Optimization Algorithms for Internet Revenue Management

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***Abstract*—Due to the development of new technology in wireless communication the amount of online media usage has been increasing significantly in recent years. As the number of online media users increases, the revenue management from online advertising becomes a complex task. In general, a revenue management system for online advertising system consists of Inference Engine and Ad Server. Inference Engine predicts users' profiles based on their historical viewing data while Ad Server allocates users' viewing (impressions) to advertising campaigns based on their target audience. In this paper, models for advertise optimization (Impression Allocation models) that can be implemented at Ad Server are introduced. Impression Allocation models maximize the revenue by optimally allocating users' impressions to advertising campaigns. Models as well as the proposed algorithms that can be used to solve the models efficiently are provided.**

***Index Terms*—Heuristic Algorithm; Optimization System; Online Advertising; Revenue Management.**

I. INTRODUCTION

With the increase in internet broadcasting and web casting services, the amount of online media usage has been increasing significantly. Ad Serving operation that assigns advertisement to users based on targeted campaigns is a crucial component for the success in internet revenue management. In general, different advertising campaigns target different demographics' groups of users. The ability to infer users' profiles and allocate impressions to advertising campaigns' targeted groups are the most important functions of Ad Serving operation. A typical Ad serving system is shown in Figure 1.

When users register themselves to the system, users' profiles are created by profile manager and stored at a profile DB. The video content is managed by a content server that distributes video content based on user's preference. The demographics of users without profile will be inferred by an inference engine that uses users' viewing history as input. In this paper, models that can be used to allocate users' viewing (impressions) to advertising campaigns are proposed as well as algorithms that can solve the models efficiently.

II. LITERATURE REVIEW

The works that relate to the development of Ad serving system are in collaborative filtering area. [1] implemented collaborative filtering by applying dimensionality reduction method. The method provides user's inference by calculating similarity between users.

[2] focused on recommendation system that uses view history in order to create adaptive agents that generate program recommendations for TV viewers. [3] developed a hybrid system for restaurant recommendation system. The proposed system integrates knowledge-based recommendation and collaborative filtering.

[4] focused on a recommendation system for books, CDs and movies. The system relies on collaborating technique which is based on a Bayesian classifier. [5] developed a TV recommendation system that uses an adaptive assistance. The assistance monitors and updates users' profiles continuously in order to create recommended programs to users.

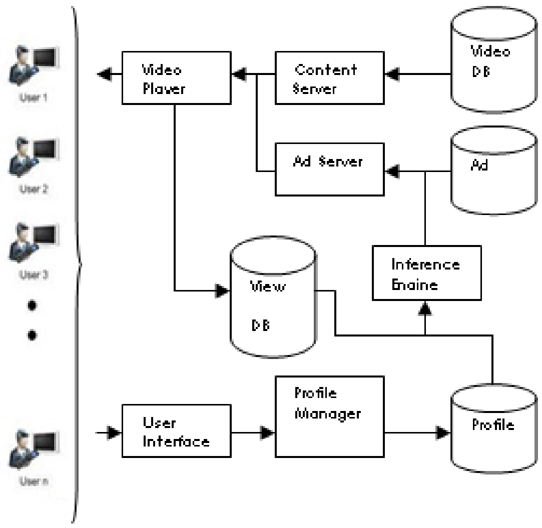

Figure 1: Ad serving system for online video provider

In the area of consumer clustering and targeted advertising, [6] developed consumer clustering and targeted advertising for digital TV. The data from the set top box (STB) were used to create clusters of consumers. A data mining technique is used to match new consumer with existing clusters, then the best match advertisement is displayed to user.

III. MATHEMATICAL MODEL

The requirements of advertising campaigns are the targeted demographics. In general, the demographics for online users consist of 732 combinations which are 2 genders (male and female), 6 age groups (<18, 18-24, 25-34, 35-44, 45-54, and 55+), and 61 genres. The genres define specific category of each video (e.g., science fiction, sports).

In this section, the models that can be implemented at Ad Server are proposed. The objective of ad requirements of advertising campaigns are the targeted demographics. In general, the demographics for online users consist of 732 combinations which are 2 genders (male and female), 6 age groups (<18, 18-24, 25-34, 35-44, 45-54, and 55+), and 61 genres. The genres define specific category of each video (e.g., science fiction, sports). serving is to allocate user viewing (impressions) to advertise campaigns that leads to maximum revenue. In general, the targeted groups for ad campaign are defined based on the combinations mentioned above (combinations of gender, age groups, and genres). In addition, there are requirement such as the frequency cap which limits the number of times that each user can view the same advertisement in a given period and the start time and end time of each advertising campaign.

A. Parameters and Decision Variables

The sets and indices used in the model are listed as follows:

- T a set of time periods indexed by t
- F a set of frequency groups (1 per 24 hours, 2 per 24 hours or no restriction) indexed by f
- G a set of demographics groups defined by combinations of gender, age groups, and genres indexed by g
- G_c a set of demographics groups that is targeted by campaign c defined by combinations of gender, age groups, and genres indexed by g
- T_c a set of time periods of campaign c indexed by t

The parameters used in the model are listed as follows:

- V_c required volume of campaign c
- Π_c number of forecasted users for targeted group g, frequency group f, and period t
- $N_{f,g,t}$ frequency capacity (per period) for campaign c
- $R_{c,g}$ revenue per impression of targeted group g from campaign c

The decision variables can be defined as follows:

- xc,f,g,t Number of impressions from targeted group g, frequency group f allocated to campaign c in period t

B. Impression Allocation Model (IAM)

In this section, the basic impression allocation model is proposed. The model is classified as a pure integer programming model where the decision variables represent the number of allocated impressions for combinations of c, f, g and t, respectively.

The objective function maximizes the total revenue of the impression allocation system which is represented as the multiplication of revenue per impression and the number of impressions allocated to each combination of c ∈ C, f ∈ F, g ∈ G, t ∈ T. Constraints (1b) limits the allocated impressions for all campaigns to the forecasted number of impressions. Constraints (2b) make sure that the number of impressions requirement of each campaign is satisfied. Constraints (3b) specify the upper bounds from the frequency requirement of each campaign. Constraints (4b) state integer requirement of decision variables.

Objective Function:

$$Maximize \sum_{c\in C}\sum_{f\in F}\sum_{g\in G}\sum_{t\in T} R_{c,g} x_{c,f,g,t}$$

Constraints:

$$\sum_{c\in C} x_{c,f,g,t} \le f * N_{f,g,t}, \forall f \in F, g \in G, t \in T \qquad (1b)$$

$$\sum_{f\in F}\sum_{g\in G}\sum_{t\in T} x_{c,f,g,t} \ge V_c, \forall c \in C \qquad (2b)$$

$$x_{c,f,g,t} \le \Pi c * N_{f,g,t}, \forall c \in C, f \in F, g \in G^c, t \in T \qquad (3b)$$

$$x_{c,f,g,t} \in Integer, \forall c \in C, f \in F, g \in G^c, t \in T \qquad (4b)$$

In the next section, an enhanced version of IAM (IAM_1) where preemptable campaigns or campaigns that the allocated impressions can be less than the specified volumes are considered. Also, since the evenly distributed of allocated impressions is preferred, the constraints that control the smoothness of the allocated impression for each campaign over the planning horizon are introduced.

C. Impression Allocation Model 1 (IAM_1)

In order to take into account preemptable campaigns, a new set C_p is introduced to the model.

- C_p A set of preemptable campaigns indexed by c

To control the smoothness of the allocated impression for each campaign, the lower and upper bounds of number of impressions are defined and introduced to IAM_1.

- $U_{c,t}$ The upper bounds of number of allocated impressions of campaign c in period t
- $L_{c,t}$ The lower bounds of number of allocated impressions of campaign c in period t

IAM_1 can be summarized as follows:

Objective Function:

$$Maximize \sum_{c\in C\cup C_p}\sum_{f\in F}\sum_{g\in G}\sum_{t\in T} R_{c,g} x_{c,f,g,t}$$

Constraints:

$$\sum_{c\in C} x_{c,f,g,t} \le f * N_{f,g,t}, \forall f \in F, g \in G, t \in T \qquad (1c)$$

$$\sum_{f\in F}\sum_{g\in G}\sum_{t\in T} x_{c,f,g,t} \ge V_c, \forall c \in C \qquad (2c)$$

$$\sum_{f\in F}\sum_{g\in G}\sum_{t\in T} x_{c,f,g,t} \le V_c, \forall c \in C_p \qquad (3c)$$

$$\sum_{f\in F}\sum_{g\in G} x_{c,f,g,t} \le U_{c,t}, \forall c \in C, t \in T \qquad (4c)$$

$$x_{c,f,g,t} \le \Pi c * N_{f,g,t}, \qquad (5c)$$

$$\forall c \in C, f \in F, g \in G^c, t \in T \qquad (6c)$$

$$x_{c,f,g,t} \in Integer, \quad \forall c \in C, f \in F, g \in G^c, t \in T \qquad (7c)$$

Constraints (3c) hold for preemptable campaigns where the volumes can be violated. Constraints (4c) and (5c) ensure that the allocated impressions are within the lower and upper bounds. Note that the objective function, constraints (1c), (2c), (6c) and (7c) remain the same.

D. Impression Allocation Model 2 (IAM$_2$)

In this section, the assumption that the available impressions can satisfy volume requirement from all campaigns is relaxed. Instead, purchasing impressions from other video publishers is allowed. A set of video publishers is denoted by H indexed by h. The cost per impression for video publisher h is:

$\phi_{h,f,g,t}$ Cost per impression from video publisher h in period t for targeted group g, frequency group f

The decision variables for number of impressions bought from publisher h is:

$y_{h,f,g,t}$ Number of impressions bought from publisher h in period t for targeted group g, frequency group f

$z_{h,c,f,g,t}$ Number of impressions bought from publisher h allocated to campaign c in period t for targeted group g, frequency group f

IAM$_2$ can be summarized as follows:

Objective Function:

$$\textit{Maximize} \quad \sum_{c\in C\cup C_p}\sum_{f\in F}\sum_{g\in G}\sum_{t\in T} R_{c,g}x_{c,f,g,t} - \sum_{h\in H}\sum_{f\in F}\sum_{g\in G}\sum_{t\in T}\Phi_{h,f,g,t}y_{h,f,g,t}$$

Constraints:

$$\sum_{c\in C} x_{c,f,g,t} \le f * N_{f,g,t}, \forall f\in F, g\in G, t\in T \tag{1d}$$

$$\sum_{h\in H}\sum_{f\in F}\sum_{g\in G}\sum_{t\in T}(z_{h,c,f,g,t}+x_{c,f,g,t}) \ge V_c, \forall c\in C \tag{2d}$$

$$\sum_{h\in H}\sum_{f\in F}\sum_{g\in G}\sum_{t\in T}(z_{h,c,f,g,t}+x_{c,f,g,t}) \le V_c, \forall c\in C_p \tag{3d}$$

$$\sum_{h\in H}\sum_{f\in F}\sum_{g\in G}(z_{h,c,f,g,t}+x_{c,f,g,t}) \le U_{c,t}, \forall c\in C, t\in T \tag{4d}$$

$$\sum_{h\in H}\sum_{f\in F}\sum_{g\in G}(z_{h,c,f,g,t}+x_{c,f,g,t}) \ge L_{c,t}, \forall c\in C, t\in T \tag{5d}$$

$$\sum_{c\in C} z_{h,c,f,g,t} \le y_{h,f,g,t}, \forall h\in H, f\in F, g\in G, t\in T \tag{6d}$$

$$x_{c,f,g,t} \le \Pi c * N_{f,g,t}, \forall c\in C, f\in F, g\in G^c, t\in T \tag{7d}$$

$$x_{c,f,g,t}, y_{h,f,g,t}, z_{h,c,f,g,t} \in \textit{Integer}, \quad \forall c\in C, h\in H, f\in F, g\in G^c, t\in T \tag{8d}$$

Variables $y_{h,f,g,t}$ are included in the objective function to represent the cost of acquiring impressions from publisher h for combination f, g and t. Constraints (1d) remain the same. Constraints (6d) introduce variables $z_{h,c,g,g,t}$ that represent the allocated number of impressions bought from publisher h to campaign c for combination f, g and t. Constratins (2d), (3d), (4d) and (5d) ensure that the required volume and bounds for each campaign is satisfied. Constraints (7d) and (8d) remains the same. It is assumed that the number of impressions from external publishers considered is large enough to satisfy volume requirement from all campaigns.

IV. Solution Methodology

The typical sizes of IAM, IAM$_1$ and IAM$_2$ grow significantly, as the problem size (number of campaigns, frequency groups, time periods or demographics) increases. Furthermore, the model needs to be solved periodically (hourly) in order to have up-to-date impression allocation solution. As a result, the computational time is crucial for the implementation of IAM or IAM$_1$ at the Ad Server. In this paper, it is assumed that the number of impressions based on available users is large enough in order to satisfy the volume requirement from all campaigns.

A. Algorithm for IAM

In this section, an efficient algorithm for solving the impression allocation model (IOPT) is proposed. Since the objective is to maximize the revenue, the impression allocation will be based on parameter $R_{c,g}$ mainly. To satisfy all the constraints imposed by IAM the proposed algorithm consists of 4 steps.

In Step 1, the campaigns are ordered based on Rc,g for each g. Then, the cumulative assigned impression for campaign c, CI$_c$ is initialized to 0 in Step 2. Step 3 initializes the allocated impression for all combinations of f ∈ F, g ∈ G, t ∈ T, AI$_{f,g,t}$, to 0. In Step 4, the impressions are assigned to each campaign if the cumulative assigned impression does not exceed the required volume. Note that the allocation is limited by the number of available impressions for each combination of f∈F,g∈G,t∈T. Algorithm IOPT is summarized as follows:

1. The campaigns are ordered based on $R_{c,g}$ for each g.
   $R_{c_{1,g}} \ge R_{c_{2,g}} \ge \ldots \ge R_{c_{|C|,g}}$
2. Initialize the cumulative assigned impression for campaign c, CI$_c$, to 0.
3. Initialize allocated impression for combinations f∈ F, g ∈ G, t ∈ T, AI$_{f,g,t}$,to 0.
   For i = 1 ,.., |C|

For each combination f∈ F, g ∈ G, t ∈ T

Do

$$x_{c_{i,g},f,g,t} = \max\{\ \min\left\{f, \pi_{c_{i,g}}\right\} * N_{f,g,t} - AI_{f,g,t}, 0\}$$

$$AI_{f,g,t} = AI_{f,g,t} + x_{c_{i,g},f,g,t}$$

$$CI_{c_i} = CI_{c_i} + x_{c_{i,g},f,g,t}$$

While $(CI_{c_i} \le V_{c_i})$

Proposition 1IOPT algorithm provides optimal solution for IAM. Proof. Using contradiction, it can be shown that the solution from IOPT is optimal. Without loss of generality, assume that the revenues from all campaigns (Rc,g) are different and when sorted they can be represented as:

$$R_{c_{1,g}} \ge R_{c_{2,g}} \ge \ldots \ge R_{c_{|C|,g}}$$

For each g ∈ G, based on a solution generated by IOPT, if there exists another solution where the impressions are allocated to campaigns with lower revenues per impression, then the current solution is not optimal. However, in the algorithm, the impression allocation gives priority to campaigns with higher revenues per impression, these results in a contradiction.

B. Algorithm for IAM$_1$

An algorithm for solving the IAM$_1$, IOPT$_1$, is proposed in this section. To satisfy all the constraints imposed by IAM$_1$, IOPT$_1$ consists of 6 steps. In Step 1, the revenues per impression of campaigns that the volumes cannot be violated, $R_{c_i}, i \in N$, and the revenues per impression of preemptable campaigns, $R_{c_i}, i \in M$, are ordered in decreasing order. Then, the cumulative assigned impression for campaign c, CI$_c$, is initialized to 0 in Step 2. Steps 3 and 4 initialize the allocated impression for all combinations (f∈F,g∈G,t∈T) AI$_{f,g,t}$ and combinations (c∈ C , t ∈ T)UAI$_{c,t}$ to 0.

In Step 5, the impressions are assigned to each campaign in set N, if the cumulative assigned impression does not exceed the required volume. Note that the allocation is limited by the number of available impressions for each combination of f∈F,g∈G,t∈T and also the lower and upper bounds, L$_{c,t}$ and U$_{c,t}$. Step 6 is similar to step 5 but considers the campaigns in set M. By using similar proof shown in proposition 1, algorithm IOPT$_1$ provides optimal solution for IAM$_1$. Algorithm IOPT$_1$ is described as follows:

1. Define N as the set of campaign that the volumes cannot be violated and M as the set of preemptable campaigns. So, for set N, the notation for campaigns once they are ordered in decreasing order is

$$R_{c_1,g} \geq R_{c_2,g} \geq \cdots \geq R_{c_{|N|},g}$$

For set M, the notation for preemtable campaigns once they are ordered in decreasing order is:

$$R_{c_1,g} \geq R_{c_2,g} \geq \cdots \geq R_{c_{|N|},g}$$

2. Initialize the cumulative assigned impression for campaign c, CI$_c$, to 0.
3. Initialize allocated impression for combinations (f∈ F, g ∈ G, t ∈ T), AI$_{f,g,t}$ to 0.
4. Initialize allocated impression for combinations (c∈ C, t ∈ T), UAI$_{c,t}$ to 0.
5. Iterate through set N, set of campaigns that the volumes cannot be violated.
   For i = 1 ,.., |N|
   For each combination f∈ F, g ∈ G, t ∈ T
   Do

$$x_{c_{i,g},f,g,t} = \max\{\ \min\left\{f, \pi_{c_{i,g}}\right\} * N_{f,g,t} - AI_{f,g,t}, 0\}$$

$$AI_{f,g,t} = AI_{f,g,t} + x_{c_{i,g},f,g,t}$$

$$CI_{c_i} = CI_{c_i} + x_{c_{i,g},f,g,t}$$

$$UAI_{c_i,t} = UAI_{c_i,t} + x_{c_i,f,g,t}$$

   While ($CI_{c_i} \leq V_{c_i}$ and $UAI_{c_i,t} \leq U_{c_i,t}$ and $UAI_{c_i,t} \geq L_{c_i,t}$)
6. Reset AI$_{f,g,t}$ and UAI$_{c,t}$ to 0, then repeat step 5 by iterating through set M, set of preemptable campaigns using *i* = 1 .., |M|.

C. Algorithm for IAM$_2$

IAM$_2$ consists of additional variables y$_{h,f,g,t}$ and z$_{h,c,f,g,t}$ that represent number of impressions from external publishers in case the available impressions of internal users are not sufficient to satisfy campaigns' volume requirement. Algorithm IOPT$_2$ is proposed in order to optimize IAM$_2$. IOPT$_2$ consists of 9 steps as shown below.

Steps 1 to 5 are the same as those from IOPT$_1$.

6. Store the list of campaigns that violate volume requirement from step 5. in list Nu. Then, calculate for $R_{c,g} - \phi_{h,f,g,t}$ every combination of c ∈ Nu, h ∈ H, f ∈ F, g ∈ G, t ∈ T. For each combination off ∈ F, g ∈ G, t ∈ T, $R_{c,g} - \phi_{h,f,g,t}$ ∈ F, g ∈ G, t ∈ T, is sorted in decreasing order and stored in list B.
7. For i = 1 ,.., |B|
   For each combinations (f ∈F,g∈G,t∈ T)
   Do

$$z_{h_i,c_{i,g},f,g,t} = \min\{V_{c_i} - CI_{c_i}, U_{c_i,t} - UAI_{c_i,t}\}$$

$$CI_{c_i} = CI_{c_i} + z_{h_i,c_{i,g},f,g,t}$$

$$UAI_{c_i,t} = UAI_{c_i,t} + z_{h_i,c_{i,g},f,g,t}$$

   While ($CI_{c_i} \leq V_{c_i}$ and $UAI_{c_i,t} \leq U_{c_i,t}$ and $UAI_{c_i,t} \geq L_{c_i,t}$)
8. Repeat steps 5 and 6 using set M, set of preemp table campaigns.
9. Calculate $y_{h,f,h,t} = \sum_{c \in N^u} z_{h,c,f,g,t}$ forever combination of h∈ H, f ∈ F, g ∈ G, t ∈ T.

Steps 1 to 5 are the same as those from IOPT$_1$. In step 6, the campaigns that violate volume requirement in step 5 are stored in list N^u and the net profits for acquiring impressions from external publishers in order to satisfy volume requirement of campaigns in list N^u are sorted in decreasing order and stored in list B. In step7, the impressions are allocated to combinations (c$_i$, f, g t) ∈ B in decreasing order of net profits until the volume requirement of all campaigns is satisfied. Step 8 repeats steps 5,6 and 7 by considering set M instead of set N. Step 9 calculates Y$_{h,f,g,t}$ for each combination h ∈ H, f ∈ F, g ∈ G, t ∈ T which is used in the objective function of IOPT$_2$.

V. COMPLEXITY OF ALGORITHM

For IOPT, the amount of work associated with ordering $R_{c_i}, i \in N$ is O(|C|log(|C|)|G|). For each campaign, determine allocated impressions for each combination of f ∈ F, g ∈ G, t ∈ T requires O(|F||G||T|) worse case enumeration. If K = max {|C|, |F|, |G|, |T|}, the amount of work for IOPT is O(K^{2}log(K)) + O(K^4) = O(K^4).

For IOPT$_1$, the amount of work for ordering campaigns in set N and M is still O(|C|log(|C|) |G|). Steps 5 and 6 of IOPT$_1$ require worse case enumeration = O(|C||F||G||T|). As a result, the amount of work of IOPT$_1$ is also O(K^4).

Considering IOPT$_2$, since steps 1-5 of IOPT$_2$ are similar to those of IOPT$_1$, the amount of work from step1 to 5 is O(|C||F||G||T|). In step6, storing the list of campaigns that violate volume requirement and calculating R$_{c,g}$ – ϕ$_{h,f,g,t}$ for every combination of c ∈ Nu, h ∈ H, f ∈ F, g ∈ G, t ∈ T require $O(|C||H|||F||G||T| + |C||H|||F||G||T| \log((|C||H|||F||G||T|)) = O(K^5+K^5\log(K^5))$ worst case enumeration, note that $K = \max\{|H|, |C|, |F|, |G|, |T|\}$.

Step 7 requires $O((|C||F||G||T|) = O(K^4)$ amount of work. As a result, the amount of work for IOPT2 is $O(K^5+K^5\log(K^5))$. The computational results for of IOPT, $IOPT_1$ and $IOPT_2$are summarized in Table 1.

Table 1
Computational results of IOPT and $IOPT_1$

Test Problems	IOPT	$IOPT_1$	$IOPT_2$
Small	90%	82%	80%
Medium	88%	80%	77%
Large	84%	71%	66%

Three types of test problems (small, medium and large) were used in the experiment. 10 test problems were used in each type. The small test problems consider 10 campaigns, 10 genres and 5 periods. The medium test problems consider 50 campaigns, 10 genres and 10 periods. The large test problems consider 100 campaigns, 60 genres and 20 periods.

The runtimes are reported in term of percentage of the runtimes when the test problems were solved via solving models IAM and IAM_1 using standard Mathematics solver (Cplex 12.5, 64 bits).

From Table 1, the amount of runtime reduction of IOPT, $IOPT_1$ and $IOPT_2$ increases as the problem size increases. This is because specialized algorithm normally performs better than the performance from standard Mathematics solver. Also, due to the increase in complexity of IAM_2 compared with IAM_1 and IAM, the amount of reduction also increases when comparing runtimes of $IOPT_2$, $IOPT_1$ and IOPT.

VI. OPERATION PLAN

In this section, the function of Ad server which assigns advertising campaigns to users when they start entering the system or start watching the videos is illustrated. Figure 2 depicts the Ad serving implementation. Each user entering the system will be assigned to advertising campaign based on the targeted demographics. However, the assignment rule depends on the allocated impressions from the impression allocation model and the accumulated error from the forecasted number of users.

In general, the operation plans attempts to follow the impression allocated by the impression allocation model. However, due to the uncertainty from the forecasted number of users in the system in particular period, $N_{f,g,t}$, the rule for assigning advertising campaign to each incoming user must be defined in order to minimize the deviation from the allocated impressions or maximize the revenue if the error from the forecasted number of users is more than a specified limit. To measure the amount of deviation of actual number of users from the forecasted number of users in each period, the actual number of users entering the system is monitored hourly in each period and the cumulative error for each combination of $f \in F$, $g \in G$, $t \in T$ is also calculated hourly. The rule for assigning advertising to users is summarized below.

At current period $t \in T$

1. Retrieve user demographics, g*.
2. IF (the accumulated forecast error < 5 percent) then

Assign advertising campaign that target demographics group g* in round-robin order (with equal weights) starting with the one with the highest revenue until the allocated impressions are satisfied.

ELSE

The weights for campaigns with higher revenue are increased as the forecast error increase. The general rule is, for every 10 percent increase in error, the weight is uniformly increased by 10 percent toward the campaigns with higher revenues.

END IF

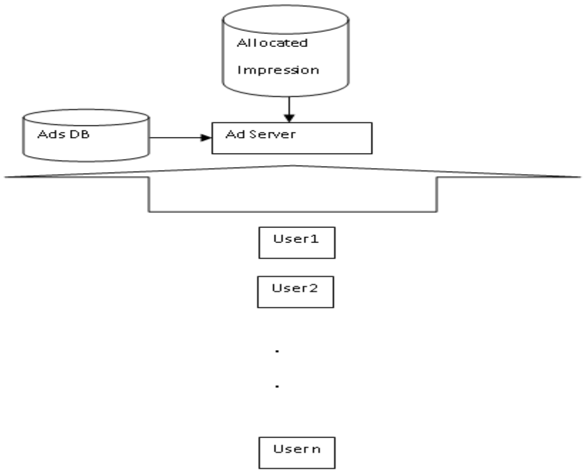

Figure 2: Ad serving implementation

Practically, the number of users entering the system in each hour is compared to the forecasted number of users. Then, the percentage error is calculated in order to adaptively adjust the weights of the campaigns that target the demographics of the user. The forecast error is accumulated hourly and if it is less than 5 percent, the weights for all advertising campaigns that target the same demographics are equal and the campaigns are chosen in round-robin starting with the one with the highest revenue. However, if the accumulated forecast error (AFE) becomes more than 5 percent, the weights are adjusted by increasing the weights of campaigns with higher revenues. The increase is set to 10 percent for every 10 percent increase in accumulated forecast error and the increase is uniformly distributed toward the campaigns with higher revenues. Next, the function of the Ad server is illustrated. In the example, the solution from the allocation model where there are 2 campaigns (c=1 and 2 with $V_1 = 100$ and $V_2 = 200$), 1 frequency level (f = 1) and 2 Demographics groups (g =1 :Male,18-25, Sports and g = 2 :Female,18-25,Sports) are considered. Without loss of generality, let's consider the case where campaigns 1 and 2 target users with the same demographics (g = 1 and g = 2) in periods t = 1 and 2. Assuming that the solution from the impression allocation model is the following: $x_{1,1,1,1} = 35$, $x_{1,1,2,1} = 35$, $x_{1,1,1,2} = 15$, $x_{1,1,2,2} = 15$, $x_{2,1,1,1} = 70$, $x_{2,1,2,1} = 70$, $x_{2,1,1,2} =30$, $x_{2,1,2,2} = 30$ and the periods are in hours. Assuming that during the first hour there are 12 users and 20 users with demographics g = 1 and g = 2 in the system. Also, assume that the revenue from campaign 1 is higher. Since the AFE at the beginning of period 1 is zero, the Ad assignment rule is to assign equal weights to campaigns 1 and 2 because they target the same demographics groups. As a result, the actual impression allocation $alloc_{1,1,1,1}$, $alloc_{1,1,2,1}$, $alloc_{2,1,1,1}$ and $alloc_{2,1,2,1} = 3$.

Table 2 summarizes the Ad assignment at the end of the first hour.

Table 2
Ad assignment at the end of the first hour

(c,f,g,t)	$x_{c,f,g,t}$	$alloc_{c,f,g,t}$	(c,f,g,t)	$x_{c,f,g,t}$	$alloc_{c,f,g,t}$
(1,1,1,1)	35	3	(2,1,1,1)	70	3
(1,1,2,1)	35	3	(2,1,2,1)	70	3
(1,1,1,2)	15	0	(2,1,1,2)	30	0
(1,1,2,2)	15	0	(2,1,2,2)	30	0

At the end of the first hour, AFE_1 is calculated by comparing the actual number of users to the forecasted number of users with demographics groups g = 1 and 2. In this example, AFE_1 = (20-12)/20 = 0.4 (40 percent), the weight of campaign 1 is increased by 40 percent in the second hour which means that the number of assigned Ad from campaign 1 should be higher. Assuming that during the second hour there are 20 users, the number of users for campaign 1 and 2 are now 14 and 6, respectively, The Ad assignment at the end of the second hour is shown in Table 3. If the accumulated hour forecast error in any period is more than 40 percent, the forecast for number of users and the impression allocation need to be regenerated. This process is automated and typically done once every day.

Table 3
Ad assignment at the end of second hour

(c,f,g,t)	$x_{c,f,g,t}$	$alloc_{c,f,g,t}$	(c,f,g,t)	$x_{c,f,g,t}$	$alloc_{c,f,g,t}$
(1,1,1,1)	35	3	(2,1,1,1)	70	3
(1,1,2,1)	35	3	(2,1,2,1)	70	3
(1,1,1,2)	15	7	(2,1,1,2)	30	3
(1,1,2,2)	15	7	(2,1,2,2)	30	3

VII. Conclusion

In this paper, the model that can be used to allocate the number of impressions to campaigns (IAM) is proposed. The model considers both the required volume and frequency from advertisers. The targeted demographics are the combination of gender, age groups and genres. An extension of IAM, called IAM1and IAM2are also introduced. In IAM1, the pre-emptible campaigns are considered in the model. Also, smoothing constraints that define lower and upper bounds of number of allocated impressions are considered. In IAM2, buying impressions from external publishers are considered in the proposed model.

Due to the size and complexity of the model, efficient algorithms for IAM, IAM_1 and IAM_2 (IOPT, $IOPT_1$ and $IOPT_2$) with polynomial complexity are also proposed. The algorithms allocate impression by considering campaigns in decreasing order of revenue per impression based on constraints defined in each case. The operational plan that can practically assign advertising campaigns to impressions is also proposed.

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