Very fast soft tissue predictions with mass tensor model for maxillofacial surgery planning systems

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Abstract. Maxillofacial surgery treats abnormalities of the skeleton of the head. Skull remodelling implies osteotomies, bone fragment repositioning, restoration of bone defects and inserting implants. In this field there is a huge demand from the surgeons to be able to predict the new facial outlook after surgery. Besides the big interests for the surgeon during planning, it is also an essential tool to improve the communication between surgeon and patient. Mass Spring Models (MSM) are widely used to simulate the deformation behavior of soft tissues. The usage of these models has however some serious disadvantages. Most important is that the MSM has no real bio-mechanical foundation. The Finite Element Model (FEM) on the other hand has a strong bio-mechanical foundation but the high computational cost and a difficult architecture make it not very useful for fast simulators. In this paper we present the usage of a Mass Tensor Model (MTM) to simulate the soft tissue deformations after bone displacement. This model tries to combine the advantages of FEM and MSM. Moreover by combining this MTM with a new static solution scheme and a dynamic threshold, we were able to reduce the calculation time seriously when compared to MSM or FEM. © 2005 CARS & Elsevier B.V. All rights reserved.

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1. Introduction

Mass Spring Models (MSM) \cite{1,2} are increasingly important to simulate the behavior of soft tissues. As compared to the Finite Element Modelling (FEM) \cite{3,4} they have some important benefits. MSM have a very easy architecture, low memory usage and are fast,
which make them very attractive for real-time and fast simulators. In the past we implemented a Tetrahedral Mass Spring Model for a fast maxillofacial planning system [5]. We showed pretty good results with this Tetrahedral MSM [6]. However this Tetrahedral MSM and Mass Springs Models more generally have also some important disadvantages. Three important disadvantages are distinguished:

1. In the Tetrahedral MSM an object is described into discrete mass points which are connected to each other by springs. A change of length of a spring induces a force in the direction of that spring. However since no volume behavior of the tetrahedron is incorporated, this may cause a possible ‘flip over’ of a spring.

2. The value of the spring constant determines the elastic behavior of the MSM. As shown in [7] there should be some kind of topology dependent mapping of Young’s Modulus to these spring constants. Although some suggestions are made for doing this [7,8] the calculated value for the spring constant is only an approximation and has no true biomechanical relevance.

3. In a MSM, there is no way to control the volume conservation during simulation, even if you extend your model with extra ‘volume springs’ [5].

These shortcomings are typical to the Mass Spring Model and are not easily remedied. FEM on the other hand is more bio-mechanically relevant and doesn’t have these problems. What we wanted was a new model that on the one hand preserves the easy architecture and fast simulation of MSM and on the other hand has the bio-mechanical relevance of FEM.

2. Materials and methods

2.1. The original model

The original Mass Tensor Model (MTM) was introduced by Cotin [9]. In the MTM the modelled object is discretized into a tetrahedral mesh. Inside every tetrahedron \( T_i \), the displacement field is defined by a linear interpolation of the displacement vectors of the four vertices of \( T_i \), as defined by the finite element theory. The linear elastic energy of tetrahedron \( T_i \) can then be expressed as a function of the displacements of the four vertices and of the two Lamé coefficients \( \lambda \) and \( \mu \) which are bio-mechanical elastic constants. The force applied at vertex \( j \) of tetrahedron \( T_i \) is defined as the derivative of this elastic energy:

\[
F_{jT_i}^T = -\frac{\partial W_{\text{elastic}}(T_i)}{\partial P_j} = \sum_{k=0}^{3} \left[ K_{jk}^{T_i} \right] u_{T_i(k)}
\]

where \( u_{T_i(k)} \) is the displacement of vertex \( k \) of tetrahedron \( T_i \) and \( K_{jk}^{T_i} \) are the stiffness tensors for this tetrahedron. These tensors are completely determined by the initial position of each vertex of the tetrahedron and the Lamé coefficients, which are material specific.

For the whole mesh, the total force \( F_j \) at a vertex \( j \) is now simply the sum of the contributions by all adjacent terahedra of this vertex \( j \):

\[
F_j = [K_{jj}]u_j + \sum_{k\in N(j)} [K_{jk}]u_k
\]
where $K_{jj}$ is the sum of tensors $K_{jj}^T$ associated to the tetrahedra adjacent to vertex $j$, $K_{jk}$ is the sum of all $K_{jj}^T$ associated with the tetrahedra adjacent to edge $(j, k)$ and $N(j)$ is the neighborhood of mass point $j$.

2.2. Direct computation of the deformation

When simulating a maxillofacial procedure, we assume that the motion of a soft tissue point that adjoins the skull, equals the motion of that part of the skull. In this way the bone related planning of the maxillofacial procedure determines the motion of some of the soft tissue points. We call these points the join points.

After displacing these join points, the new rest position for all the other points of the mesh (the free points) can be found by time integrating the Newtonian motion equation[9] for these free points. However as already shown in[5] this solution scheme can lead to a slow convergence and errors in the estimation of the final rest position. In our maxillofacial surgery planning system this rest position is more important than the exact animation of the deformation. Therefore we combined this original Mass Tensor Model with a new static constraint to directly estimate the deformation without calculating the animation.

The new rest position is now found by demanding that in each free point the total force should be zero when the object is in rest.

$$u_{j}^{new} = \arg\min_{u_j} F_j(u_j)$$

To solve this optimization problem, we use a local steepest gradient method where we try to minimize the total force in each free point in every iteration step. So in iteration step $p$ we get:

$$F_j(u_j^p) = 0 \rightarrow u_j^p = [K_{jj}]^{-1} \left( \sum_{k \in N(j)} [K_{jk}] u_k^{p-1} \right).$$

Since all $K_{jj}$ and $K_{jk}$ are only depend on the initial mesh configuration and the two biomechanical constants $\lambda$ and $\mu$, we can pre-compute all $[K_{jj}]^{-1}$ and $K_{jk}$. Hence the optimization problem will be restricted to a series of matrix–vector multiplications and matrix summations.

2.3. Dynamic cut-off

A second improvement was introduced to the original Mass Tensor Model. The linear tetrahedral MTM has of course only a limited accuracy. It is unthinkable that we could

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Fig. 1. The two models used for validation.
simulate the real behavior of living tissue by such a model. Moreover our application, the maxillofacial planning system, demands only a finite accuracy. This accuracy is determined by the fact that deformations are significant when they are visible to the human eye. As a consequence, it is needless to iterate until the total force in each point becomes exactly zero. Very small forces give rise to very small displacements which are much smaller than the requested accuracy. Therefore we can stop the iterations early.

To determine when the iterations should stop we look at the mean gradient of the displacement field over a number of iterations. If this gradient becomes small in every point, we can conclude that we must be very close to the real solution in which the total force in each point should be zero.

3. Results

To validate the new MTM we used a virtual test cube and a real patient data set (see Fig. 1). For each set-up we compared the simulation results with those generated with a MSM [5] and FEM [4] simulator.

A cubic tetrahedral mesh of $10 \times 10 \times 10$ mm was used, containing 53,380 tetrahedra and 10,368 mesh nodes. During simulation the cube was elongated by displacing one side of the cube with 4 mm, while the other side was fixed. For the patient model we started from patient CT data to generate a tetrahedral mesh. Subsequently boundary conditions for simulation were derived from the bone-related planning using the Maxilim software as discussed in [6].

In Table 1 we show the differences between MSM, FEM and the novel MTM for a Young’s Modulus of 3000 Pa and a Poisson Coefficient of 0.45. For both set-ups the maximum and mean distance between the corresponding mesh points of the three models are shown in the first and second column of the table. In these experiments the FEM result was considered to be the ground truth. The MTM gives more or less the same simulation result as FEM. Between MSM and FEM however there remain some differences. Particularly in the middle area of the cube where the model should contract due to the volume conservation, errors remain rather large.

<table>
<thead>
<tr>
<th></th>
<th>Cube</th>
<th>Patient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_{\text{mean}}$ (mm)</td>
<td>$D_{\text{max}}$ (mm)</td>
</tr>
<tr>
<td>FEM</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MSM</td>
<td>0.964</td>
<td>1.88</td>
</tr>
<tr>
<td>MTM</td>
<td>0.082</td>
<td>0.47</td>
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</tbody>
</table>

$D_{\text{mean}}$ and $D_{\text{max}}$ are the mean and maximum distance between the corresponding points of the deformed model and the deformed FEM. The last column shows time needed to simulate this deformation.

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If we calculate for the MTM of the patient the maximum distance over all mass points between the final position of each mass point and the position of that point in iteration n, we get a graph as shown in Fig. 2. It is observed that after 160 iterations this maximum


distance becomes smaller then 0.5 mm, which is an acceptable error for maxillofacial planning systems. Therefore iterations can be stopped early, as discussed in Section 2.3. This induces a very small simulation time as shown in the third column of Table 1.

4. Conclusion

In this work we proposed the usage of a Tetrahedral Mass Tensor Model to simulate the new facial contour after maxillofacial surgery. This new model has more bio-mechanical relevance than the Mass Spring Model we used before. Moreover by combining the original Mass Tensor Model with a new dynamic threshold to stop the iterations early, we were able to achieve very small simulation times. We strongly believe that these fast simulation times are necessary when the surgeon wants to use the soft tissue prediction during planning. Furthermore since the final prediction has only a limited accuracy, this threshold doesn’t introduce new significant errors.

In the near future we will validate this new model on a database of several patients who had a maxillofacial surgery. We will compare the predicted facial contour with the real post operative result like described in Ref. [6]. Afterwards we hope to refine the parameters of our Mass Tensor Model to improve the prediction result and to ensure a certain prediction accuracy for typical maxillofacial procedures.

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References


