Fractal research of pathological tissue images

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Abstract

A novel texture detecting analysis for medical pathological tissue images was developed by fractal Brown model. According to fractal Brown random field model, a discrete fractal random field for image texture detection on a definite scale could be derived from the Brown model. Using fractal dimensions in partial region of the image and gray difference between adjacent pixels and relevant Hurst coefficient, a new medical image texture detecting analysis that could reflect the image feature was brought forward. The detecting results had been acquired. The results indicate that the texture detecting method by the Brown model is remarkable.

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1. Introduction

Generally, some of medical images are very vague because of certain pathological situations and objective reasons, which bring a definite difficulty to diagnose. It is needed to enhance the edges and textures of the image, so that doctors can easily analyze diseases from these images.

Distinction between texture detection and edge detection is that edge detection usually uses differential operator, which needs pixel gray to have leap change, while texture detection only needs difference in gray value.

With the development of fractal theory, it becomes the focus of recent research to medical image processing and recognition. The thesis is based on premise that discrete fractal random model derived from fractal theory and statistical feature of image texture on a definite scale satisfy the Brown model condition. The whole missions could be into three steps:

(1) Using fractal dimensions to detect texture of image partial region and having three definitions.

(2) In accordance with gray between relevant Hurst coefficient of fractal dimensions and adjacent pixels, the texture of detected image can be reconstructed to reflect texture roughness and intensity information.

(3) When calculating Hurst coefficient, the best scheme could be chosen by algorithm complicacy with detecting effect while moving rectangular window in different scales.

Comparing with other texture detecting methods, such as variance-based, absolute difference-based, and information entropy-based technique, fractal texture detection contains fractal dimension that has significant relativity with human’s roughness sense. The larger fractal dimension indicates more roughness of the image surface and vice versa. Therefore, fractal function can be used to describe texture model and fractal dimension can also reflect the characteristic of image texture.

On the other hand, current fractal researches mostly focused on theoretic discussions and few investigations had applied it on computer-based image fractal texture research. Hence, our investigation defined original value image, based on which texture detecting images with texture intensity, roughness, and self-similarity information were studied.

2. Fractal Brown model of image analysis

2.1. Fractal Brown random field model

An example of statistical fraction is fractal Brown movement, which is a non-smooth random procedure. Mandelbrot
put forward that Brown movement could describe a procedure phenomenon that was of self-similitude, which uses fractal Brown movement in describing spatial distributive random, fractal Brown random can be acquired.

**Definition 1.** Provided that $H$ satisfies $0 < H < 1$, $b_0$ is an arbitrary real number. If random value satisfies:

\[
\begin{align*}
B_H(0, \omega) &= b_0 \\
B_H(s, \omega) &= \frac{1}{\Gamma(H + 0.5)} \int_{-\infty}^{\infty} [t - s]^{-H - 0.5} - [(-s)]^{-H - 0.5} dB(s, \omega) + \int_0^t (t - s)^{-H - 0.5} dB(s, \omega)
\end{align*}
\]

Then $B_H(s, \omega)$ is called fractal Brown movement (FBM), where $H$ is Hurst coefficient, $b_0$ the initial value, and $H = 1/2$ is common Brown movement.

**Definition 2.** Suppose $B_H(t)$ is Gauss random field, while $0 < H < 1$, arbitrary $t$ and $\Delta t$ satisfy:

\[
Pr \left( \frac{|B_H(t + \Delta t) - B_H(t)|}{\Delta t^{0.5}} < y \right) = F(y)
\]

$B_H(t)$ is called isotropy FBRF, where $F(y)$ is zero average Gauss random variable distributive function, $Pr()$ the probability measurement, and $||$ expresses pattern number. Fractal Brown random field $B_H(t)$ has character as follows:

\[
\begin{align*}
E[|B_H(t + \Delta t) - B_H(t)|] &= E[B_H(t + 1) - B_H(t)] || \Delta t ||^H, \\
E[|B_H(t + \Delta t) - B_H(t)|^2] &= E[B_H(t + 1) - B_H(t)]^2 || \Delta t ||^{2H}
\end{align*}
\]

where $E$ is a pattern number, Pentland [2] has proved that gray images reflected by most natural scenery surface belonged to FBRF in a certain scale and FBRF is served as descriptive model of nature scenery image.

**Definition 3.** If $t$ and $\Delta t$ take discrete value of $n$ and $m$, respectively, then $B_H(n) - B_H(m)$ is called discrete fractal Brown random (DFBR) field.

It is known from above definition that fractal Brown random field is non-smooth, and corresponding discrete DFBR increment is of statistical smooth self-similitude which DFBR field satisfies:

\[
\begin{align*}
E[|B_H(n + m) - B_H(n)|] &= E[|B_H(n + v) - B_H(n)|] \\
\cdot ||m||^H, \\
E[|B_H(n + m) - B_H(n)|^2] &= E[|B_H(n + v) - B_H(n)|^2] \cdot ||m||^{2H}
\end{align*}
\]

where $v$ is a vector. It can be seen from above formula that one-order and two-order absolute matrix are isotropy in any direction. DFBR field model is an effective model that describes natural scenery self-similitude, whose local statistic feature can effectively coincide with local statistical character of the image region. Consequently, DFBR is used to describe mathematic model of the image region, and parameter $H$ can express self-similitude of same image region (i.e. symmetrical degree of gray surface), and fractal dimension $D$, which corresponds to gray surface of the image region, can be obtained from the parameter $H$:

\[
D = D_T + 1 - H
\]

where $D_T$ is the topology dimension of the image region, $D_T = 2$.

**2.2. Definition of parameter $H$ and estimation**

Gray surface of the image region satisfies DFBR field model, and suppose $I(x_0, y_0)$ expresses gray value at $(x_0, y_0)$ of the image, we can get the property of DFBR field model as follows:

\[
E[|I(x, y) - I(x_0, y_0)|] = E[|I(x_1, y_1) - I(x_0, y_0)|] || \Delta r^H
\]

where $\Delta r = \sqrt{(x - x_0)^2 + (y - y_0)^2}$ and $\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} = 1$.

If defining:

\[
r = \sqrt{(x - x_0)^2 + (y - y_0)^2}, \quad \Delta I(r) = |I(x, y) - I(x_0, y_0)|
\]

Then the above formula can be rewritten as follows:

\[
E[\Delta I(r)] = E[\Delta I(1)] r^H, \quad r > 1
\]

Taking the logarithm for two sides, we can get:

\[
H(r) = \frac{\log E[\Delta I(r)] - \log E[\Delta I(1)]}{\log(r)}
\]

We can know from the definition and the character of DFBR field model that DFBR field is of the smooth course and satisfies the average ergodicity, which gives:

\[
\langle \Delta I(r) \rangle = \frac{1}{N_r} \sum_{r=1}^{N_r} \Delta I(r) = E[\Delta I(r)]
\]

where $N_r$ is the number of pixels whose distance to point $(x_0, y_0)$ is $r$. Therefore, formula (7) can be rewritten as follows:

\[
H(r) = \frac{\log(1/N_r) \sum_{r=1}^{N_r} |I(x, y) - I(x_0, y_0)|}{\log(r)} \\
H(r) = \frac{\log(1/N_r) \sum_{r=1}^{N_r} |I(x, y) - I(x_0, y_0)|}{\log(r)}
\]

Parameter $H$ can be directly calculated from formula (9). As this calculating procedure needs to adjudge distance in turn, hence rectangular region moving window $n \times n$ can be set. When calculating $H$ value of gray difference between center point and pixel point within the window $n \times n$ is calculated. When $n$ is large, the important details would miss and computation cost increases; and when $n$ is small, the ability of anti-disturbance decreases. In normal case, $n$ is set to 3, 5, or 7.
3. Fractal detecting of image texture

3.1. Estimating fractal number of the image

A box dimension algorithm is used to calculate fractal dimension number here. The general idea is to transform two-dimension gray image into a gray surface of three-dimension embedded space. The core of the algorithm is to count \( N(\varepsilon) \), and the dimensions of cubic box (with each side of \( \varepsilon \) ) needed to overlay the whole image gray surface, then fractal dimension number to be calculated out in accordance with Eq. (10) \([3,4]\) is

\[
\log N(\varepsilon) = \log a - D \log \varepsilon
\]

where \( a \) is the proportion coefficient and \( \varepsilon \) is measuring scale.

For each side \( \varepsilon \) of the cubic box, the formula of calculating total box number \( N(\varepsilon) \) is as follows:

\[
N(\varepsilon) = \sum_{i=1}^{A} \sum_{j=1}^{B} n(i, j)
\]

where \( n(i, j) \) is the gray surface, \( A \) and \( B \) are respectively the side length, \( i \in \{1, 2, \ldots, A\}\), \( j \in \{1, 2, \ldots, B\}\).

Fig. 1 shows the pathological original image of gastric tubular adenocarcinoma selected to process. The pixel matrix dimension is \( 350 \times 500 \), and it comes from common medical teaching pictures. As a typical example, it is not used to extract edges of the image as usual, but we use it to accomplish the experiment.

According to the formula of calculating the total box number (Eq. (11)), processing results to pathological original image is shown as Table 1.

Calculated by minimized two multiple imitating of \( \log N(\varepsilon) \sim \log(\varepsilon) \), the fractal dimension number of the image is 2.2479.

Although fractal dimension number of the image is not directly used to detect texture, it could reflect toughness of the image. Because both pathological tissue and normal tissue of the image have different fractal dimension number, fractal dimension number could be served as one of medical diagnosis basis \([5,6]\).

<table>
<thead>
<tr>
<th>( \varepsilon )</th>
<th>( N(\varepsilon) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>3,371,551</td>
</tr>
<tr>
<td>1.4</td>
<td>1,649,845</td>
</tr>
<tr>
<td>1.8</td>
<td>933,129</td>
</tr>
<tr>
<td>2.2</td>
<td>590,013</td>
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<tr>
<td>2.6</td>
<td>401,288</td>
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<tr>
<td>3.0</td>
<td>286,848</td>
</tr>
</tbody>
</table>

3.2. Criterion of fractal dimension number of texture detection

DFBR field model is an effective model describing nature scenery self-similitude, whose local statistical feature can effectively coincide with local statistical character of the image region. Thereby, from fractal theory it is known that the actual fractal dimensions should be between 2 and 3. Due to \( D = D_r + 1 - H \), \( D_r \) expresses that topology dimension of the image is 2. For every pixel of the image, if it is located within image texture, the corresponding parameter estimation \( H \) should be between 0 and 1, i.e. \( 0 < H < 1 \); conversely, if the pixel is located outside of texture or located at edges of texture, the consistency of the DFBR field would be damaged, therefore the corresponding parameter value \( H \) would overstep the theory range, i.e. \( H \leq 0 \) or \( H \geq 1 \) \([7]\).

Thus, corresponding criterion of detecting texture is: when \( 0 < H < 1 \), the point \((x, y)\) is the pixel within texture image, so the point is reserved; and when \( H \leq 0 \) or \( H \geq 1 \), the point \((x, y)\) is outside the texture image, and is cancelled. The detecting result of the criterion is expressed by binary image as follows:

- When \( 0 < H < 1 \), the point \((x, y)\) is black, and gray scale is 128.
- When \( H \leq 0 \) or \( H \geq 1 \), the point \((x, y)\) is white, and gray scale is 255.

The stripes of texture are already detected in binary image. To represent the intensity of texture, and to display more clearly the details of texture, it is also needed to endue corresponding gray values to the point of the texture, as detected texture result is expressed in the form of 256 grades of gray image value.

An easier way is adopting corresponding point of gray value in original image. The reason is that texture image and original image have the same size. There is a mapping between corresponding pixels of the two images. Therefore, we only need to replace black pixels of binary image with corresponding pixels gray value of original image. This image is called “original value image”.

The original value image uses different gray values to describe texture, and detecting effect is improved much more than binary image. In addition to the representing stripes of texture, many thin textures can be distinguished. Though the original value image can reflect the stripes of texture, it could not reflect intensity and roughness of texture, and it cannot distinguish the different self-similitude of texture. Hence, Section 4 of the thesis is experimental texture detecting result in accor-

Fig. 1. A pathological original image of gastric tubular adenocarcinoma.
dance with intensity information of texture, roughness of texture and self-similitude in turn.

4. Experimental results and analysis

4.1. Detecting results of original value image and texture intensity information

Intensity information of texture is related to difference of adjacent gray values. The more different gray values are, the stronger corresponding texture information is. Inversely, the less different gray values are, the weaker corresponding texture information is. To describe intensity information better, different colors can be used to distinguish different gray value [8]. The stronger texture can be expressed by darker color, and the weaker texture can be expressed by brighter color.

While getting $H$, the expressions would be used as follows:

$$\log \left( \frac{1}{N_r} \sum_{r} |I(x, y) - I(x_0 - y_0)| \right) \sim \log(r)$$

(12)

where symbol ‘$\sim$’ expresses logarithm conjunction between $H$ and $r$.

As point data are reconstructed by least-square method, the slope of the fitted line expresses parameter $H$, and the cutting distance of reconstructed beeline reflects gray difference of center point to adjacent point. Consequently, the cutting distance of reconstructed rectilinear can be transferred to gray value of texture image pixel points [9] to represent intensity information of texture.

Suppose the cutting distance is $B$, the gray degrees of the image are expressed by integers from 0 to 255, and the shifts between $B$ and gray values of the pixels are

$$G(x, y) = \left\lfloor \frac{B(x, y) - \min(B)}{\max(B) - \min(B)} \times 255 \right\rfloor$$

(13)

where $B(x, y)$ is the $B$ value of the point$(x, y)$ of the original image, $\max(B)$ and $\min(B)$ express, respectively, maximum and minimum of whole $B$ values, $G(x, y)$ is gray value at point$(x, y)$ of texture image.

Due to the distribution of $B$ values, the contrast of the image derived directly from formula (13) is not great, so result images need to be enhanced through histogram equalization. When taking $7 \times 7$ template, the original value image and texture intensity information image are shown as Fig. 2(a) and (b).

4.2. Detecting result of texture roughness and self-similitude

Besides intensity information, texture has roughness information and self-similitude information, which reflect stripes of the texture image [10]. Roughness and self-similitude of the image are related to fractal dimension of the local region. Since textures with the same roughness and self-similitude information have the same fractal dimension, another method to construct texture image is mapping roughness or self-similitude correspond to gray degree, i.e. using fractal dimension numbers to acquire gray values. Each pixel point in the image has accordingly parameter $H$, and there is a direct relation between parameter $H$ and fractal dimension: $H = 3 - D$, thus parameter $H$ can be used to correspond to gray value of pixel points in texture image, and express the region of similar $H$ value with the same gray level. Different texture may have the same fractal dimension, therefore the same gray level may reflect to more than one texture.

Since parameter $H$ would be taken range from 0 to 1, correspondingly gray grades of the image are arranged from 0 to 255, the converting between parameter $H$ and pixel gray value can be calculated by formula as follows:

$$G(x, y) = \lfloor H(x, y) \times 256 \rfloor$$

(14)

where $H(x, y)$ is the parameter $H$ of the point$(x, y)$ in the original image, $[x]$ expresses integer which is not larger than $x$, and $G(x, y)$ is the gray value of point$(x, y)$ in the texture image. While taking $7 \times 7$ rectangular template, the detecting image representing texture roughness and self-similitude are shown as Fig. 3.

In Fig. 3, the textures that have similar roughness are described by similar gray values, so it is not as clear as Fig. 2. The reason is that fractal dimension values of the textures are relatively more concentrated and most fractal dimension numbers of the texture are not too much different from each other. But it
can detect tiny textures, and the textures in regions where gray value changes slowly in the original image are also detected. In all, the detecting accuracy is higher than Fig. 2(b).

This is because of the same kind texture adopting the same gray value, so it makes interlaced different textures easy to distinguish, especially for the same texture. However, when these textures are detected, the noise of the image also makes much more disturbance to the detected results, especially in the center region of the image the stripes of the texture are hardly distinguished from the noise.

Since DFBR field model is based on gray difference from center point to surrounding pixel point, it is considered that gray surface of the same image region have self-similitude in statistics level. While detecting texture, any region in line with DFBR field model is considered texture region of the image. Since noise existing in the image often has the same size with fine texture, although it is irregular, it probably has self-similitude opposite to the surrounding. Therefore, while detecting fine textures, noise would have significant influence on image.

4.3. Influence of different size rectangular moving template to detecting texture

As solving parameter \(H\), we need to set an \(n \times n\) moving template (\(n\) is an odd number more than 1) and to calculate the average value of the absolute values of the gray difference between a pixel inside the template and the center pixel. While \(n\) is taken larger, many important details would be lost and calculating cost would increase; and while \(n\) is taken smaller, it would make bad anti-disturbance. Because \(n\) affects the result profoundly, the choice of \(n\) properly is very important in texture detecting.

It is known from the calculating of \(H\) that for an \(n \times n\) rectangular window the cost of calculating gray difference value is \(n \times n - 1\). If \(n = 9\), calculating every \(H\) value would need to calculate 80 pairs gray difference value, which computing amount are considerable. Computing procedure is not only complex, but also it would spend extra plenty of time. It is not advisable. Therefore choosing \(n = 3, 5, 7\) is better decisions. Figs. 4 and 5 respectively give various texture detecting result images of \(3 \times 3\) and \(5 \times 5\) rectangular moving template for comparison.

From above results we can see: the smaller the scale is, the detecting is more sensitive, but it is more disturbed by noise. The larger the scale is, the stronger anti-disturbance is gained, but many details would be lost and calculation is more complex. Besides, in judging criterion of edge crossing point, the choice of gray threshold would also influence the texture detecting results.
5. Conclusion

The thesis used fractal theory in image texture detecting, and used discrete fractal Brown random field model to detect medical image textures, and then texture roughness and self-similitude have acquired. By processing image to original image, different texture edges have been enhanced. Comparing with other texture detecting method, this could provide a new way and thread [11,12].

Because fractal texture detection tends to be disturbed by noise, many fine textures cannot be distinguished unclearly. Although Gauss low pass filter could reduce the noise in the image, at the same time it would also smooth the images. We need to design a special noise filter and an enhance method aiming at the image fractal feature to improve the effect of fractal texture detection. This needs further analysis research.

6. Summary

With the development of fractal theory and the contribution of medical images in pathological diagnosis, the applied research of fractal theory in the field of medical image analysis and identification becomes an important study subject in recent years. According to the theory of fractional Brownian random (FBR) field model, we designed an effective model algorithm, which is used to detect the texture of medical pathological tissue image. On the basis of discrete fractional Brown random field model which is derived from FBR field model and the precondition that statistic texture of image in accord with this model in proper scale, texture detecting results of the original value, the intensity information, the roughness and self-similitude are extracted according to the Hurst coefficient reflecting the local region fractal dimension of image and the difference of gray degree of adjoining pixels. Comparing the detecting effect with different scales of rectangle moving window, we estimate corresponding Hurst coefficient and the complexity of algorithm among these, to choose the optimal scale of the moving window. We found that original image should be enhanced to improve the detecting effect before the fractal method applied.

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References


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