COMPARISON OF NONLOCAL MEANS DESPECKLING BASED ON STOCHASTIC MEASURES

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ABSTRACT

This work presents the use of stochastic measures of similarities as features with statistical significance for the design of despeckling nonlocal means filters. Assuming that the observations follow a Gamma model with two parameters (mean and number of looks), patches are compared by means of the Kullback-Leibler and Hellinger distances, and by their Shannon entropies. A convolution mask is formed using the p-values of tests that verify if the patches come from the same distribution. The filter performances are assessed using well-known phantoms, three measures of quality, and a Monte Carlo experiment with several factors. The proposed filters are contrasted with the Refined Lee and NL-SAR filters.

Index Terms— adaptative filters, information theory, nonlocal means, radar imaging, synthetic aperture radar.

1. INTRODUCTION

Nonlocal Means (NLM) filters were introduced by Buades et al. [1] and, since then, have been applied in a number of challenging situations. The main idea consists in forming a local convolution mask where each non-negative value is proportional to the similarity between the central observation and each of the other values. Such similarity has been estimated using a number of models and approaches, among them the Euclidean distance and measures of statistical closeness.

Speckle noise is characteristic of images obtained by means of coherent illumination, as is the case of SAR – Synthetic Aperture Radar imagery. Its granular appearance, and the fact that it cannot be adequately described by the usual additive Gaussian model, make it a nuisance. Nonlocal speckle filters have been proposed in the literature using likelihood functions [2, 3], hypothesis tests [4, 5], Bayesian approaches [6, 7], and wavelet shrinkage [8].

In this work we compare two strategies for specifying the weights of NLM speckle filters based on measures stemming from the Information Theory framework, namely those proportional to the p-values of goodness-of-fit tests built by comparing densities (through the Kullback-Leibler and Hellinger distances) and Shannon entropies, both under the Gamma model for the return. The former were used in [9] for selecting Nagao-Matsuyama windows. The latter, to the best of the authors’ knowledge, have not been used for despeckling purposes.

2. METHODS

2.1. Proposed filters

The model we consider for the return is the Gamma law with mean $\lambda$ and $L$ looks, denoted $\Gamma(L, L/\lambda)$, and characterized by the density

$$f_Z(z; L, \lambda) = \frac{L^L}{\lambda^L L!} z^{L-1} \exp\{-Lz/\lambda\},$$

where $z, L, \lambda > 0$. It is usually assumed that $L \geq 1$, but this restriction is relaxed to account for possible departures from the textureless situation, for which this model is valid.

The maximum likelihood estimator of the parameters $\theta = (L, \lambda)$ based on the random sample $Z = (Z_1, \ldots, Z_n)$ of independent identically distributed $\Gamma(L, L/\lambda)$ observations is
given by the sample mean \( \hat{\lambda} = n^{-1} \sum_{i=1}^{n} Z_i \), and by the unique solution of \( L - \psi(L) - \ln \hat{\lambda} + n^{-1} \sum_{i=1}^{n} \ln Z_i = 0 \), where \( \psi \) is the digamma function. The Shannon entropy of a \( \Gamma(L, L/\lambda) \)-distributed random variable is \( H_\lambda(\theta) = - \ln L + \ln \lambda + L + (1 - L)\psi(L) + \ln \Gamma(L) \).

Two geometric parameters are important in the design of a nonlocal mean filter: the estimation and search windows. The former is used to define the region over which data are extracted to estimate the parameters, while the latter stipulates how far from the central pixel such samples are sought. We define these two quantities using radii: the estimation radius \( E_r \) and the search radius \( S_r \).

We use a squared patch of pixels of side \( 2E_r + 1 \) pixels centered on each pixel to compute \( \hat{\lambda} \) (notice that the sample size decreases near the edges of the image). In this way, a new image is built where in each coordinate we have the ML estimate of the parameters that produced the observation.

The next step is computing the tests statistics with each pair of estimates that lie within a search area of radius \( S_r \). For this, we use (2), (3), and (4). The theory behind the main results we use relates stochastic distances and entropies with goodness-of-fit test statistics; details can be found in Ref. [10].

The Kullback-Leibler and Hellinger test statistics are given by

\[
S_{KL}(\hat{\theta}_1, \hat{\theta}_2) = (E_r + 1/2)^2 \left( \frac{\hat{L}_1 - \hat{L}_2}{2} \right)
\]

\[
\left( \log(\hat{\lambda}_1/\hat{\lambda}_2) - \log(\hat{L}_1/\hat{L}_2) + \psi(\hat{L}_1) - \psi(\hat{L}_2) \right) + \frac{\hat{L}_1 (\hat{\lambda}_2/\hat{\lambda}_1 - 1) + \hat{L}_2 (\hat{\lambda}_1/\hat{\lambda}_2 - 1)}{2},
\]

and

\[
S_H(\hat{\theta}_1, \hat{\theta}_2) = 4(2E_r + 1)^2 \left( 1 - \Gamma \left( \frac{\hat{L}_1 + \hat{L}_2}{2} \right) \right) \left( \frac{2}{\hat{L}_1 \hat{\lambda}_2 + \hat{L}_2 \hat{\lambda}_1} \left( \frac{\hat{L}_1}{\hat{L}_1 \hat{\lambda}_2} + \frac{\hat{L}_2}{\hat{L}_2 \hat{\lambda}_1} \right) \right)
\]

\[
\left( \frac{\hat{L}_1 + \hat{L}_2}{\Gamma(\hat{L}_1) \Gamma(\hat{L}_2)} \right),
\]

respectively. Using maximum likelihood estimators for \( \theta_1 \) and \( \theta_2 \), these two statistics follow, asymptotically, a \( \chi^2 \) distribution under the null hypothesis (\( H_0 \)) of same underlying distribution for \( Z_1 \) and \( Z_2 \). Similarly, the test statistic derived from the Shannon entropy is given by

\[
S_S(\hat{\theta}_1, \hat{\theta}_2) = (2E_r + 1)^2 \left[ \frac{(H_S(\hat{\theta}_1) - \bar{\tau})^2}{\sigma_S^2(\hat{\theta}_1)} + \frac{(H_S(\hat{\theta}_2) - \bar{\tau})^2}{\sigma_S^2(\hat{\theta}_2)} \right],
\]

where

\[
\sigma_S^2(\theta) = \frac{[(1 - L)\psi(1)(L) + 1 - \frac{1}{L}][1 - L]}{\psi(1)(L) - \frac{1}{L}},
\]

\[
\bar{\tau} = (2E_r + 1)^2 \frac{H_S(\hat{\theta}_1)\sigma_S^2(\hat{\theta}_2) + H_S(\hat{\theta}_2)\sigma_S^2(\hat{\theta}_1)}{\sigma_S^2(\theta_2) + \sigma_S^2(\theta_1)},
\]

and \( \psi(1) \) is the polygamma function of order 1. Under \( H_0 \), \( S_S(\hat{\theta}_1, \hat{\theta}_2) \) asymptotically follows a \( \chi^2 \) distribution. The finite-sample behavior of these test statistics has been assessed, under the complex Wishart model for full polarimetric SAR data, in [11, 12].

Following the discussion by Torres et al. [5], the weights of the convolution matrix \( w_\eta(p) \) are obtained by a soft transformation of the \( p \)-values of the observed statistics as a function of \( \eta \), the significance of the test, as follows:

\[
w_\eta(p) = \begin{cases} 1 & \text{if } p \geq \eta, \\ \frac{1}{2}p - 1 & \text{if } \frac{1}{2} < p < \eta, \\ 0 & \text{otherwise}. \end{cases}
\]

After that, the weights in the search window are scaled to sum one. It is worth noticing that, since the Gamma distribution is convolution-invariant, the model holds even after applying the filter. Therefore, the NLM filter is defined stipulating the kind of test, its significance, the number of iterations, and the search and estimation radii.

We implemented the three aforementioned tests (\( S_{KL}, S_H, \) and \( S_S \), see https://github.com/rgrimson/StochasticFilters), and compared them with the improved Lee filter [13] and the NL-SAR filter [3].

### 2.2. Filter quality assessment

We used the two phantoms shown in Fig. 1 to assess the quality of the filters. The phantoms were speckled with three looks and filtered with the five aforementioned filters. For the three proposed filters we varied significance (\( \eta \in \{0.90, 0.95, 0.99\} \)), number of iterations (1, 2, 3) and search radius (2, 3, 5). The estimation radius was set to 1. For the NL-SAR filter we used constant search and estimation radius of 5 and 2, respectively. Three quality measures were computed for each result: the Universal Quality Index \( Q \) [14], the radiometric preservation (\( RP \), one minus the normalized relative error of the mean, equal to 1 if the mean is preserved) and the estimated number of looks (\( ENL \)). The measures \( RP \) and \( ENL \) were computed on several evaluation regions per phantom, and the worst value was retained as the final result. In the case of the Zhong’s phantom, five evaluation regions were defined inside homogeneous areas (one for the background and one for each square) excluding the small corner reflectors. For the stripes phantom, two evaluation regions were defined (one for the background and one for the large stripe). All these regions
3. RESULTS AND DISCUSSION

The results suggest that the best proposed filter is the one based on the Hellinger test statistic, due to its highest ENL, \( RP \) closest to 1 and \( Q \) closest to 1. Visual inspection of the filtered images is consistent with this: border preservation, texture and preservation of small corner reflectors is better for the Hellinger filter than for the others.

The Hellinger filter performance is similar for different significance levels in the Stripes phantom, whereas a decrease in \( Q \) is observed when increasing significance level in the Zhong’s phantom. For both phantoms, \( RP \) is nearly constant and close to 1, and better \( ENL \) and \( Q \) are obtained with increasing number of iterations and with increasing search radius (the latter was seen particularly in the Zhong’s phantom). \( ENL \) is much greater for the stripes filter than for the Zhong’s one. The optimum parameter settings for the Hellinger filter were: three iterations, search radius = 5, \( \eta = 0.99 \).

The significance level is the least influential parameter for the other two filters. The Kulback-Leibler filter has also a good performance, and a similar behavior than the Hellinger filter when varying iterations, significance level and search radius. Comparing the same parameter settings, this filter’s performance is worse than Hellinger’s, especially regarding \( RP \) and \( Q \). However, \( ENL \) is better for the Kulback-Leibler filter than for the Hellinger filter in the case of the Zhong’s phantom. The optimum parameter settings for the Kulback-Leibler filter were: three iterations, search radius = 5, \( \eta = 0.99 \).

Lastly, the Shannon filter shows worse performance than the Hellinger and Kulback-Leibler filters, with the smallest \( Q \), \( RP \) and \( ENL \) values. The performance of this filter decreases with the number of iterations and with the search radius. The optimum parameters for the Shannon filter were: one iteration, search radius = 5, \( \eta = 0.90 \).

The optimum parameters of the three proposed filters were used to compare their overall quality, and to contrast them with the quality of the Refined Lee and the NL-SAR filters (Fig. 2). The Hellinger filter is the best among the proposed filters. The Shannon filter has a similar performance than the Refined Lee filter. The NL-SAR filter seems to be better than the filters here proposed, particularly for the \( Q \) index.

4. CONCLUSIONS

The main conclusion is that there is not a single filter able to cope with all possible situations and to produce uniformly best results. The filters here proposed bring new tools for de-speckling: those stemming from statistical tests based on the comparison of stochastic distances and entropies. These tests are based on maximum likelihood estimates, a possible source of problems as these estimators are susceptible to suffering from the inherent contamination of data in image processing.

This work will continue by the use of other inference techniques, and other stochastic distances. An important issue that was not considered here is the computational complexity of these filters.

5. REFERENCES


Fig. 2. Quality measure results for each filter and phantom, derived from a Monte Carlo experiment (100 replications). Filter names are coded as follows: H (Hellinger), KL (Kullback-Leibler), S (Shannon), RL (Refined Lee), NL-SAR.