F). For low rise building, roofing area is the major source of heat gain because the surface is the most affected area by the solar radiation [1, 2]. The main mode of heat transfer from outdoor via the roof is conduction. The rate of heat transfer and the thermal impact is dependent on the thermophysical properties of roofing materials [3, 4]. Thus the selection on the type of roofing materials is important so that the heat gain through the roof can be minimized. There are many factors that influence the heat gain inside the building and these affect the thermal comfort and building energy consumption. In Malaysia, the installation of the radiant barrier is a standard practice for roof in residential building but not the thermal insulation. Therefore, studies were conducted on thermal and energy performances of the application of conductive thermal insulation at the roof pitch and ceiling as published in [3, 5, 6]. The findings concluded temporal and spatial benefits of the insulation. According to Dominguez *et al*. [7], the energy savings can vary greatly depending on the roof insulating properties.

Besides the type of roofing materials, the heat gain is also related to the roof pitch angles. Ambient and surface temperature data were collected from an empirical field investigation in the campus of Universiti Teknologi MARA Shah Alam [8]. The polycarbonate and polyfilled roofing materials were installed on free-standing structures in natural ventilation at three different pitch angles of 0˚, 25˚ and 45˚. She concluded that higher roof pitch angles gives better heat transfer for both polycarbonate and polyfilled materials. Thus, present study aims to extend the study initiated by [8]. Focusing on the experimental surface temperature data taken for 45˚ pitch angle, a general functional model to represent the daily conductive heat transfer is developed. Since temperature data are generally periodic, the situation is modeled using the Fourier series expansion. In addition, the relationship between the conductive heat transfer for both glazed roofing materials and the ambient temperature are also investigated.

Fourier series expansion had been shown to be a great method in predicting pollutant transport based on stochastic advection dispersion pollutant equation [9]. In another study, Fadhilah *et al.*[10] has used Fourier series to exhibit seasonal fluctuation of rainfall process. Numerical harmonic analysis, which is a discrete analog to Fourier analysis, was successfully used to analyze the profile of dengue fever incidences in Shah Alam [11]. Fourier series

Proceedings of the 21st National Symposium on Mathematical Sciences (SKSM21)
AIP Conf. Proc. 1605, 452-457 (2014); doi: 10.1063/1.4887631

had been applied in many areas but not in profiling the conductive heat transfer in the roof space. Therefore, the objectives of this paper are to obtain the Fourier series for the conductive heat transfer for two types of glazed roofing materials, namely polycarbonate and polyfilled, and also to determine the relationship between the ambient temperature and the conductive heat transfer for these materials. The mathematical equation obtained will provide a general functional representation for further analysis, particularly in thermal and energy performance in Malaysian buildings.

METHODOLOGY

Ambient and surface temperature data were collected from an empirical field investigation for two consecutive days whereby the roofing materials were installed on free-standing structures in natural ventilation [8]. The conductive heat transfer for the glazed roofing materials is calculated using the Fourier's Law [12]. The characteristics of glazed roofing materials are shown in Table 1.

TABLE (1). The characteristics of glazed roofing materials

Characteristic	Polycarbonate	Polyfilled
Thickness (m)	0.001	0.003
Thermal Conductivity (W/Mk)	0.189	0.0079
Area (m^2)	0.36	0.132

Fourier series enables any waveform to be expressed into a number of sinusoidal waves with different frequencies [13, 14]. In this study, the 24-point harmonic analysis is considered for the average daily conductive heat transfer. Therefore, the conductive heat transfer $Q(t)$, which is piecewise continuous function on $[0,24]$ and $Q(t+24)=Q(t)$ can be expressed as

$$Q(t)=\frac{a_o}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi t}{12}+b_n\sin\frac{n\pi t}{12}\right) \quad (1)$$

where $a_0=\frac{1}{12}\sum_{n=1}^{24}Q(t_n)$ $\quad a_n=\frac{1}{12}\sum_{n=1}^{24}Q(t_n)\cos nT_n$ $\quad b_n=\frac{1}{12}\sum_{n=1}^{24}Q(t_n)\sin nT_n$ $\quad$ for $T_n=\frac{\pi t_n}{12}$

In these equations, the term $n = 1$ is known as the first harmonic, $n = 2$ is the second harmonics and so on. Thus, the conductive heat transfer up to the n^{th} harmonics is given by

$$Q_n(t)=Q_{n-1}(t)+a_n\cos nT_n+b_n\sin nT_n \quad (2)$$

The average daily profile for the conductive heat transfer for polycarbonate, *QC(t)* and polyfilled, *QF(t)* are shown in Figure 1 and 2 respectively.

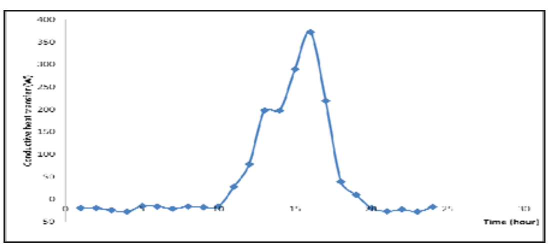

FIGURE 1. Daily conductive heat transfer for polycarbonate

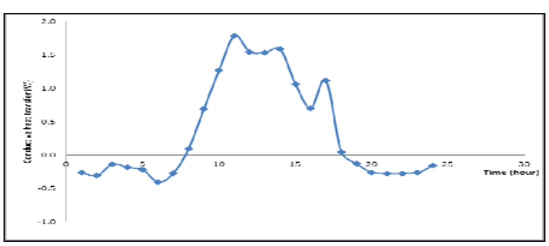

FIGURE 2. Daily conductive heat transfer for polyfilled.

The conductive heat transfer up to the n^{th} harmonics ($QC_n(t)$ and $QF_n(t)$) and the Fourier coefficients a_0, a_n and b_n for n = 1, 2,…, 15 are calculated. For each glazed roofing material, the minimum Mean Square Error (*MSE*) is chosen to determine the appropriate number of harmonics for the Fourier series expansion. The *MSE* is given by

$$MSE = \frac{\sum_{i=1}^{n}\left(Q_i - \hat{Q}_i\right)^2}{n} \quad (3)$$

where Q_i denote actual or experimental values; $\hat{Q}_i$ denote calculated or model values and n is the number of data sets. Meanwhile, coefficient of determination (R^2) is used to determine the best-fit equation that represents the relationship between the ambient temperature and the conductive heat transfer for the roofing glazed materials.

RESULTS AND DISCUSSION

Aforementioned, the first objective of this paper is to obtain the Fourier series for the conductive heat transfer for polycarbonate and polyfilled glazed roofing materials using the 24-point harmonics analysis. Following that, the second objective is to determine the relationship between the ambient temperature and the conductive heat transfer for these materials using the curve-fitting techniques.

Fourier series for Polycarbonate and Polyfilled Glazed Materials

The Fourier coefficients a_0, a_n and b_n for n = 1, 2,…, 15 for both glazed roofing materials are calculated using (2) and are shown in Table 2.

TABLE (2). The Fourier Coefficient for glazed roofing materials

n	Fourier Coefficient (Polycarbonate)		Fourier Coefficient (Polyfilled)	
	a_n	b_n	a_n	b_n
0	93.6514		0.6917	
1	-86.3784	-84.8564	-0.9053	-0.2183
2	-4.8829	85.3890	0.3908	0.1570
3	40.5867	-24.8537	-0.0673	0.0969
4	-14.6187	-14.5028	0.0014	-0.1409
5	-11.8104	14.4602	0.0032	-0.0119
6	17.2850	6.9543	-0.0146	0.0448
7	-5.3103	-16.1715	0.1061	0.0160
8	-8.6680	9.1456	-0.0496	-0.0831
9	10.5851	0.9222	-0.0098	0.0793
10	-2.9531	-0.8861	0.0599	0.0167
11	4.5293	-0.9097	0.0214	-0.0372
12	-5.0123	0.0000	-0.0817	0.0000
13	4.5293	0.9097	0.0214	0.0372
14	-2.9531	0.8861	0.0599	-0.0167
15	10.5851	-0.9222	-0.0098	-0.0793

Consequently, the *MSE* for conductive heat transfer of $QC_n(t)$ and $QF_n(t)$ for $n = 1, 2, ..., 15$ are calculated using (3) and tabulated in Table 3. It is found that the minimum value of *MSE* for both glazed materials are obtained for n = 11. These show that the conductive heat transfers for polycarbonate and polyfilled can be appropriately represented the Fourier series up to the 11^{th} harmonics as in (4) and (5) respectively.

$$\begin{aligned} QC_{11}(t) &= QC_{10}(t) + \mathrm{a}_{11}\cos 11\mathrm{T} + b_{11}\sin 11\mathrm{T} \\ &= QC_{10}(t) + 4.5293\cos\frac{11\pi t}{12} - 0.9097\sin\frac{11\pi t}{12} \end{aligned} \quad (4)$$

$$\begin{aligned} QF_{11}(t) &= QF_{10}(t) + \mathrm{a}_{11}\cos 11\mathrm{T} + b_{11}\sin 11\mathrm{T} \\ &= 8.3006 + 0.0214\cos\frac{11\pi t}{12} - 0.0372\sin\frac{11\pi t}{12} \end{aligned} \quad (5)$$

TABLE (3). *MSE* for conductive heat transfer up to the 15^{th} harmonics

n	MSE (Polycarbonate)	MSE (Polyfilled)
1	5652.3246	0.1249
2	1994.7646	0.0362
3	862.2740	0.0293
4	650.2562	0.0193
5	475.9646	0.0193
6	302.3984	0.0182
7	157.5398	0.0124
8	78.1521	0.0077
9	21.7048	0.0045
10	16.9518	0.0026
11	**6.2808**	**0.0017**
12	6.2808	0.0017
13	16.9518	0.0026
14	21.7048	0.0045
15	78.1521	0.0077

The Fourier series for the conductive heat transfer *Q(t)* can be written in a standard form as

$$Q(t) = A_0 + \sum_{n=1}^{n=\infty} A_n \sin(\omega t + \phi_n) \tag{6}$$

Therefore, equation (6) is used to express the appropriate Fourier series obtained for the conductive heat transfer for both roofing glazed materials. Thus, the Fourier series $QC_{11}(t)$ and $QF_{11}(t)$ in standard form are

$$QC_{11}(t) = 46.8257 + \sum_{n=1}^{11} A_n \sin(\omega t + \phi_n) \tag{7}$$

$$QF_{11}(t) = 0.3459 + \sum_{n=1}^{11} A_n \sin(\omega t + \phi_n) \tag{8}$$

where the values of A_n and ϕ_n for n = 1, 2, …, 11 are shown in Table 4. Figure 3 and 4 illustrate the comparison between the actual data and the generated 11^{th} order harmonics Fourier series expansion for polycarbonate, $QC_{11}(t)$ and polyfilled, $QF_{11}(t)$ respectively. The Mean Absolute Percentage Error (MAPE) for $QC_{11}(t)$ and $QF_{11}(t)$ are calculated to be 10.02% and 16.88% respectively. MAPE is defined as the absolute percentage error calculated on the fitted values [15]. Therefore, this indicates that modelling conductive heat transfer for polycarbonate and polyfilled using Fourier series up to 11^{th} harmonics is adequate.

TABLE (4). Fourier Coefficient for $QC_{11}(t)$ and $QF_{11}(t)$

	$QC_{11}(t)$		$QF_{11}(t)$	
n	A_n	ϕ_n (°)	A_n	ϕ_n (°)
1	121.0861	45.5093	121.0861	45.5093
2	85.5285	-3.2728	85.5285	-3.2728
3	47.5918	-58.5183	47.5918	-58.5183
4	20.5921	45.228	20.5921	45.228
5	18.6704	-14.5028	18.6704	-14.5028
6	18.6315	68.0835	18.6315	68.0835
7	17.0211	18.1788	17.0211	18.1788
8	12.6006	-43.4692	12.6006	-43.4692
9	10.6252	85.0208	10.6252	85.0208
10	3.0832	73.2977	3.0832	73.2977
11	4.6197	-78.6434	4.6197	-78.6434

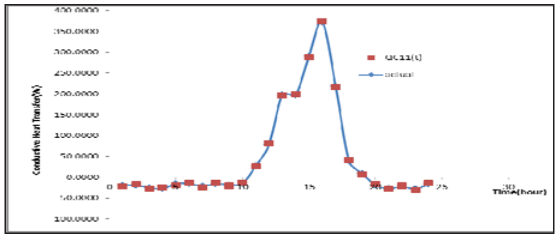

FIGURE 3. Comparison between actual data and $QC_{11}(t)$

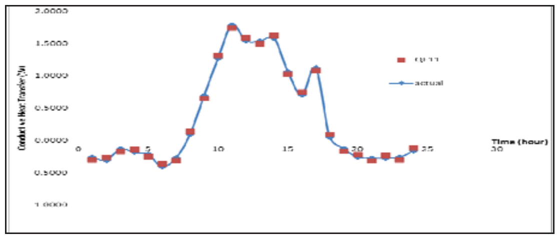

FIGURE 4. Comparison between actual data and $QF_{11}(t)$

Based on Figure 3 and 4, it is observed that the conductive heat transfer for polycarbonate is high between 3.00 – 4.00 p.m and low between 3.00 – 4.00 a.m and 9.00 – 11.00 p.m while the conductive heat transfer for polyfilled is high between 11.00 a.m – 2.00 p.m and low between 6.00 – 7.00 a.m and 1.00 – 2.00 a.m. The rate of heat transfer and the thermal impact is dependent on the thermophysical properties of roofing materials. Thus, the high amount of heat transfer means that the heat is trapped inside the roofing materials that will lead to thermal discomfort.

Relationship between ambient temperature and conductive heat transfer for glazed materials

The relationship between the ambient temperature and the conductive heat transfer for both glazed roofing materials is determined using the "trendline" for curve fitting in Microsoft Excel. Several mathematical equations were explored in order to determine the best coefficient of determination (R^2). The value of R^2 closest to 1 is chosen. It is found that the conductive heat transfer for both glazed roofing materials are linearly related to the ambient temperature. For polycarbonate, the linear equation is given by

$$QC(t) = 26.33\ \mathrm{T}_{C-ambient} + 619.98 \tag{9}$$

For polyfilled, the linear equation is given by

$$QF(t) = 0.2723\ \mathrm{T}_{F-ambient} + 6.7608 \tag{10}$$

The coefficient of determination for polycarbonate and polyfilled are given by R^2= 0.8255 and R^2= 0.7279 respectively. This means that 82.55% of the data obtained for polycarbonate and 72.79% of the data for polyfilled can be explained by these linear equations. Some "missing data" for ambient temperature of polycarbonate are noted. This is shown in Figure 7 and 8. Based on the gradient of the graphs as shown in (9) and (10), lower heat transfer is indicated through polyfilled. Thus polyfilled would have a lower thermal impact compared to polycarbonate.

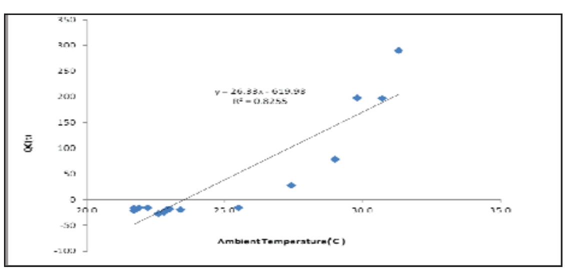

FIGURE 5. Relationship between conductive heat transfer and ambient temperature for Polycarbonate

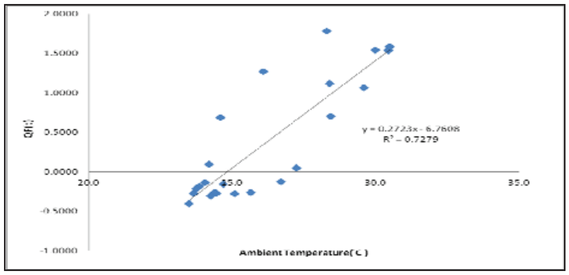

FIGURE 6. Relationship between conductive heat transfer and ambient temperature for Polyfilled

CONCLUSION

Based on the 24-point harmonic analysis, the eleventh order harmonics is found to generate an adequate Fourier series expansion for both glazed roofing materials. In addition, there exists a linear relationship between the ambient temperature and the conductive heat transfer for both glazed roofing materials. Based on the gradient of the graphs, lower conductive heat transfer is indicated through polyfilled. Thus polyfilled would have a lower thermal impact compared to polycarbonate. This analysis is based only on the temperature data recorded from a simple experiment under natural ventilation [8]. Therefore, recommendations for further work would include extending the duration of the experiment for recording the temperature data and considering factors that might affect the accuracy of the recorded data.

Nevertheless, this work has illustrated the application of Fourier series and numerical harmonic analysis to the temperature data related to the conductive heat transfer for glazed roofing materials. The Fourier series expansion obtained gives a general functional mathematical model for further mathematical analysis that can enhance estimation and inference of the situation.

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