A fuzzy approach for the evaluation of competences

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Abstract

Maintaining and developing competitive advantages in a constantly changing market requires the creation and exploitation of a network of adequate competences. Competence management necessitates both clear formalisation and evaluation of the pertinence of competences. This depends on several factors and involves subjective elements. In this paper, a methodological approach for evaluating competences is proposed. This new approach is mainly based on fuzzy logic which handles the integration of subjective elements. The approach is demonstrated within a real problem in the transportation industry.

Keywords: Competence management, resources based competition, fuzzy logic, multi-criteria approach, linguistic variables.

1 Introduction

Enterprises today operate in a constantly evolving market, characterised by short product life cycles and increasing requirements for flexibility. A key issue to remain competitive is the ability to respond rapidly to changing market opportunities by quickly providing the required combination of design and manufacturing capabilities. Maintaining and developing this competitive advantage requires the creation and exploitation of a network of adequate competences. Accordingly, competence management is a key issue in strategic as well as tactical management.
Managing competences requires the ability to estimate an organization’s competence value which relies on several factors and involves subjective evaluation. One main challenge is to propose an approach able to evaluate four characteristics for the resources involved in the development of a competence. The other challenge consists of being able to characterize resources by relying on subjective values that are not subject to precise measurements.

The aim of this paper is to propose a methodological approach for evaluating competences and integrating various points of view. Following a brief taxonomy of terms and definitions (section 2), the paper provides the objectives and motivations of a fuzzy approach for the evaluation of the competences (section 3). Section 4 and 5 report the proposed fuzzy approach and a discussion of the results. Section 6 presents the application of the methodology within an industrial case and the corresponding results follow in section 7. Finally, section 8 presents the conclusion and further avenues of research in this field.

## 2 Concepts and definitions

The proposed approach for the evaluation of competences is based on a specific formalism where the competence $C(A_i)$ is the ability to efficiently combine non-material resources (knowledge, know-how, etc.) and material resources (instruments, machines etc.) in order to satisfy an activity $(A_i)$ (figure 1). A competence $C(A_i)$ is formally defined as (9):

$$\langle C(A_i), [R_{\text{NM}}], [N_{\text{ext}}], [J_u], [R_M], [J_v], \text{type} \rangle$$

![Fig. 1. Description of the different competences.](image)
Within **figure 1**, $C(A_i)$ is a specific competence, $[R_{NM}]$ is an array of terms representing the different attributes (identification, label, description and type) of the non-material resources required by the activity ($A_i$) or acquired by the actor involved in this activity - $[N_{ctx}]$ is an array which indicates the skill level of the non-material resource for the execution of the activity in a given context ”$ctx$”, $[I_u]$ is an array of terms representing the values of the indicators $I_u$ for each non-material resource ($u \in [1 \ldots n]$ where $n$ is the number of evaluating indicators of the non-material resources), $[R_M]$ is an array where the terms are the different attributes of the material resources (identification, label, description and type) required by the activity ($A_i$) or acquired by the organization involved in the activity, $[J_v]$ is an array the terms of which are the values of the indicators $J_v$ for each material resource ($v \in [1 \ldots m]$ where $m$ is the number of evaluating indicators of the material resources), - *type* is an attribute representing the type of the competence defined by the following set of values [uc (unit competence), ic (individual competence), cc (collective competence)]. The evaluation of the arrays $[I_u]$ and $[J_v]$ will be based on the competence evaluation approach presented in section 4.

### 3 Objectives and motivations of a fuzzy approach for the evaluation of competences

To seek a competitive advantage, the aspects that should be taken into account are the operational, tactical and strategic importance of each competence, the availability of competences within and outside the enterprise, and the difficulty to transfer them. It is thus necessary to model several factors in the development of an approach to estimate the value of competences. It should in particular, allow the decision makers to answer different questions such as: "Which competences having a high value for the enterprise need to be developed?" To address this issue, several studies ([13]; [12]; [16]) in strategic management have treated the question "How to maintain a competitive advantage?". In the middle of the 1990s, the resource-based view of the firm ([4]; [13]; [14]; [15]; [17]) was presented as a dominant theoretical model for strategic management ([2]). This model upholds that competitive advantage of a firm is not built on the exploitation of a dominant and protected position in a market, but in the evaluation of the resources within the enterprise. The proposed approach is based on this postulate and states that the significance of resources can be evaluated on the basis of four characteristics:

1. **Value**: measures the significance of the contribution of a resource to the value of the product for the customer ([5]; [2]) or to the execution of processes or activities.
2. **Scarcity**: measures a resource’s accessibility for enterprises (1).

3. **Imitation**: measures the difficulty to reproduce a resource. Indeed, if the resource is difficult to imitate, competitors will avoid copying it (7).

4. **Replacement**: measures the difficulty of replacing one resource by another. If the resource has no easily accessible substitutes, this preserves its value (1) (2).

The main challenge is to propose an approach able to evaluate these four characteristics for the resources involved in the development of a competence. Unfortunately, due to their nature, it is difficult to precisely measure these characteristics and they are specific for each enterprise. This evaluation challenge can be addressed through the definition of criteria and indicators. It will then be possible to aggregate criteria to build specific indicators for each enterprise. Moreover, the evaluation of the resource criteria should consider internal and external references to guarantee the realism of the estimated value of the resources and thus, the competences. This can be fixed by defining internal references (processes, activities, etc.) and external references (product, market, etc.) for the evaluation of criteria.

A second challenge is the ability to handle non-precise subjective values for evaluating criteria that characterize resources. In this case, the mechanism for evaluating competence can be described by the global indicator ACI (Aggregated Competence Indicator) that is related to the four characteristics. This indicator is a qualitative evaluation rather than a numerical value. A fuzzy logic approach (18) for describing the different criteria and building an estimation engine for the indicators is proposed. In the next section, we demonstrate the suitability of the fuzzy-based mechanism for evaluating competences based on criteria and indicators, and for configuring the inference engine.

## 4 Proposed fuzzy-approach

The global system for evaluating competence is composed of two levels. The approach has been originally proposed in (10) and consists of (figure 2):

1. At the first level, the estimation of the different indicators for a given competence defines the sources of advantages based on the four characteristics. These indicators will be calculated from the evaluation of resources involved in developing the competence.

2. A second level for the estimation of the global indicator ACI for a given competence. Its value depends on the criteria and indicators used at the first level.
Fig. 2. Global model for the evaluation of a competence.
Consider the following definitions and symbolisms used in the next sections:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR(_{j,k})</td>
<td>Represents the (j^{th}) criterion relative to the characteristic (k) where (k \in {\text{Value, Scarcity, Imitation, Replacement}}) and (j \in {1, \ldots, m}) where (m) is the number of criteria for this same characteristic (k).</td>
</tr>
<tr>
<td>Ind(_k)</td>
<td>Represents an aggregated indicator of the different criteria relative to the same characteristic (k) where (k \in {\text{Value, Scarcity, Imitation, Replacement}}).</td>
</tr>
<tr>
<td>ACI</td>
<td>Aggregated Competence Indicator is an aggregated indicator of the competence. It explains the notion of &quot;criticality(^2) of competence in an aggregate way starting from the criteria CR(_{j,k}) and the indicators Ind(_k).</td>
</tr>
<tr>
<td>&lt; CR(_{j,k}) &gt;</td>
<td>Represents the value of the criterion CR(_{j,k}) for a given competence.</td>
</tr>
<tr>
<td>&lt; CR(_{i,j,k}) &gt;</td>
<td>Represents the value of the criterion CR(_{j,k}) for the resource (i) in a given competence.</td>
</tr>
<tr>
<td>&lt; Ind(_k) &gt;</td>
<td>Represents the value of the indicator Ind(_k) for a given competence.</td>
</tr>
<tr>
<td>(X_i)</td>
<td>(X_i) is a decision variable: (X_i = 1) if the resource (i) is selected for the evaluation of the criterion CR(_{j,k}), else (X_i = 0).</td>
</tr>
</tbody>
</table>

Table 1
Definition of used terms and symbols.

4.1 Estimation of the different indicators - Level 1

Step 1 (Level 1): This first step consists of defining five fuzzy sets relative to linguistic values \{\text{Very low, Low, Medium, High, Very High}\}. These sets are described by means of a membership functions and scales of measurement (or universe of discourse) for each criterion CR\(_{j,k}\) retained.

- Example: Consider the criterion CR\(_{3,\text{Scarcity}}\): Difficulty to access to a resource externally for the characteristic Scarcity. For this criterion, the scale of time is used for evaluation. The five linguistic values \{\text{Very low, Low, Medium, High, Very High}\} that constitute the fuzzy sets of this universe of discourse, are described by means of a membership function \(\mu(x)\). For each of the five linguistic values, upper limits \((\mu = 1)\) and lower limits \((\mu = 0)\) are defined. By connecting each of the obtained points \((\mu = 1)\) and \((\mu = 0)\) with a linear function, a membership function is defined.

\(^2\) The "criticality" of competence is a property that defines its degree of being of the highest importance of this same competence.
For instance, in figure 3 the linguistic value \{High\} is defined by \(\mu = 1\) for 3 months and \(\mu = 0\) for 1 month and for 1 year.

![Fig. 3. Example of the membership function for the criterion CR_{3,Scarcity}.](image)

Step 2 (Level 1): The second step consists of "fuzzifying\(^3\)" all values of the different criteria for each of the resources \(i\) combined in the competence by using the membership function relative to each criterion CR\(_{j,k}\).

![Fig. 4. Example of "Fuzzification" of < CR\(_{3,\text{Scarcity}}\) > for the criterion CR\(_{3,\text{Scarcity}}\).](image)

Step 3 (Level 1): The third step consists of "defuzzifying\(^4\)" all the previously obtained fuzzy sets, in order to determine a numerical value \(< CR_{i,j,k}\) (on a 0 to 10 scale) for each resource \(i\) and each of the criteria CR\(_{j,k}\). This is done by using the membership function described in figure 5 and the centroid method as a "defuzzification" method.

The value \(< CR_{j,k}\) of the estimated competence criterion CR\(_{j,k}\) is then determined by calculating the average of the values \(< CR_{i,j,k}\) of criteria obtained for each of the resources \(i\) (or a part of them) combined in the construction of this competence.

\(^3\) "Fuzzify" means associating a numerical value to a fuzzy set.  
\(^4\) "Defuzzify" means associating a fuzzy set to an interpretable numerical value.
This value $< \text{CR}_{j,k} >$ is given by equation 2.

$$< \text{CR}_{j,k} > = \frac{\sum_{i=1}^{n} < \text{CR}_{i,j,k} > \cdot X_i}{\sum_{i=1}^{n} X_i} \quad (2)$$

where $n$ is the number of resources combined in the competence and evaluated by $\text{CR}_{j,k}$.

![Fig. 5. Membership function of "Defuzzification".](image)

**Step 4 (Level 1):** This step consists of a new "fuzzification" of the values $< \text{CR}_{j,k} >$ for each of the criteria $\text{CR}_{j,k}$ obtained previously for the competence and of the activation of rules of the inference engine. This step is described in the following parts:

- The **first part** consists of "fuzzifying" the input values $< \text{CR}_{j,k} >$ obtained previously for each criterion $\text{CR}_{j,k}$. For each input (every value $< \text{CR}_{j,k} >$ of the criterion obtained in the **Step 3 (Level 1)**), it is necessary to determine its membership degree in each fuzzy set described in figure 6. The input is always a numerical value limited to its definition domain (here it is an interval between 0 and 10). In this manner, every input (different criterion of evaluation of the competence required by the inference engine) is fuzzyfied over all the qualifying membership functions described in figure 6.

- Example: **Figure 6** shows the result of the "fuzzification" step for a criterion $\text{CR}_{j,k}$ over its membership function. In this case, the criterion $\text{CR}_{j,k}$ is rated 5.50, which corresponds to Medium and High in the given graphical definition, or to $\mu = 0.70$ for the Medium membership function and $\mu = 0.30$ for the High membership function.
This means:

"Criterion CR_{j,k} is Medium to degree 0.70 and High to degree 0.30"

Fig. 6. Example of "Fuzzyfication" of the < CR_{j,k} > of the criterion CR_{j,k}.

In the same manner, each value < CR_{j,k} > of the different criteria for evaluating a competence required by the inference engine, are "fuzzified" over a membership function. For this example, each criterion is the same as shown by figure 6).

- The second part consists of choosing a fuzzy logic aggregation operator for the rule premise step.

Once the inputs (here the value < CR_{j,k} > of the different criteria obtained in the previous step) have been fuzzyfied, the degree to which each part of the antecedent part has been satisfied for each rule is determined. If the antecedent part of a given rule has more than one part, a fuzzy aggregation operator AND ($\mu_{A \wedge B} = \min\{\mu_A; \mu_B\}$) is applied to obtain a number that represents the result of this antecedent part.

- Example: Consider the indicator Ind_{Scarcity} for the characteristic Scarcity evaluated on the basis of 3 criteria: CR_{1,Scarcity}, CR_{2,Scarcity} and CR_{3,Scarcity}. The membership degrees of these 3 criteria to the term Medium in the antecedent part of the rule (CR_{1,Scarcity} is Medium to membership degree 0.70, the criterion CR_{2,Scarcity} is Medium(0.45) and the criterion CR_{3,Scarcity} is Medium(0.30)) are respectively 0.70, 0.45 et 0.30. Consider the following rule (See figure 7) where the different criteria are linked by a conjunction AND. In this case, the t-norme defining the fuzzy intersection is the Zadeh’s operator $\mu_{A_1 \wedge A_2}(x) = \mu_{A_1}(x) \wedge \mu_{A_2}(x) = \min(\mu_{A_1}(x), \mu_{A_2}(x))$ where the minimum of 3 values is taken into account.

- The third part consists in applying an implication method in order to carry out reasonings with fuzzy sets as the following logic statement:

  IF (CR_{1,Scarcity} is Medium) AND (CR_{2,Scarcity} is Very High) THEN (Ind_{Scarcity} is High).
The two fuzzy implications (Mamdani and Assilian)

Considering the statement **IF** ($x \in A$) **AND** ($y \in B$) **THEN** ($z \in C$), the difficulty lies in determining the fuzzy set of $z$ satisfying the implication, knowing $x$, $y$ and the membership functions of $A$, $B$ and $C$.

A fuzzy implication is a relation $\mathcal{R}$ between two sets $X$ and $Y$ qualifying the truth degree of the proposition: **IF** ($x \in A$) **THEN** ($y \in B$), where $A$ and $B$ are fuzzy subsets of $X$ and $Y$ respectively. The membership function $f_{\mathcal{R}}$ of this relation depends on the membership functions $f_A$ and $f_B$ of $A$ and $B$. The two fuzzy implications (Mamdani and Assilian) are the most used.

**Example:** Consider the following rule:

**IF** ($\text{CR}_1 \text{Scarcity is Medium}) \text{AND} (\text{CR}_2 \text{Scarcity is Medium}) \text{AND} (\text{CR}_3 \text{Scarcity is Medium}) \text{THEN} (\text{Ind}_\text{Scarcity is Medium}).$

The type of reasoning is: **IF** ($x \in A$) **AND** ($y \in B$) **AND** ($z \in C$) **THEN** ($w \in D$). In this case, the obtained fuzzy subset $S$ has the membership function $f_S(w) = f_{\mathcal{R}}(f_A(x) \times f_B(y) \times f_C(z)), f_D(w))$ where $f_{\mathcal{R}}$ is the membership function of a relation of fuzzy implication (here $\mathcal{R}_M$ is a fuzzy implication of Mamdani) and $\times$ a triangular norm (here the Zadeh’s conjunction operator: **AND**). The obtained result of the fuzzy implication for this type of rule is illustrated in **figure 8** and has the following membership function $f_S(w) = \min(\min(f_A(x), f_B(y), f_C(z)), f_D(w))$. 

Fig. 7. **AND** aggregation operator applied to the antecedent part of the rule.

Fig. 8. Implication operator **AND** to the consequent part of the rule.
- The **fourth part** consists of aggregating all the outputs of each rule. Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set. Consider a set of rules, \( P_1, P_2, \ldots, P_n \)

\[
\text{IF } (x \in A_1) \land (y \in B_1) \land (z \in C_1) \text{ THEN } (w \in D_1) \quad P_1
\]

\[
\text{IF } (x \in A_2) \land (y \in B_2) \land (z \in C_2) \text{ THEN } (w \in D_2) \quad P_2
\]

\[
\vdots
\]

\[
\text{IF } (x \in A_n) \land (y \in B_n) \land (z \in C_n) \text{ THEN } (w \in D_n) \quad P_n
\]

In order to determine the resulting fuzzy subset of the conjunction of these \( n \) rules, it is necessary to use an **aggregation** operator \( \Lambda \) allowing the synthesis of the solutions for each \( P_i \). The fuzzy subset \( S \) resulting from the rules is defined by the membership function:

\[
f_S(w) = \Lambda(f_{S_1}(w), f_{S_2}(w), \ldots, f_{S_n}(w)).
\]

The rules that are not concerned (for the \( j \text{th} \) rules if \( f_{A_j}(x) = f_{B_j}(y) = f_{C_j}(z) = 0 \) don’t have to be taken into account in the synthesis. But if \( f_{A_j}(x) = f_{B_j}(y) = f_{C_j}(z) = 0 \) implies \( f_{A_j}(x) \ast f_{B_j}(y) \ast f_{C_j}(z) = 0 \) and thus \( f_{S_j}(w) = f_R(0, f_{D_j}(w)). \) If \( R \) is a Mamdani’s implication, then \( f_{S_j}(w) = 0 \), otherwise \( f_{S_j}(w) = 1. \)

It is necessary to use as operator of aggregation \( \Lambda \) an operator admitting 0 like neutral for the first case and 1 for the other cases. We will use the operator **max** due to the choice of Mamdani’s implication. The method of Mamdani or called **min-max** will be used; its characteristics are the operator **min** as operator of conjunction \( \ast \) and the operator **max** as operator of aggregation \( \Lambda \). The fuzzy subset \( S \) resulting from the system of the \( n \) rules is defined by the membership function:

\[
f_S(w) = \max(f_{S_1}(w), f_{S_2}(w), \ldots, f_{S_n}(w)) \text{ where } f_{S_j}(w) = \min(f_{A_j}(x), f_{B_j}(y), f_{C_j}(z), f_{D_j}(w)).
\]

- **Example:** In the three following figures (figure 9, figure 10 and figure 11), three rules are illustrated in order to show how the output of each rule is combined, or aggregated, into a single fuzzy set with the Mamdani’s method using the operator **max** as aggregation operator \( \Lambda \).

**Rule 1:** IF (CR\(_{1, \text{Scarcity}}\) is Medium) AND (CR\(_{2, \text{Scarcity}}\) is Medium) AND (CR\(_{3, \text{Scarcity}}\) is Medium) THEN (Ind\(_{\text{Scarcity}}\) is Medium).

![Rule 1](image)

Fig. 9. Illustration of Rule 1.
The criterion CR<sub>1</sub>, Scarcity is Medium to degree 0.8, the criterion CR<sub>2</sub>, Scarcity is Medium to degree 0.5 and the criterion CR<sub>3</sub>, Scarcity is Medium to degree 0.3. The function of the indicator Ind<sub>Scarcity</sub> resulting of the fuzzy implication is presented by the figure 9 \((\min(0.8; 0.5; 0.3) = 0.3)\).

**Rule 2:** IF (CR<sub>1</sub>, Scarcity is High) AND (CR<sub>2</sub>, Scarcity is High) AND (CR<sub>3</sub>, Scarcity is High) THEN (Ind<sub>Scarcity</sub> is High).

The criterion CR<sub>1</sub>, Scarcity is High to degree 0.3, the criterion CR<sub>2</sub>, Scarcity is High to degree 0.6 and the criterion CR<sub>3</sub>, Scarcity is High to degree 0.9. The function of the indicator Ind<sub>Scarcity</sub> resulting from the fuzzy implication is presented by figure 10 \((\min(0.1; 0.6; 0.9) = 0.1)\).

![Rule 2](image)

Fig. 10. Illustration of Rule 2.

**Rule 3:** IF (CR<sub>1</sub>, Scarcity is Medium) AND (CR<sub>2</sub>, Scarcity is Medium) AND (CR<sub>3</sub>, Scarcity is High) THEN (Ind<sub>Scarcity</sub> is High).

The criterion CR<sub>1</sub>, Scarcity is Medium to degree 0.8, the criterion CR<sub>2</sub>, Scarcity is Medium to degree 0.5 and the criterion CR<sub>3</sub>, Scarcity is High to degree 0.9. The function of the indicator Ind<sub>Scarcity</sub> resulting of the fuzzy implication is presented by the figure 11 \((\min(0.8; 0.5; 0.9) = 0.5)\).

![Rule 3](image)

Fig. 11. Illustration of Rule 3.

Now that the membership functions of the fuzzy subset resulting from each rule has been determined; it is then necessary to determine the fuzzy subset resulting from the three rules by aggregating the various results with the operator \(\max\) as shown in (figure 12).

- The fifth part and last step consists of ”defuzzifying” the fuzzy set obtained previously. The most popular methods of ”defuzzification” are the center of gravity method and method of the maximum. Finding the best choice amongst all the ”defuzzification” methods cannot be accomplished in a systematic way.
However, since the maximum method has a major disadvantage of being highly sensitive, the center of gravity method is retained, as it is illustrated in figure 13. Indeed the obtained results are much more stable as reflected in the low variations of the fuzzy subset of figure 12.

Fig. 12. Fuzzy subset resulting from the 3 rules.

Fig. 13. "Defuzzification" of the fuzzy subset resulting from the 3 rules.

4.2 Evaluation of the Aggregated Competence Indicator - Level 2

The Level 2 consists of determining the Aggregated Competence Indicator ACI on the basis of Ind<sub>k</sub>.

Step 1 (Level 2): consists of "fuzzifying" the output values < Ind<sub>k</sub> > previously obtained in the Level 1 for each of the indicators Ind<sub>k</sub> where k ∈ {Value, Scarcity, Imitation, Replacement}. For each output of the Level 1 (each value < Ind<sub>k</sub> > of the indicator Ind<sub>k</sub>) we determine its membership degree for each fuzzy set by means of the membership functions of figure 14. The input is a numerical value defined on [0, 10]. The figure 14 shows, for an indicator Ind<sub>k</sub>, the result of its "fuzzification" through its membership function.

Step 2 (Level 2): This step follows the same procedure as explained in parts two, three and four of Step 4 (Level 1).

This step consists of defining an inference engine for the calculation of the "Criticality" of the competence. A competence, having the characteristics k ∈ {Value, Scarcity, Imitation, Replacement} with a certain degree, is evaluated with respect to the "Criticality" with different nature and intensity.
Using a predicate in the form of rules (based on fuzzy logic) makes it possible to describe gradual situations of "Criticality". However, a major disadvantage remains since there exists a certain empiricism in the choice of rules. The rules will be expressed as following:

\[ P_1 : \text{IF} \ (\text{Ind}_{\text{Value}} \in A_1) \ \text{AND} \ (\text{Ind}_{\text{Scarcity}} \in B_1) \ \text{AND} \ (\text{Ind}_{\text{Imitation}} \in C_1) \ \text{AND} \ (\text{Ind}_{\text{Replacement}} \in D_1) \ \text{THEN} \ (\text{ACI} \in E_1) \]

\[ P_2 : \text{IF} \ (\text{Ind}_{\text{Value}} \in A_2) \ \text{AND} \ (\text{Ind}_{\text{Scarcity}} \in B_2) \ \text{AND} \ (\text{Ind}_{\text{Imitation}} \in C_2) \ \text{AND} \ (\text{Ind}_{\text{Replacement}} \in D_2) \ \text{THEN} \ (\text{ACI} \in E_2) \]

\[ \ldots \]

\[ P_n : \text{IF} \ (\text{Ind}_{\text{Value}} \in A_n) \ \text{AND} \ (\text{Ind}_{\text{Scarcity}} \in B_n) \ \text{AND} \ (\text{Ind}_{\text{Imitation}} \in C_n) \ \text{AND} \ (\text{Ind}_{\text{Replacement}} \in D_n) \ \text{THEN} \ (\text{ACI} \in E_n) \]

where \( A_n, B_n, C_n, D_n, E_n \in \{\text{Very Low}, \text{Low}, \text{Medium}, \text{High}, \text{Very High}\} \)

**Step 3 (Level 2):** This step consists of the "defuzzification" of the fuzzy set previously obtained in Step 2 (Level 2). The input of the process of "defuzzification" is thus a set and the output is a numerical value.

- **Example:** In figure 15, the indicators \( \text{Ind}_{\text{Value}} \) and \( \text{Ind}_{\text{Scarcity}} \) are combined in a global indicator defining the "Criticality" of the competence called \( \text{ACI} \). The last fuzzy based engine block renders the mechanism:

  "IF (a competence is important (to a certain degree)) AND (and rare (to a certain degree)) THEN (its value encapsulated in the global indicator ACI is high (to a certain degree))"
5 Discussion

In the present work, the capability of expert fuzzy systems is used to efficiently map input variables (criteria for the resources) to output variables (indicators of the competence). Indeed, the proposed model does not require any explicit mathematical relationships between the input and output variables. Once the design of the fuzzy rule base has been automated, the remaining computation is mechanical and simple to carry out.

The proposed methodology requires the choice of various criteria (such as for the determination of the indicators) and of several parameters (such as for defining the membership functions). While this implies a significant initial effort, it also presents the advantage of flexibility. Moreover, the proposed fuzzy-based approach permits the incorporation and combination of various criteria and indicators, which may affect the "Criticality" of a competence. This allows for the definition of different evaluation points of view which is useful for management decisions.

The proposed approach could reveal some sensitivity with respect to the criteria, rules and parameters that must be defined (evaluation framework). This point has not been studied so far but should be considered in future work. However, within a given evaluation framework (for example within a single enterprise) the coherence and stability of the methodology is guaranteed.

6 Application to an industrial case

This section presents an industrial application of the approach for the evaluation of the competences.

The objectives is to illustrate the mechanism of evaluation of the required unit competence $C_{r_{cu}}(A)$ in an engineering process at a producer of railway vehicles. Only the non-material resources of the required unit competence are taken into account.
By applying the aggregated competence indicator ACI, the industrial case focuses on the calculation of the importance of the unit’s competences (and non-material resources mobilized) required by this engineering process. The results obtained will be analyzed for roughly 60 unit competences within the considered process.

6.1 Evaluation of a unit competence

Consider the activity \((A)\): **Assemble a front wall**. The unit competence required by the activity \((A)\): \(C_{cu}(A)\) is constituted of nine non-material resources such as following.

\[
C_{cu}(A) = \begin{cases} 
R_{NM}^{(A|R_1)} &: \text{To know the security rules} \\
R_{NM}^{(A|R_2)} &: \text{To know the alarm procedures} \\
R_{NM}^{(A|R_3)} &: \text{To know the techniques of assembly} \\
R_{NM}^{(A|R_4)} &: \text{To know the cold welding} \\
R_{NM}^{(A|R_5)} &: \text{To be able to read technical drawings} \\
R_{NM}^{(A|R_6)} &: \text{To aim at positioning a metal sheet} \\
R_{NM}^{(A|R_7)} &: \text{To know how to adjust and use a welding set} \\
R_{NM}^{(A|R_8)} &: \text{To know locksmith’s trade} \\
R_{NM}^{(A|R_9)} &: \text{To know how to use an overhead travelling crane} 
\end{cases}
\]

The evaluation methodology of unit competence \(C_{cu}(A)\) is based on the four following criteria:

1. **CR\(_1\),Value** = **Degree of Importance**
   This criterion is related to the concept of Value and indicates if the non-material resource is essential, necessary or desirable in the development of the competence.

2. **CR\(_1\),Scarcity** = **Difficulty of internal access**
   This criterion is related to the concept of Scarcity and estimates the time required to access the non-material resource within the enterprise.

3. **CR\(_1\),Replacement** = **Difficulty of external access**
   This criterion is related to the concept of Replacement and estimates the time required to access the non-material resource in the labour market.

4. **CR\(_1\),Imitation** = **Training duration**
   This criterion is related to the concept of Imitation and estimates the time required to acquire the non-material resource and make it operational.

These criteria constitute the measuring instruments of each indicator in order to obtain the aggregated competence indicator \(ACI\).
In this case, the indicator of Imitation “\( \text{Ind}_{\text{Imitation}} \)” is evaluated only on the basis of the criterion \( \text{CR}_{1,\text{Imitation}} \) describing the Training duration.

### 6.2 Evaluation of the indicator \( \text{Ind}_{\text{Imitation}} \)

This section demonstrates the methodology for the \( \text{Ind}_{\text{Imitation}} \) indicator. The other indicators will be estimated by following the same steps. The methodology for evaluating this indicator respects the different steps of Level 1.

**Step 1 (Level 1):** Consider the criterion \( \text{CR}_{1,\text{Imitation}} \): Training duration used for the estimation of the indicator \( \text{Ind}_{\text{Imitation}} \) for the Imitation competence. It is thus necessary for each linguistic term \{Very Low, Low, Medium, High, Very High\}, used to represent \( \text{CR}_{1,\text{Imitation}} \), to define typical values in order to build the specific membership function of this criterion. This membership function is shown in figure 16.

![Fig. 16. Membership function for the criterion CR\(_{1,\text{Imitation}}\).](image)

**Step 2 (Level 1):** This step consists of "fuzzifying" all values of the criterion \( \text{CR}_{1,\text{Imitation}} \) for each of the resources mobilized in the unit competence \( C'_{cu}(A) \) by using the membership function described in figure 16. The "fuzzified" values are given in table 2.

**Step 3 (Level 1):** This step consists of "defuzzifying" all the values previously obtained in order to determine for each of the resources \( i \ (i \in \{1, 2, ..., 9\}) \) a numerical value \( < \text{CR}_{1,\text{Imitation}} > \) (scale 0 to 10) of the criterion \( \text{CR}_{1,\text{Imitation}} \) by using the membership function described in figure 17 and the method of centroid as "defuzzification" model.

![Fig. 17. Membership function used for "Defuzzification".](image)
The value \( < \text{CR}_{1, \text{Imitation}} > \) of the criterion \( \text{CR}_{1, \text{Imitation}} \) for the unit competence \( C_{ru}^c(A) \) is obtained by calculating the average of each "defuzzified" value for each of the resources mobilized in this same competence (see equation 2).

The "defuzzified" values of the criterion \( \text{CR}_{1, \text{Imitation}} \) for each of the resources are presented in table 4.

<table>
<thead>
<tr>
<th>Resource</th>
<th>Description</th>
<th>Training duration</th>
<th>( &lt; \text{CR}_{1, \text{Imitation}} &gt; )</th>
<th>( &lt; \text{CR}_{1, \text{Imitation}} &gt; &lt;&lt;F&gt;&gt; )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>( R_{NM}^{r(A</td>
<td>R_1)} )</td>
<td>- 6</td>
<td>-</td>
</tr>
<tr>
<td>R2</td>
<td>( R_{NM}^{r(A</td>
<td>R_2)} )</td>
<td>- 6</td>
<td>-</td>
</tr>
<tr>
<td>R3</td>
<td>( R_{NM}^{r(A</td>
<td>R_3)} )</td>
<td>10 weeks</td>
<td>Medium(0.25)</td>
</tr>
<tr>
<td>R4</td>
<td>( R_{NM}^{r(A</td>
<td>R_4)} )</td>
<td>40 weeks</td>
<td>High(0.30)</td>
</tr>
<tr>
<td>R5</td>
<td>( R_{NM}^{r(A</td>
<td>R_5)} )</td>
<td>10 weeks</td>
<td>Medium(0.25)</td>
</tr>
<tr>
<td>R6</td>
<td>( R_{NM}^{r(A</td>
<td>R_6)} )</td>
<td>8 weeks</td>
<td>Medium(0.50)</td>
</tr>
<tr>
<td>R7</td>
<td>( R_{NM}^{r(A</td>
<td>R_7)} )</td>
<td>1 week</td>
<td>Very Low(1.00)</td>
</tr>
<tr>
<td>R8</td>
<td>( R_{NM}^{r(A</td>
<td>R_8)} )</td>
<td>50 weeks</td>
<td>High(0.05)</td>
</tr>
<tr>
<td>R9</td>
<td>( R_{NM}^{r(A</td>
<td>R_9)} )</td>
<td>1 week</td>
<td>Very Low(1.00)</td>
</tr>
</tbody>
</table>

Table 2
"Fuzzified" values for the criterion \( \text{CR}_{1, \text{Imitation}} \) for each of the resources mobilized in the unit competence \( C_{ru}^c(A) \).

The indicator \( \text{Ind}_{\text{Imitation}} \) is estimated on the basis of the single criterion \( \text{CR}_{1, \text{Imitation}} \). Consequently, \( < \text{Ind}_{\text{Imitation}} > \) is equal to \( < \text{CR}_{1, \text{Imitation}} > \). Using equation 2 for \( n = 9 \), the value of the indicator \( < \text{Ind}_{\text{Imitation}} > = 5.39 \) (see table 3). Note that following the proposed approach, there is no Step 4 since the indicator is based on the single criterion \( \text{CR}_{1, \text{Imitation}} \).

\(^5<<F>>\) indicates that the value of the criterion \( \text{CR}_{1, \text{Imitation}} \) for each of the resources is "Fuzzified"

\(^6\) - The resource cannot be evaluated by the criterion \( \text{CR}_{1, \text{Imitation}} \).
Value of Indimitation for the unit competence $C_{cu}^r(A)$

<table>
<thead>
<tr>
<th>Competence</th>
<th>Training duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{cu}^r(A)$</td>
<td>$\sum_{i=1}^{n} \frac{&lt;CR_{imitation}^i &gt; \cdot X_i}{\sum_{i=1}^{n} X_i}$</td>
</tr>
</tbody>
</table>

Table 3
Value of the criterion $CR_{1,Imitation}$ and the indicator $Ind_{imitation}$ for $C_{cu}^r(A)$.

6.3 Estimation of the other criteria $CR_{1,Value}$, $CR_{1,Scarcity}$ and $CR_{1,Replacement}$

The estimation of the other criteria $CR_{1,Value}$ (Degree of importance), $CR_{1,Scarcity}$ (Difficulty of internal access) and $CR_{1,Replacement}$ (Difficulty of external access) is based on the same methodology by using equation 2 for each characteristic.

<table>
<thead>
<tr>
<th>Indicator: Indimitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Res. Des.</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$R_1$</td>
</tr>
<tr>
<td>$R_2$</td>
</tr>
<tr>
<td>$R_3$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$R_4$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$R_5$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$R_6$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$R_7$</td>
</tr>
<tr>
<td>$R_8$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$R_9$</td>
</tr>
</tbody>
</table>

Table 4
"Defuzzified" values for the criterion $CR_{1,Imitation}$ for each of the resources mobilized in the unit competence $C_{cu}^r(A)$.

$^7 <<D>>$ indicates that the value of the criterion $CR_{1,Imitation}$ for each resource is "Defuzzified"
The numerical values of the indicators are given in the **table 5**. These are used to estimate the aggregated competence indicator (ACI).

<table>
<thead>
<tr>
<th>Results for the 4 indicators for the unit competence $C^r_{cu}(A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comp.</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>$C^r_{cu}(A)$</td>
</tr>
</tbody>
</table>

Table 5
Results obtained for the 4 indicators for the unit competence $C^r_{cu}(A)$.

6.4 **Evaluation of the aggregated competence indicator: ACI**

The evaluation of the aggregated indicator of the competence ACI follows the steps described in section 4.2. It consists of combining the four values of the indicators $<\text{Ind}_{\text{Value}}>$, $<\text{Ind}_{\text{Scarcity}}>$, $<\text{Ind}_{\text{Replacement}}>$ and $<\text{Ind}_{\text{Imitation}}>$ (**table 5**), based on a fuzzy inference engine.

**Step 1 (Level 2):** This step consists in ”fuzzifying” the four values of the indicators $<\text{Ind}_{\text{Value}}>$, $<\text{Ind}_{\text{Scarcity}}>$, $<\text{Ind}_{\text{Replacement}}>$ and $<\text{Ind}_{\text{Imitation}}>$ for the competence $C^r_{cu}(A)$. For each indicator value in **table 5**, it is necessary to determine their degree of membership in each fuzzy set described in **figure 18**. The obtained results after ”fuzzification” of the different indicators are given in **table 6**.

![Membership function for the indicators Ind_{Value}, Ind_{Scarcity}, Ind_{Replacement} and Ind_{Imitation}](image)

**Fig. 18.** Membership function for the indicators Ind_{Value}, Ind_{Scarcity}, Ind_{Replacement} and Ind_{Imitation}
Results of "Fuzzification" of the 4 indicators $\text{Ind}_k$

<table>
<thead>
<tr>
<th>Indicator</th>
<th>$&lt; \text{Ind}_k&gt;$</th>
<th>$&lt; \text{Ind}_k&gt;$ &lt;&lt;F&gt;&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Ind}_{\text{Value}}$</td>
<td>6.41</td>
<td>Medium (0.15)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>High (0.85)</td>
</tr>
<tr>
<td>$\text{Ind}_{\text{Scarcity}}$</td>
<td>4.85</td>
<td>Low (0.09)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Medium (0.91)</td>
</tr>
<tr>
<td>$\text{Ind}_{\text{Replacement}}$</td>
<td>3.56</td>
<td>Low (0.86)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Medium (0.14)</td>
</tr>
<tr>
<td>$\text{Ind}_{\text{Imitation}}$</td>
<td>5.39</td>
<td>Medium (0.77)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>High (0.23)</td>
</tr>
</tbody>
</table>

Table 6
"Fuzzified" values of the 4 indicators $\text{Ind}_k$ for the unit competence $C_{cu}^r(A)$.

Step 2 (Level 2): This step consists of defining an inference engine based on rules for the calculation of the aggregated competence indicator $\text{ACI}$. The inferences link the obtained values of the four indicators to the output variable by linguistic rules. In this case, the fuzzy system has four inputs $\text{Ind}_{\text{Value}}$, $\text{Ind}_{\text{Scarcity}}$, $\text{Ind}_{\text{Replacement}}$ and $\text{Ind}_{\text{Imitation}}$ transformed into linguistic variables in the previous step and one output ($\text{ACI}$). Each variable (4 inputs and 1 output), subdivided in five subsets Very Low, Low, Medium, High, Very High according to figure 18, is linked by the following type of linguistic rules:

\[
\begin{align*}
\text{IF} & \quad (\text{Ind}_{\text{Value}}) = \text{Very Low} \\
\text{AND} & \quad (\text{Ind}_{\text{Scarcity}}) = \text{Very Low} \\
\text{AND} & \quad (\text{Ind}_{\text{Replacement}}) = \text{Very Low} \\
\text{AND} & \quad (\text{Ind}_{\text{Imitation}}) = \text{Very Low} \quad \text{THEN ACI} = \text{Very Low} \\
\downarrow \\
\text{IF} & \quad (\text{Ind}_{\text{Value}}) = \text{Very High} \\
\text{AND} & \quad (\text{Ind}_{\text{Scarcity}}) = \text{Very High} \\
\text{AND} & \quad (\text{Ind}_{\text{Replacement}}) = \text{Very High} \\
\text{AND} & \quad (\text{Ind}_{\text{Imitation}}) = \text{Very High} \quad \text{THEN ACI} = \text{Very High}
\end{align*}
\]
Assuming that the two indicators (Ind\textsubscript{Valeur}) and (Ind\textsubscript{Scarcity}) have the same influence on the level of "Criticality" of the competence according to the indicator ACI, the inference matrix given in the table 7 presents the rules linking these two indicators to the output ACI.

<table>
<thead>
<tr>
<th>(Ind\textsubscript{Scarcity})</th>
<th>Very Low</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
<th>Very High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Low</td>
<td>V_Low</td>
<td>V_Low</td>
<td>Low</td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td>Low</td>
<td>V_Low</td>
<td>Low</td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
</tr>
<tr>
<td>Medium</td>
<td>Low</td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>High</td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
<td>High</td>
<td>V_High</td>
</tr>
<tr>
<td>Very High</td>
<td>Medium</td>
<td>High</td>
<td>High</td>
<td>V_High</td>
<td>V_High</td>
</tr>
</tbody>
</table>

Table 7
Example of inference matrix linking (Ind\textsubscript{Value}) and (Ind\textsubscript{Scarcity}) to (ACI).

Step 3 (Level 2): This step consists of the "defuzzification" of the fuzzy set previously obtained by following the activation rules (rules connecting the four indicators Ind\textsubscript{k} and the indicator ACI) from the inference engine, for each "fuzzified" value of < Ind\textsubscript{k} >. In this case, the single fuzzy set obtained after treatment of the results of table 6 is illustrated by the figure 19.

![Fig. 19. Fuzzy set obtained by the application of the rules connecting the four Ind\textsubscript{k} and the indicator ACI.](image-url)
The result of the aggregated competence indicator (ACI) obtained for the unit competence $C_{cu}(A)$ is given in table 8.

<table>
<thead>
<tr>
<th>Competence</th>
<th>ACI</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{cu}(A)$</td>
<td>Centroid</td>
<td>5.60</td>
</tr>
</tbody>
</table>

Table 8
Value of the aggregated competence indicator (ACI) for $C_{cu}(A)$.

7 Analysis of results

The results of the "Criticality" study for some 60 unit competences mobilized in engineering processes are presented. This "Criticality" is evaluated on the basis of the Aggregated Competence Indicator where the calculation rests on the four indicators $Ind_{Value}$, $Ind_{Scarcity}$, $Ind_{Replacement}$ and $Ind_{Imitation}$. The goal of this analysis is to identify and analyse the origin of the "Criticality" for the competences stemming from the characteristics of the non-material resources. The results can be illustrated by the distribution of the unit competences according to their "Criticality" in a two dimensional diagram $Ind_k$ versus ACI (see figure 20). In this diagram, the relative position of a unit competence with respect to the main diagonal gives an indication on the origin of the "Criticality" (given by the ACI). If it is below the diagonal, the contribution of $Ind_k$ to the competence "Criticality" (ACI) is relatively weak; If it is above the diagonal, it is relatively strong.

![Diagram](image)

Fig. 20. Unit competence distribution on the two dimensional diagram, $Ind_k$ versus ACI.
The figure 21(a) and 21(b) show that the "Criticality" is mainly due to the dominating characteristics "degree of importance" and "training duration" in this case. On the contrary, the figure 21(c) shows that "scarcity" has a relatively weak contribution. Finally, in the figure 21(d) we can observe that the difficulty of external access has a balanced contribution to ACI.

![Diagram](image)

Fig. 21. Indk versus ACI of the studied process.

The most critical unit competences (with a high ACI) are located on the right side of figure 21. The previous analysis indicates in particular a strong contribution of the "degree of importance" (IndValue) to the ACI. It can be explained by the technical knowledge usually required in engineering activities.

The contribution of the "training duration" (IndImitation) on the ACI of the most critical competences is high (see figure 21b). It can therefore be concluded that "training" is a key issue for this enterprise.

For the most critical competences, the "Criticality" appears not to be dominated by the contribution of a specific indicator but rather due to a similar contribution of all of them.
In superposing the four figures 22(a to d), we obtain figure 22 (ACI represented in X-coordinate, Ind_k represented in Y-coordinate). In this figure, a competence is represented by four symbols (full triangle, empty triangle, empty square, star) distributed vertically.

![Figure 22](image)

**Fig. 22.** Influence of the different indicators on the ACI.

Let consider 2 unit competences UC1 and UC2 on the figure 22:

- **UC1**: Superposition of ▲, △, □ and ◆: It can be noticed from the analysis of figure 22 that the "Criticality" of the unit competence UC1 is mainly due to the "importance" of the resources which it mobilizes (Ind_value), to the "difficulty of external access" to these resources (Ind_replacement), to the "difficulty of internal access" to these same resources (Ind_scarcity) and to the "training duration" (Ind_imitation).

- **UC2**: Superposition of ▲ and of ◆: In a similar way, it can be noticed that the "Criticality" of the unit competence UC2 is due to the "importance" of the resources which it mobilizes (Ind_value) and to the fact that these resources require long "training durations" (Ind_imitation).
8 Conclusion

The approach proposed in this paper constitutes a first attempt for a generic description of the mechanisms for a quantitative evaluation of competences. The methodology is useful for modeling different aspects and interactions that add values to the competence. Linguistic variables normally make it extremely difficult to quantify these aspects. The proposed methodology permits the integration of linguistic variables and their transformation into numerical ones that can then be used for a quantitative determination of the competence criticality. Moreover, the proposed fuzzy-based methodology is flexible enough to incorporate additional criteria combined in multiple ways, which may affect the evaluation of the competence. Thus, it will allow decision makers to understand and analyse, from different points of view, the criticality of each competence. The use of a fuzzy-based approach in the field of competence evaluation seems a promising field of investigation. Moreover a comparison between different enterprises, using non identical evaluation frameworks, should be pursued.

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References


