

## Buckling-induced smart applications: recent advances and trends

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## Topical Review

# Buckling-induced smart applications: recent advances and trends

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Received 23 November 2014, revised 5 March 2015

Accepted for publication 24 March 2015

Published 18 May 2015



CrossMark

### Abstract

A paradigm shift has emerged over the last decade pointing to an exciting research area dealing with the harnessing of elastic structural instabilities for ‘smart’ purposes in a variety of venues. Among the different types of unstable responses, buckling is a phenomenon that has been known for centuries, and yet it is generally avoided through special design modifications. Increasing interest in the design of smart devices and mechanical systems has identified buckling and postbuckling response as a favorable behavior. The objective of this topical review is to showcase the recent advances in buckling-induced smart applications and to explain why buckling responses have certain advantages and are especially suitable for these particular applications. Interesting prototypes in terms of structural forms and material uses associated with these applications are summarized. Finally, this review identifies potential research avenues and emerging trends for using buckling and other elastic instabilities for future innovations.

Keywords: buckling, snapping, elastic instability, smart structures, smart materials

(Some figures may appear in colour only in the online journal)

## 1. Introduction

The study of structural elastic stability [1, 2], a field considered among the most mature in mechanics, has been experiencing a paradigm shift over the last decade in terms of rekindling the popularity of the study of elastic instabilities (EIs) of many kinds [3], such as snapping, buckling, wrinkling, crumpling, phase transitions, and cavitation. Among these, buckling is a phenomenon that has been known for centuries—ever since an equation to determine the critical buckling of a column was derived by Leonhard Euler. Such a phenomenon is generally avoided through special design modifications, but new and emerging applications consider such behavior to be favorable. Recent studies of buckling, and postbuckling in general, have tried to transform this effect from a negative into a positive. A research field has thus emerged to harness such elastic unstable events for their potential contribution to ‘smart’ purposes. It is a highly multidisciplinary problem that includes elements of applied

mathematics, biology, material science, mechanics, physics, engineering, sustainability, etc. The emergence of a research community studying elastic instabilities from this new perspective was showcased in a special issue of *Soft Matter* [4]. Buckling-induced instabilities are thus being increasingly explored as a desirable opportunity, and a new journal called *Extreme Mechanics Letters* featured studies of elastic instabilities in natural and engineering systems in its launch issue [5].

An overview of this topic has not been written yet, and it is worthwhile to review and showcase this interesting and still not fully exposed research trend. The main objective of this review is therefore to present a broad perspective of research efforts during the past decade with respect to buckling-induced smart applications and their prototypes. Section 2 discusses a number of major applications by way of explaining why buckling responses have certain advantages that fit their purposes. Interesting prototypes in terms of structural forms and material uses associated with buckling-



Figure 1. Research publications on the use of elastic instability for different purposes.

induced applications are summarized in sections 3 and 4, respectively. Section 5 identifies potential research avenues and emerging trends for using the buckling response in future innovations.

## 2. Why buckling? smart applications

For centuries, buckling has been a major concern in the design of all slender structural elements and systems due to the resultant capacity reduction, associated large deformations, and catastrophic failure. The structural response beyond the first bifurcation, i.e., the postbuckling response, usually attracts interest only in terms of residual capacity but is otherwise not given much attention and is thought to be of little practical use in most cases. This is because it is difficult to predict and control due to its high sensitivity to initial conditions, namely, geometric imperfections. Other than load-carrying capacity, two encouraging features of many buckling phenomena are high-rate motion and sudden energy release. Within the elastic region, structures are capable of snapping from their initial shape into a buckled shape accompanied by a significant amount of energy release from the system, represented as a load drop in the response curve. These two features make buckling an ideal mechanism for adaptive and smart applications. Increasing studies are demonstrating the use of such behavior across disciplines, which can be divided into two main categories: energy-related and motion-related applications (see figure 1).

### 2.1. Energy-related applications

Energy-related applications can be further categorized into energy production and energy dissipation. Energy released from buckling events can be used in the design of energy harvesters and sensors for micro-electromechanical systems (MEMS), whereas the energy dissipated by buckling events can be useful for the design of absorbers, dampers, stabilizers, and isolators.

Buckling events have been shown to be a promising source for nonlinear energy harvesters by featuring the generation of high-frequency impulses over a large frequency interval, self-tuning features, and the capability of adapting to variable acceleration levels [6]. Energy harvesting (EH) is an attractive technique for a wide variety of self-powered microsystems. Compared with macro EH technology (such as the kW or MW level of power generated by energy plants), micro EH technology focuses on the development of alternative solutions for the conventional battery [7]. The source for the EH technology can come from mechanical, thermal, frictional, photic, chemical, or biological sources, and the corresponding harvested power ranges from mW to  $\mu$ W. Energy harvesting from mechanical vibration has gained considerable interest in recent years by converting mechanical energy to electrical energy [8]. A smart energy harvester should be able to respond with large-amplitude motions to increase power generation as well as EH efficiency. The harnessing of buckling events offers advantages in the design of an EH device because the nonlinear behavior can be excited at high-energy orbits from low-frequency broadband vibrations at which linear harvesters are usually only weakly excited. Buckling events explored for use in nonlinear harvesters include the snap-through mechanism in laterally loaded clamped-clamped beams and bistability in thin laminated composite plates. In these applications it has also been found that the excitation frequency of a harvester becomes less sensitive to frequency changes compared with linear harvesters, such as a cantilever excited at its natural frequency [9]. Such buckling-induced prototypes have proved suitable for many MEMS applications because nonlinear response can broaden their application bandwidth, modify device performance, and control resonant frequencies.

The benefit of buckling events has been the catalyst for much recent research into MEMS applications. With advances in integrated circuit technology, a wide range of MEMS applications has been discovered and the production costs have been significantly reduced [10]. The use of buckling-

induced response has been demonstrated to be reliable and capable of delivering useful power to MEMS devices. However, MEMS devices are susceptible to the well-known ‘pull-in’ instability if the applied electrostatic voltage reaches a critical level. Since it is widely accepted that ‘pull-in’ instability is unavoidable, an increasing number of studies have explored the feasibility of using ‘pull-in’ instability to design special sensors and actuators, as pointed out in a recent review [11]. More examples and prototypes of buckling-induced MEMS systems are presented in section 3.

Increasing research activities have focused on the development of wireless sensors and sensor networks through the use of buckling and snapping for structural health monitoring (SHM) and non-destructive evaluation (NDE). A comprehensive review [12] has shown that researchers began to explore self-powered wireless sensors because power operation based on batteries was a major drawback. The emerging concept of wireless sensors with actuation interfaces is an important step in the evolution of wireless sensor technology because it closes the gap between traditional SHM and NDE methods. Recent efforts have demonstrated the capability of active sensing due to the actuation from buckling events in different structural types such as wires [13], beams [14], strips [15], and cylinders [16]. The actuation provides the interaction between the wireless sensors and the system which they are attached to or embedded in. With advances in smart materials, such as piezoelectric materials, power harvesting (conversion to electrical energy) becomes possible. Buckling events provide the function of modifying the input to smart materials by serving as a trigger. Under this concept, active sensors will prove to be a promising technology in the future.

Promising endeavors in the use of buckling events for energy dissipation have also been conducted, but to a lesser extent than for energy production purposes. A major reason is that postbuckling response is usually associated with failure. Yet recent studies have explored the possibility of taking advantage of such a response for the design of absorbers [17–19], dampers [20, 21], and isolators [22–25]. It should be noted that in all of these applications the structural elements suffering buckling and postbuckling response are in the elastic response region. Plastic buckling along with progressive failure mechanisms are outside the scope of this review.

## 2.2. Motion-related applications

The high rate of motion associated with snap-through buckling is another promising feature that has been investigated to develop novel design concepts for actuators and micro-optical switching in MEMS devices. Small perturbations can generate sudden snapping behavior in elastic elements that enables the structure to dynamically change its configuration. The onset of the snap-buckling transition can provide actuation output in terms of out-of-plane displacements, which provide a new route for designing materials and structures at multiple scales with switchable functionalities, morphogenesis, etc. The key merit of using snap buckling is that local

and global buckling (which in some cases may happen together) in structural elements can reduce the actuation force during shape recovery, and in most cases no additional stabilizing element is required during the transformation between the equilibrium states. Such findings are included in a number of recent reviews of morphology in materials and structures. Kuder *et al* [26] showed the increasing interest in using buckling-induced response in terms of multistability to design morphing features in materials and devices. Friedman *et al* [27] showed that snap-through instabilities can lead to the development of self-deploying or self-locking structures depending on the stability of their packed configuration. Huang *et al* [28] discussed switching in polymeric materials and their structures due to instability/collapse phenomena in their shape changes. More buckling-induced mechanisms and structures are expected to be explored with advances in novel fabrication technologies.

## 3. Structural prototypes

The use of buckling and snapping events is diverse, and this section summarizes various manifestations of structural prototypes related to recent smart applications that have been designed and experimentally investigated. Based on their configuration, structural prototypes can be categorized into classic forms and emerging forms. From a geometrical point of view, the buckling phenomenon can be found in all slender members (as shown in table 1), including one-dimensional members (arches, beams, columns, rods, wires, etc), two-dimensional members (membranes, plates, shells, etc), and three-dimensional members (cylinders, tubes, spheres, etc). Some notes of interest regarding these structural forms are that (1) laterally loaded beams and bistable plates have been thoroughly studied for the purposes of energy harvesting; (2) cylinders/columns/tubes have been heavily investigated for energy dissipation purposes; (3) 3D forms are more favorable for use in motion-based applications such as morphing structures, deployable structures, and adaptive structures; and (4) novel prototypes have been proposed under each form type to address multiple potential applications. The following section highlights some of the examples listed in table 1 to identify the changing role of some classic buckling phenomena and to present emerging efforts regarding new forms with a view to potential applications.

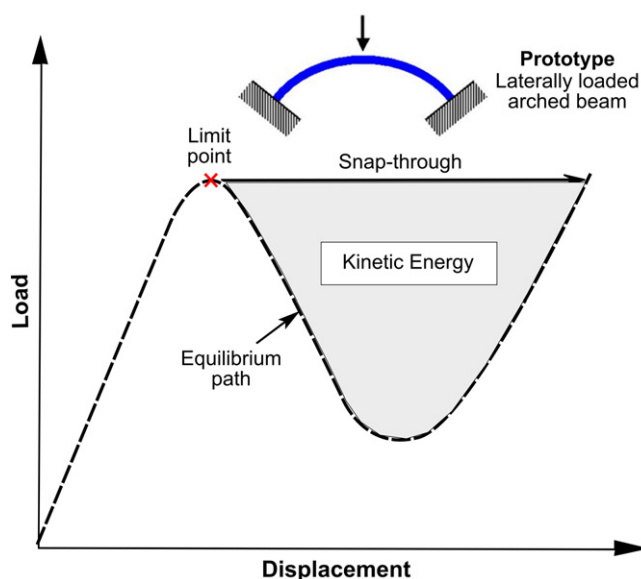
### 3.1. Classic forms for energy-related applications

The loss of stability in a structure depends on the prescribed form and its geometrical imperfections. In most cases, snap-through buckling occurs at a limit point rather than at a bifurcation point. Figure 2 shows a classic structure that is known to exhibit unstable behavior once it is subjected to external loading. A laterally loaded beam/arch/strip is the simplest 1D structural form reported in energy-related applications [9, 14, 29, 75, 77, 130–132]. This phenomenon is familiar to engineers and has only one limit point on the initial loading path. Physically, external loading at the critical state

**Table 1.** Existing structural prototypes and their use in buckling-induced applications.

	1D forms		2D forms		3D forms			Hybrid	Review
	Arch beam strip	Bar rod wire	Plate sheet surface	Membrane	Cylinder column tube	Sphere	Origami		
Proposed prototype	[29, 30]	[31–33]	[34–38]	[39, 40]	[41, 42]	[43, 44]	[45]	[46]	
<b>Energy-based application</b>									
Energy harvester	[15, 47–52]		[53, 54] [9] <sup>a</sup>	[55]	[56, 57]			[58]	[6, 9, 59]
Sensor	[14, 60]	[13]	[61]		[16]			[62]	
Damper Absorber	[18, 19]	[13]			[20, 21, 63–67] [17, 68] <sup>a</sup>			[69]	
Isolator			[70]		[22, 23]			[24, 71, 72]	
<b>Motion-based application</b>									
Actuator	[73–80]		[81–86]	[87–91]	[92, 93]	[94]		[95]	
Adaptive structure	[96]		[97–99]	[100]					[101, 102]
Deployable structure	[103]		[104]		[105]	[106]	[107–110]	[111, 112]	[27]
Morphing structure	[96, 113]	[114]	[100, 115–120]	[100]	[121, 122]	[122–125]	[126–128]	[111]	[26, 28, 129]

<sup>a</sup> Topical review of a specific structural form for a single application.



**Figure 2.** Schematic snap-through buckling of a laterally loaded arch beam.

leads to snap-buckling along the loading direction. Another common structural form for energy harvesting is the bistable panel/plate. Snap-through buckling in a bistable system can also cause large-amplitude motion due to its transition from one stable state to another, which can significantly improve the efficiency of energy harvesters. Compared with a laterally loaded beam, the snap-through behavior of a bistable plate depends on both its bending and its in-plane stiffness. A recent review [9] highlights the increasing body of literature on energy harvesting from bistable plates. Buckling in 3D forms (such as cylinders and spheres) has been far less studied for energy harvesting purposes. One of the reasons is that postbuckling behavior in cylinders and spheres is usually an asymmetric bifurcation response that exhibits random behavior. Predicting and controlling such behavior is more difficult due to its high imperfection sensitivity.

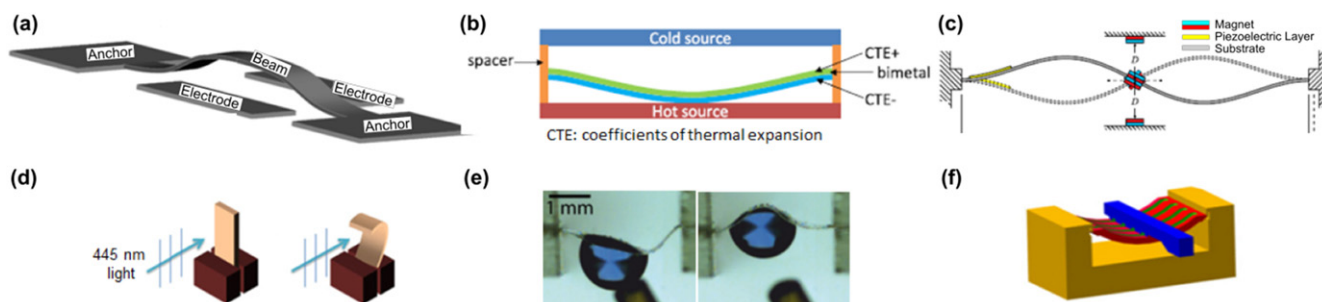
Many recent studies [35, 49, 51, 52, 54, 74, 76, 79, 121, 133] have explored various techniques to trigger snap buckling triggered by non-mechanical loading for the development of MEMS devices or energy harvesters (see figure 3). Figure 3(a) shows snap-through buckling of a curved double-clamped beam from the electrostatic actuation of a fringing field engendered by symmetrically located electrodes [76]. Figure 3(b) demonstrates a semi-flexible strip made of 115  $\mu\text{m}$ -thick curved bimetals designed to snap at 47  $^{\circ}\text{C}$  and to snap back at 42.5  $^{\circ}\text{C}$ , whereby a hysteretic thermal gradient can trigger snap-through behavior between the two positions [51]. Figure 3(c) displays a steel buckled beam subjected to a magnetic levitation system repelled by top and bottom magnets to trigger the second buckling mode and thus enhance energy harvesting at low frequency and under small-excitation conditions [52]. Two classes of azobenzene-functionalized polymers (figure 3(d)) have been investigated with a view to designing contactless photo-initiated snap-through

events in bistable arches, leading to orders-of-magnitude enhancement in the actuation rates ( $\sim 102 \text{ mm s}^{-1}$ ) and powers ( $\sim 1 \text{ kW m}^{-3}$ ) under moderate irradiation intensities ( $\ll 100 \text{ mW cm}^{-2}$ ) [78]. For micro-device design, capillarity-induced snapping of elastic beams and solvent-induced snapping of a beam (see figures 3(e)–(f)) are reported in [74] and [79], respectively. However, one of the noted drawbacks in these systems is that the external source may not always be available and/or suitable for applications at different scales. For example, it is difficult to integrate permanent magnets at the nanoscale, and magnetic fields can strongly interact with other components in electronic devices [48]. In addition to triggering from external sources, structural prototypes using smart materials offer the features of self-tuning and self-adapting; more discussion of this aspect of device development is presented in section 4.

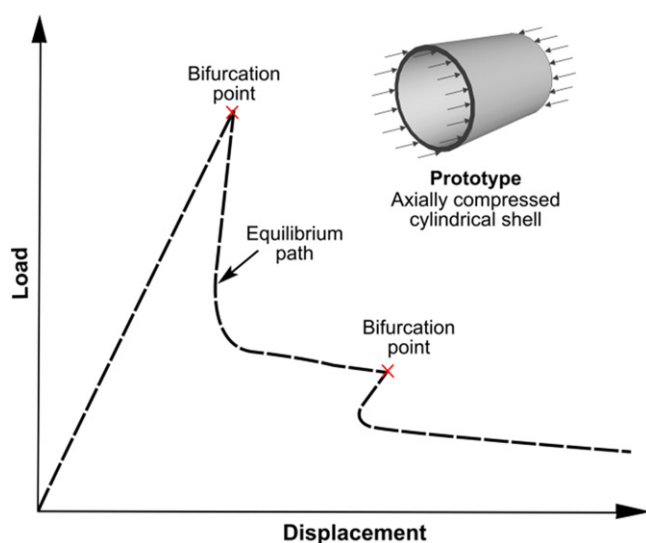
Another well-documented form class with postbuckling behavior is axially loaded structures. Such prototypes have not been extensively studied for energy harvesting purposes, but they have been used for energy dissipation devices such as absorbers, dampers, and isolators. For these devices, 3D forms, particularly cylinders and tubes, are more frequently used as a structural prototype than 1D and 2D forms. Figure 4 shows the load-deformation curve of an axially compressed cylindrical shell, which exhibits a postbuckling response with multiple bifurcation points (also termed mode transitions) due to changes in the deformed geometry after each critical point. Cylindrical shells can attain their postbuckling regime due to the natural transverse deformation restraint provided by their geometry. Dealing with such complex nonlinear behavior has proved to be a challenging task due to high imperfection sensitivity, and thus this prototype form suffers from practical limitations in the design of thin-walled structures. Three-dimensional prototypes may therefore gain more attention if the static and dynamic response from mode branch switching in their postbuckling response can be modified and tailored.

Many experimental and numerical studies have provided valuable knowledge with respect to characterizing the elastic postbuckling response of axially loaded structures. With increased understanding of response in the postbuckling regime, a wide variety of structural forms have been explored for energy dissipation purposes. For example, the postbuckling response of axially compressed cylindrical shells (figure 5(a)) can occur at a lower energy level, and efforts to characterize this response experimentally and numerically have been motivated by the interest of using the residual strength as a safeguard [63, 65, 134]. A recent study [69] may inspire using a group of cylinders connected by ligaments for use as an energy absorber (figure 5(b)). The bifurcation and snap-through collapse of a variety of thin-walled structures can be found in a comprehensive review [68]. An inspiring study [22] has investigated several vertical vibration isolation designs by using Euler buckling springs (figure 5(c)). A recent endeavor [70] uses a composite bistable plate as part of a vibration isolator subjected to harmonic base excitation (figure 5(d)). Another interesting feature in prototypes displaying buckling events is that they give rise to negative stiffness, which can potentially be used for the design of





**Figure 3.** Snap-through buckling of arch/beam forms triggered by various sources other than mechanical loading: from (a) electrode-induced (reprinted from [76], copyright (2011), with permission from Springer), (b) thermal-induced (reprinted from [51], copyright (2013), with permission from IOP Publishing), (c) magnetic-induced (reprinted from [52], copyright (2013), with permission from AIP Publishing), (d) photo-induced (reprinted from [78], copyright (2013), with permission from PNAS), (e) capillarity-induced (reprinted from [74], copyright (2013), with permission from IOP Publishing), and (f) solvent-induced (reprinted from [79], copyright (2013), with permission from the American Physical Society).



**Figure 4.** Schematic postbuckling response of an axially compressed cylinder.

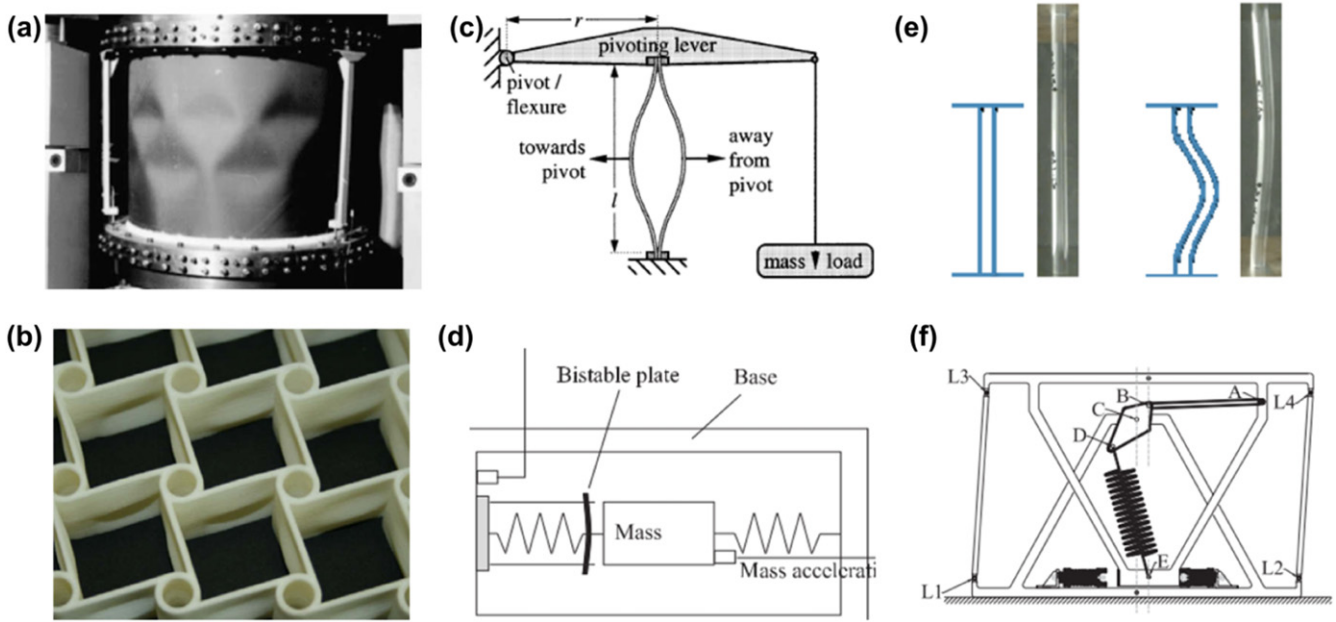
advanced dampers. For example, an interesting damping system (figure 5(e)) was designed with negative stiffness and large hysteresis by using the snap buckling of a column with flat ends [20]. A negative stiffness device using snap buckling under a similar framework was recently attempted for use as seismic protection for structures (figure 5(f)) [71, 72]. Similar buckling-induced responses in various structural prototypes can also be used as isolators.

### 3.2. Classic forms for motion-related applications

One of the key features of snap buckling is its actuation capacity. Many classic buckling phenomena have been incorporated into the design of energy harvesters and MEMS devices. Figure 6(a) shows a widely used example of using an arch-shaped beam as an actuator [73]. In this case, a magnetic field normal to the chip surface creates a Lorentz force to trigger snap-through buckling of the arch-shaped leaf spring. Similar snapping behavior can also be found in many plate designs [129] for adaptive and morphing structures

(figure 6(b)). Brinkmeyer *et al* [100] explored the snap-through buckling of an isotropic spherical dome to design morphing structures with pseudo-bistable behavior (meaning that snap buckling occurs slowly due to the use of a viscoelastic material). This dome can recover its original state without further actuation due to the use of a macro polymer composite (figure 6(c)).

Nature has long made use of elastic instabilities for functionality, and this has generated an important venue for research over the last decade. For example, a recent study [135] found that the mosquito fascicle is laterally supported throughout the penetration process such that the Euler buckling load is increased by a factor of 6. The results showed the importance of lateral supports for modifying the stability of elastic structures, which is similar to man-made laterally constrained systems. Within the subject of this review, bio-inspired concepts and biomimetics have been used in the development of structures and mechanisms with two major features: snapping and folding. With inspiration from the Venus flytrap's fast closure [136], a biomimetic responsive surface was developed that can snap from one curvature to another [81], a hingeless flapping device (figure 6(d)) was developed and patented under the name Flectofin [82], and a flytrap-inspired robot and novel actuation mechanism were designed [137]. Other recent biomimetic lessons on snapping behavior include the design of an initially flat pod valve that turns into a helix, which was inspired from the mechanical process of seed pods opening in *Bauhinia variegata* [138], and the development of a dragonfly-inspired flapping strip with piezoelectric patches [113]. Folding is another interesting mechanism that can be useful for the design of deployable and morphing structures. Leaf-folding patterns have been investigated to create new fold patterns for thin membrane structures [39]; a multistable composite cylindrical lattice structure (figure 6(e)) was designed to mimic the bistable behavior of the virus bacteriophage T4 [139]; and the design of a bistable cell (figure 6(f)) capable of reversibly unfolding from a flat configuration to a highly textured configuration was inspired by natural systems through origami design principles [110].



**Figure 5.** Harnessing buckling-induced behavior for different energy dissipation purposes. (a) Postbuckling of a CFRP cylindrical shell (reprinted from [63], copyright (2003), with permission from Elsevier). (b) Buckling of a cylinder group connected by ligaments (reprinted from [69], copyright (2010), with permission from Elsevier). (c) Buckling of a column used for an isolator (reprinted from [22], copyright (2002), with permission from Elsevier). (d) Bistable plate used as an isolator (reprinted from [70], copyright (2013), with permission from Elsevier). (e) Buckling of a column designed for a damper (reprinted from [20], copyright (2013), with permission from Elsevier). (f) Spring-based negative stiffness device for seismic protection (reprinted from [71, 72], copyright (2013), with permission from ASCE).

### 3.3. Exploration of emerging forms

Beyond the extensively studied classic forms, structural form-finding for new prototypes are part of a parallel research venue for harnessing snap-through buckling for potential applications. Buckling-induced applications have sparked advancements in the development of novel forms that can attain elastic instabilities. A recent review [33] concluded that many interesting problems within the topic of snap-through instability remain to be investigated, including (1) the mathematical complications in modeling buckling and post-buckling in thin structures, (2) the mechanical instability of materials associated with inhomogeneity and nonlinearity, (3) new phenomena due to coupling between geometric and material nonlinearities, and (4) the usefulness of mechanical instabilities for broad engineering applications.

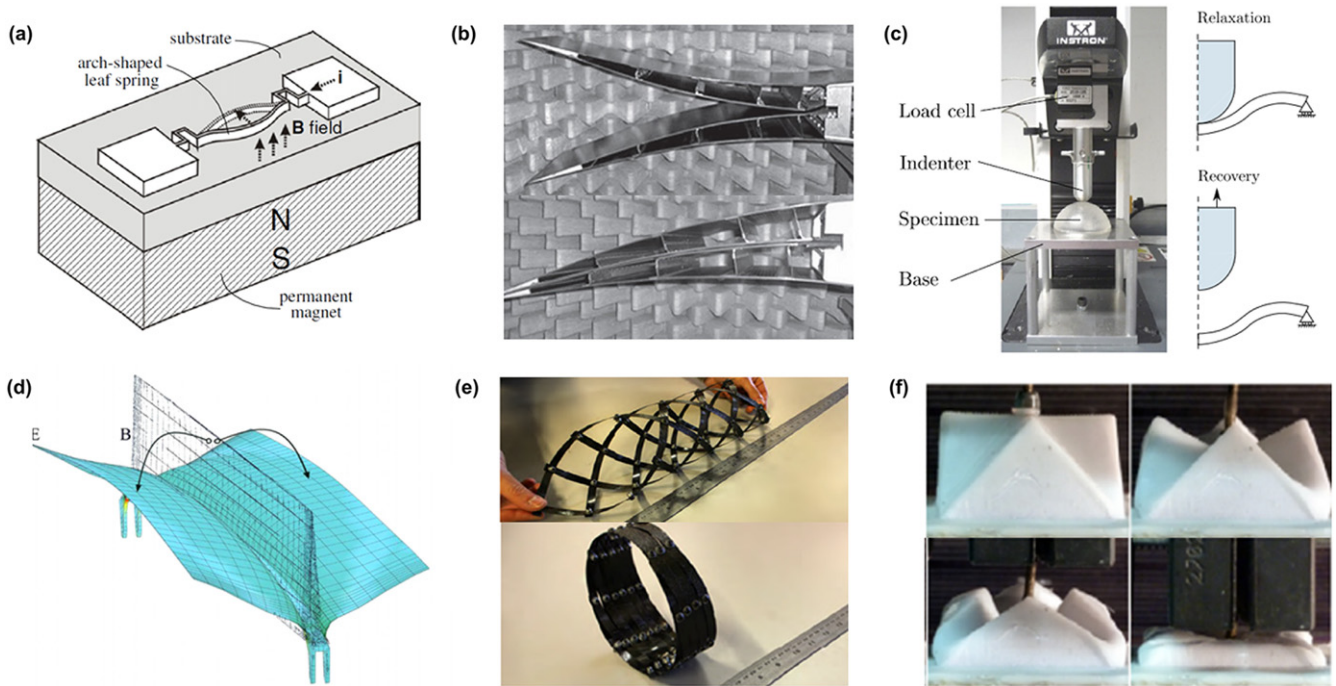
The first step for using a specific unstable event is to identify the desirable features of a given response. Recent contributions within this arena have considered elastic instabilities in a variety of structural forms, ranging from 1D to 3D, with respect to the opportunity and feasibility of using them as structural prototypes, as shown in figure 7. Featured examples include slackening of a twisted thin rod [31] (figure 7(a)), buckling of an elastic planar rod penetrating into a sliding sleeve [37], snapping of an elastic arched beam inspired by a popper [140], snapping of a simple stretched bi-strip of elastomers [114], the buckling and folding of over-curved rings [126], shrinking and buckling of thin sheets with non-Euclidean metrics [43, 123], buckling of a planar sheet with a negative-curvature liquid interface [141], the 3D shape of a sheet with a series of prescribed concentric curved folds

[45], multistability in spontaneous helical ribbons [30] (figure 7(b)), the postbuckling of a thin cylindrical shell under torsional loading to fold to a flat 2D form [107] figure 7(c)), buckling-induced encapsulation of a spherical shell patterned with a regular array of circular voids [106] (figure 7(d)), the secondary buckling instability of an initially spherical elastic capsule [44], periodic beam lattices [142], and periodic porous structures [143].

## 4. Material uses

This section summarizes various endeavors regarding buckling-induced applications from a material point of view. A recent review [28] divided materials for morphing application into two categories: having the capability of shape change and having the capability of shape memory. A material may have these two features separately or simultaneously. It can be seen from section 3 that buckling response in either energy-based or motion-based applications can be triggered suddenly, in an elastic manner, or gradually, in a viscous-elastic fashion. When an external stimulus is not available a smart structure should be able to snap back to the original shape with the use of shape-memory materials. Table 2 summarizes the research efforts with respect to various material uses for smart purposes. The applications involving shape changing materials and shape-memory materials are presented in the following subsections, whereas elastomers and graphene are discussed in section 5.2.





**Figure 6.** Harnessing snap-buckling instabilities for motion-based purposes. (a) Snap-through buckling of beam for actuator (reprinted from [73], copyright (2004), with permission from IOP Publishing). (b) Snapping behavior of adaptive morphing trailing edge (reprinted from [129], copyright (2013), with permission from John Wiley and Sons). (c) Snap-through buckling of a dome with self-recovery ability (reprinted from [100], copyright (2012), with permission from Elsevier). (d) A biomimetic design of a hingeless flapping device using snap buckling (reprinted from [82], copyright (2012), with permission from IOP Publishing). (e) Multistable cylindrical lattices inspired from the bistable behavior of a virus (reprinted from [139], copyright (2013), with permission from Elsevier). (f) Cell deployable by using snap-buckling behavior (reprinted from [110], copyright (2013), with permission from IOP Publishing).

#### 4.1. Shape changing materials

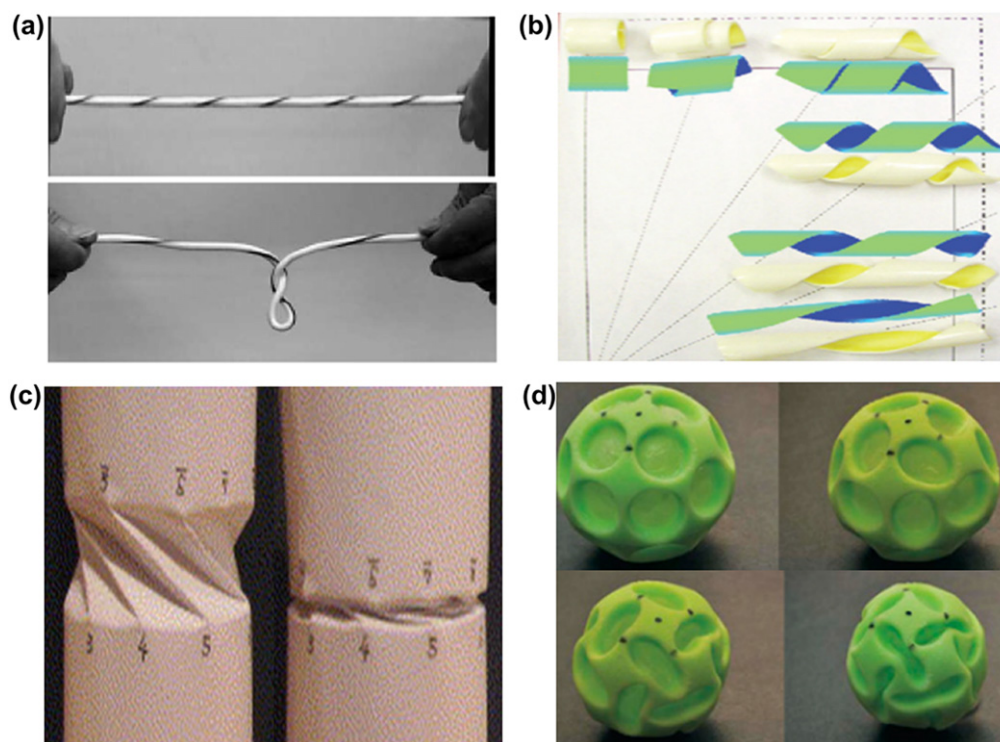
The four major reported materials with the capability of shape change are metals (including metalloids), fiber-reinforced composites, polymers, and piezoelectric materials. Conventional metals (such as steel and aluminum) are still a material option for many motion-based applications. The major drawbacks of these isotropic materials are their lack of property tailoring features and that they are usually used as a base material requiring the aid of some shape-memory material for smart applications. Research studies of bistable shells [171] have proved that, for a given configuration, a structure made from isotropic materials has only one stable configuration, whereas it can have two stable configurations by introducing material anisotropy.

Fiber-reinforced composites have been widely used for smart structures due to the possibility of varying their stiffness via selection of the individual ply material properties, fiber orientation, and laminate stacking sequence. Although anisotropic coupling effects are known to reduce the buckling capacity of many structural forms and are generally avoided through special laminate designs, their influence has been seen as favorable for new and emerging smart applications like the many pursued in this review. Figure 8 showcases four examples using composites for different smart purposes. Winkelmann [18] explored an energy absorbing mechanism by using composite bistable strips. The waiting links, made of ultra-high molecular weight polyethylene (figure 8(a)), were

designed to provide a redundant load path and provide a higher ductility to the mechanism through snap-buckling behavior. Shaw *et al* [70] experimentally investigated a passive vibration isolator with the concept of high static stiffness but low dynamic stiffness via the snap-buckling behavior of a composite bistable plate (figure 8(b)). The nonlinear behavior was shown to support substantial load and to significantly reduce the natural frequency of the system. Figure 8(c) presents an experimental study of the tristable response of a doubly curved orthotropic shell, and it shows the use of curvature and anisotropy to achieve multistability [98]. Lachenal *et al* [101] investigated an adaptive structural concept using a multistable twisting grid of carbon fiber reinforced plastic (CFRP) strips (figure 8(d)). More interesting studies of the bistability and multistability of composite structures for morphing and adaptive application can be found in the review by Daynes and Weaver [102].

The potential of polymeric materials for smart applications is similar to that of fiber-reinforced composites, particularly in the development of micro devices and mechanisms. Two major types of polymers used for buckling-induced applications are glassy polymers and rubbery polymers. A comprehensive review of instability phenomena in polymeric materials and their structures can be found in [28].

Finally, piezoelectric materials have received the most attention for buckling-induced applications due to their ability to directly convert applied strain energy to usable electric energy, particularly for integration into microscale devices.



**Figure 7.** Explorations of structural prototypes for smart applications. (a) Twisting buckling of a thin rod (reprinted from [31], copyright (2005), with permission from Springer). (b) Multistability in a helical ribbon (reprinted from [30], copyright (2014), with permission from AIP Publishing). (c) Postbuckling of an origami-inspired cylinder (reprinted from [107], copyright (2005), with permission from Elsevier). (d) Buckling-induced deployable sphere, Buckliball (reprinted from [106], copyright (2012), with permission from PNAS).

Several reviews [7, 59, 149, 150] have shown that the high compliance associated with piezoelectric materials is attractive for the design of energy harvesters, actuators, and self-powered sensors. Another interesting research area is the active control of snap-buckling instabilities in structures through piezoelectric actuator-induced loading. Similar concepts have been explored for various structural prototypes, and they have shown the possibility of instability control in columns [172, 173] and beams [131, 174]. Further discussion of the hybrid use of piezoelectric materials with other materials for buckling-induced applications is presented in section 5.

#### 4.2. Shape-memory materials

As discussed in section 3, self-adapting and self-recovering are preferable features for buckling-induced applications so that a structure can have snap-back behavior after snap buckling. Shape-memory alloys (SMAs) are one of the well-known candidates among many smart materials for this purpose. The shape recovery of SMAs can be induced by either a stress or a temperature change, which results in transformation between its two main phases due to its different crystal structure. It has been reported that SMAs can regain their original shape after being deformed well beyond 6–8% strain and that magnetic shape-memory alloys (MSMAs) can demonstrate reversible strain levels up to even 10% [151]. SMAs have a number of unique properties that make them appealing for a wide variety of applications in the biomedical

and aerospace industries, including large energy dissipation capacity, good fatigue and corrosion resistance, large damping capacity, and availability in many configurations [146, 155]. Advances in and challenges of using SMAs can be found in a recent review paper [168]. Another promising candidate for buckling-induced applications is shape-memory polymers (SMPs). Compared with thermal-induced and mechanical-induced SMAs, SMPs can also be activated by electricity, light, moisture, and even certain chemical stimulus. The shape recovering capacity of SMPs can reach up to 400% in recoverable strain, and more important, they cost less than SMAs. More smart applications using SMPs can be found in two recent comprehensive reviews [166, 167].

## 5. Discussion: research trends

Previous sections have shown four major aspects of the efforts regarding buckling-induced applications (see figure 9), including application purposes, application scales, structural prototypes, and material prototypes. This section presents emerging trends within these four aspects in harnessing buckling and other elastic instabilities.

### 5.1. In applications

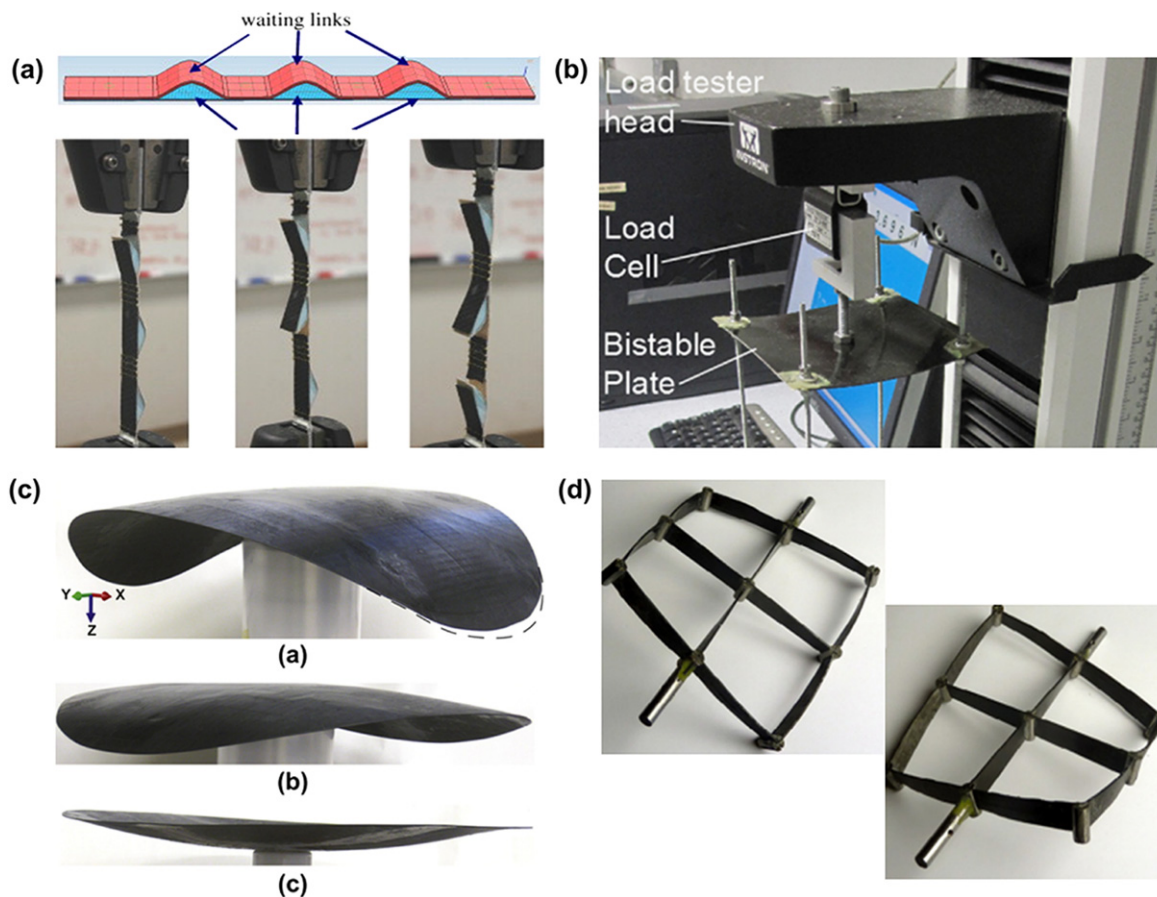
The spectrum of smart applications covered in section 2 has profiled the increasing interest in buckling responses, and the ongoing trend is to use the fundamental aspects behind

**Table 2.** Harnessing snap-through instabilities for smart purposes by material use.

	Shape changing materials				Emerging soft materials		
	Metals	Fiber-reinforced composites	Polymers	Piezoelectrics	Shape-memory materials	Elastomers	Graphene
Actuator	[73, 76, 87] [77] <sup>b</sup>	[83, 85, 86, 117, 118]	[74, 78, 79, 100]	[144, 145]	[84] [146] <sup>a</sup>	[89, 91, 147, 148]	[90]
Sensor	[62]	[13]	[14]	[60, 144]			
Energy harvester	[49, 51, 133]	[15, 53, 61]	[54, 58]	[47, 48, 50, 52, 56, 57], [7, 59, 149, 150] <sup>a</sup>	[151, 152]	[55, 148, 153, 154]	
Absorber Damper Isolator	[20, 69]	[63, 65, 70, 134]	[21]	[21]	[155] <sup>a</sup>		
Morphing structure	[42, 100, 104]	[98, 111, 119, 121] [101, 102] <sup>a</sup>	[106] [28] <sup>a</sup>	[113, 116]	[26] [156] <sup>a</sup>	[81, 157, 158]	[124, 159, 160]
Deployable structure	[161]	[82, 161]	[110]		[162]	[114]	
Prototype	[34, 46]	[41]	[140]	[29]	[163]	[164, 165]	
Topical review					[166–168]	[169, 170]	

<sup>a</sup> Topical review of a specific application.<sup>b</sup> A metalloid (specifically, silicon).





**Figure 8.** The use of snap buckling in fiber-reinforced composites for different smart purposes. (a) A bistable composite strip for energy absorption (reprinted from [18], copyright (2013), with permission from Elsevier). (b) A spring mechanism incorporating a bistable composite plate for vibration isolation (reprinted from [70], copyright (2013), with permission from Elsevier). (c) Three stable states for an orthotropic shallow shell with constant curvature (reprinted from [98], copyright (2013), with permission from Elsevier). (d) A multistable twisting grid of carbon fiber reinforced plastic strips (reprinted from [101], copyright (2013), with permission from SAGE Publications).

energy-related and motion-related applications for diverse purposes. Another interesting trend is the use of elastic instabilities across multiple scales, particularly at the nanoscale. This section discusses these two trends.

Beyond the fundamental design of actuators and energy harvesters, a number of recent endeavors have shown the promising potential of using buckling-induced behavior for multiple other purposes (see figure 10). Figure 10(a) presents a bistable compliant mechanism for a MEMS-based accelerometer in which the mechanism can switch from one position to another when the force on the accelerometer exceeds a threshold value [62]. Figure 10(b) shows a half-section view of a curved water-soluble polyethylene beam that is designed to increase the rate of solvent transport. Snap buckling of the beam is triggered by the release of internal elastic energy due to the swelling and shrinking processes when the solvent diffuses into the water-soluble polyethylene [74]. Such a mechanism can also be used in the design of artificial muscles. The schematic picture in figure 10(c) shows a new experimental approach to determine the bending rigidity of graphene using the snap-buckling instability in a prebuckled graphene membrane. Such a concept offers an interesting method for predicting the properties of nanoscale

materials [90]. For example, Sadeghian *et al* [175] experimentally estimated the Young's modulus of a silicon nanocantilever from the instability induced by electrostatic pull-in forces. This approach has substantial advantages over other methods used for the characterization of nanoelectromechanical systems. Figure 10(d) shows an experimental setup to examine the snap-through instability of a membrane subjected to an axially symmetric load simulated through water inside an acrylic column. This prototype was developed to harness the nonlinear behavior of snap buckling for the design of a ventricular assist device for better transmission of pneumatic load to the blood [87, 88]. A sensor relying on the elastic buckling of a thin wire (figure 10(e)) was designed and tested for memorizing peak strains in structures to identify damage. Such a mechanical-memory mechanism is a promising concept for damage index sensors [13]. Figure 10(f) shows a biomimetic responsive surface inspired by Venus flytrap leaves, which can snap from concave to convex states [81]. The sphere snaps by swelling an elastic network with an organic solvent to develop an osmotic stress. Such snapping surfaces can impact a variety of applications, such as coatings, adhesives, and drug delivery.

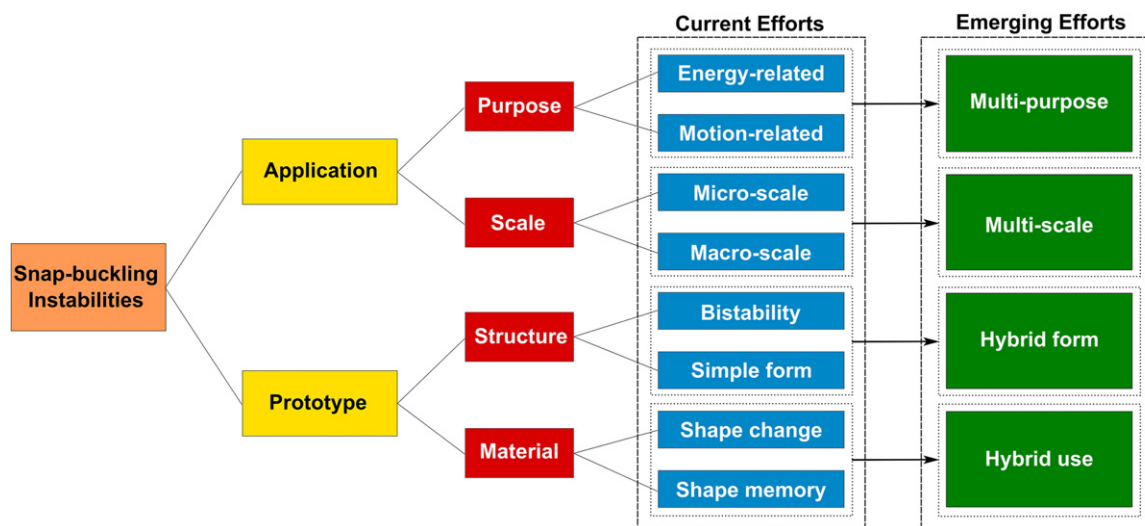


Figure 9. Concept map of the advances in buckling-induced applications.

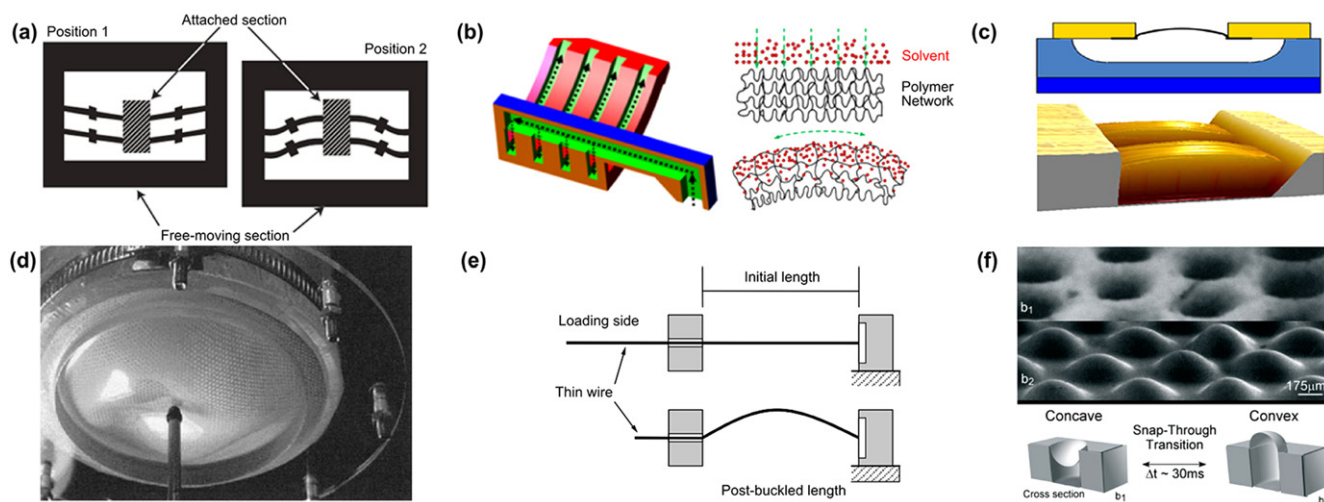
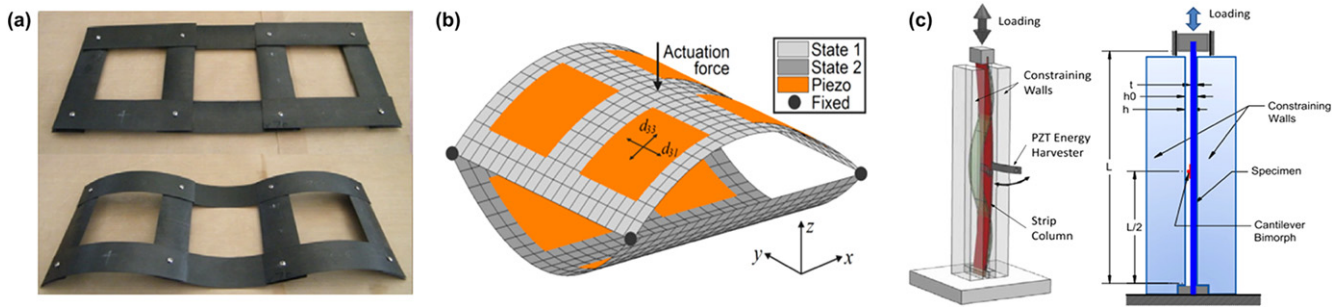


Figure 10. Harnessing of elastic instabilities for diverse purposes. (a) A bistable compliant mechanism for a MEMS-based accelerometer (reprinted from [62], copyright (2007), with permission from IOP Publishing). (b) A curved water-soluble polyethylene beam to increase solvent transport (reprinted from [74], copyright (2010), with permission from IOP Publishing). (c) A method of determining material properties using snap buckling (reprinted from [90], copyright (2012), with permission from American Chemical Society). (d) A ventricular assist device using snap buckling (reprinted from [87], copyright (2003), with permission from Elsevier). (e) A smart sensor with memorizing peak strain for damage identification (reprinted from [13], copyright (2003), with permission from IOP Publishing). (f) A biomimetic responsive surface based on snap-through buckling of domes (reprinted from [81], copyright (2007), with permission from John Wiley and Sons).

Harnessing buckling has been explored for many microscale applications, yet the emerging trend is to extend the instability principles for uses across multiple scales, particularly the nanoscale. For example, dramatic mechanically triggered transformations in 2D periodic elastomeric structures using unstable events have been used to transform phononic band gaps [157]. In a similar vein, microstructure elastic buckling responses have been used to trigger abrupt changes in the phononic properties of nanomaterials [176], to create unidirectional negative Poisson’s ratio behavior in materials with periodic microstructures [177], and to assist the nanoscale assembly of complex structures [178]. A similar idea [179] has been numerically investigated by using a two-phase transition to design a viscoelastic material with negative

elastic modulus to be used as inclusions in a positive elastic modulus matrix. Elastic instabilities have been found in many nanoscale structures [160, 180], such as nanowires and nanotubes. The major issue at such a scale is surface effects, and they should be taken into account as part of the total energy potential of the system [33]. At the same time, the success of microscale buckling-induced applications has been extended toward the discussion of possibly using similar concepts for large-scale applications. For energy-related applications, an increasing amount of published literature has shown the great potential of self-powered smart sensors based on snap buckling for wireless structural monitoring networks in buildings and bridges [15, 60, 181–184]. For motion-induced application, a number of reviews have identified





**Figure 11.** Structural prototypes for obtaining multistable behavior. (a) A bistable-composites concept for energy absorption (reprinted from [111], copyright (2013), with permission from Elsevier). (b) Bistable composites attached with four piezoelectric patches (reprinted from [53], copyright (2012), with permission from AIP Publishing). (c) A bilaterally constrained axially loaded CFRP strip (reprinted from [15], copyright (2014), with permission from IOP Publishing).

opportunities for using buckling behavior to design morphing components for different structures, such as automobiles [102], wind turbine blades [129], and infrastructure [27].

### 5.2. In prototypes

Future buckling-induced smart applications require the development of prototypes with tailored geometric configurations and material distributions. The snap-through buckling response in the structural prototypes presented thus far shows no more than two stable states. Research efforts are emerging to explore the possibility of achieving multistable events through four different approaches: (1) varying material and geometrical properties, (2) using hybrid systems, (3) using hybrid material combinations, and (4) adding constraints.

A study [171] has identified that an isotropic cylindrical shell may be stable only in its initial configuration but will exhibit second or further stable configurations if made from fiber-reinforced composites. Tailoring laminated composite to achieve tristability has been investigated for multiple shell forms, such as corrugated shells [104], orthotropic shells [185], double-curved shells [98], and shells with piezoelectric patches [116].

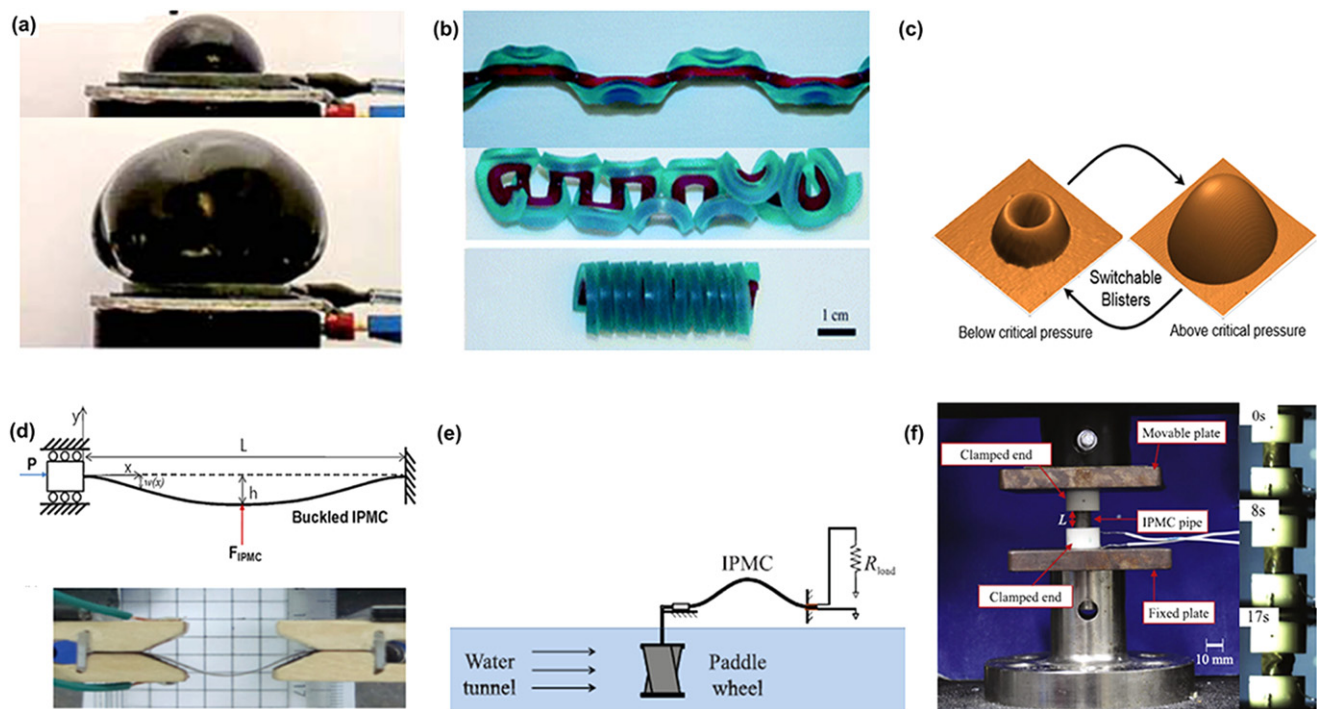
More multistable mechanisms have been reported by using hybrid material systems. A quadri-stable compliant mechanism has been designed and tested by using a bistable curved beam embedded in an arch beam [46]. A bistable switching mechanism has been achieved through a hybrid system of curved strips attached to a center beam [58]. A multistable lattice structure (figure 11(a)) has been developed based on a tristable lattice cell made from bistable laminates [111]. Hybrid carbon fiber/E-glass/epoxy cylindrical shells have been shown to obtain multiple mode transition in the postbuckling regime with a certain degree of control [41]. Finally, rather than proposing an actual physical model, recent studies have explored the design of smart structures by connecting multiple unstable configurations through a representative model, namely, the use of heteroclinic connections between unstable equilibria [186, 187].

Buckling and postbuckling response has also been studied by the combination of two materials for achieving sustainable snapping events, such as shape-memory alloy wire

actuators embedded in laminated composites [84, 188] and composite plates with piezoelectric actuators and sensors [144, 189]. Many studies have successfully investigated snap buckling in structural prototypes with piezoelectric patches and applied them to the design of novel adaptive multistable composites. Figure 11(b) shows a prototype of bistable composite plates with four piezoelectric patches such that the bistable structure can snap back to its initial configuration without the need of an external actuation force [53]. A similar effect has been achieved by using another emerging smart material to provide the actuation load—macro-fiber composites (MFC) [85, 86, 116]. MFC bonded onto other structures is also being used for developing energy harvesting prototypes due to its large deformation capacity and high energy conversion efficiency compared with traditional piezoelectric materials [47].

The use of lateral constraints has also been shown to be effective for achieving sustained snap-through and snap-back response in the far postbuckling regime. For example, the disordered packing behavior of an elastic rod constrained by a cylindrical chamber has been studied to develop the prototype for a folding mechanism [32]. Bilaterally constrained axially loaded strips, which can exhibit snap-through behavior between multiple buckling modes [190], have been shown to be efficient for devices that may harvest energy from pseudo-static motions [15]. Such behavior has been exploited for energy harvesting purposes (figure 11(c)). Inner constraints have also been shown to be a viable mechanism to modify, and potentially tailor, the postbuckling response of axially compressed composite cylindrical shells [41].

Searching structural forms to achieve snapping and buckling can be assisted by design optimization methods. Topology optimization techniques have been developed to seek the best material/stiffness distribution for a given objective [191], and their fundamental principles are well documented [192]. A recent review [193] showed that topology optimization has been extensively used to solve a variety of multidisciplinary problems in the last decade. Topology optimization methods with a buckling response objective are also emerging, thus shifting the concept from the search of maximum strength or stiffness under static loading demands to problems dealing with nonlinear large



**Figure 12.** The potential use of emerging soft material for buckling-induced applications. (a) A bistable compliant mechanism for a MEMS-based accelerometer (reprinted from [89], copyright (2012), with permission from RSC Publishing). (b) Snapping a simple stretched bi-strip of elastomers (reprinted from [114], copyright (2012), with permission from RSC Publishing). (c) A graphene blister with a switchable snap-buckling feature (reprinted from [124], copyright (2013), with permission from American Chemical Society). (d) A double-clamped IPMC actuated beam (reprinted from [208], copyright (2010), with permission from Elsevier). (e) An energy harvesting concept with two IPMC sliding cranks (reprinted from [54], copyright (2013), with permission from SAGE Publications). (f) An experimental study of buckling of IPMC pipes (reprinted from [16], copyright (2013), with permission from IOP Publishing).

deformations for smart purposes. Numerical algorithms that can track the nonlinear structural response following a path that exhibits snap-through behavior have been developed [194–198]. Recent efforts have also showcased the use of topology optimization to find material layouts that lead to larger displacement and energy release from snap-through behavior for different purposes, such as actuators [61, 73, 83, 145, 199], energy harvesters [53, 56, 200], dampers [67], multistable compliant mechanisms [201, 202], thermoelectric generators [203], and flutter control [36]. Thus, topology optimization is a promising technique for finding new structural and material layouts to meet a targeted elastic unstable response.

Another growing research trend pertains to the use of soft and superelastic materials for triggering elastic instabilities. It should be noted that the following discussion is not focused solely on buckling-induced but rather EI-induced applications in general. Two major materials reported in the literature for this purpose are dielectric elastomers (DEs) and graphene. DEs are ideal candidates for high-performance and low-cost applications due to their characteristics of light weight, ease of fabrication, and low cost. General behavioral theory and applications of DEs can be found in [204] and [169], respectively. Research studies [89, 164, 205] have shown that DEs are capable of having large voltage-induced snap-through instabilities (figure 12(a)), and their potential has been demonstrated for different purposes, such as actuators

[91, 147, 165, 206], energy harvesters [55, 148, 153], soft generators [154], and morphing in elastomer systems (figure 12(b)) [114]. More endeavors toward harnessing snap-through instabilities using DEs can be found in two recent reviews [170, 207].

Graphene is a monolayer of carbon atoms densely packed in a honeycomb crystal lattice [159]. Studies [124, 158, 160] have recognized that the morphology of graphene can lead to a sudden snap-through transition and that it has great potential for the development of graphene-based interfaces and devices (figure 12(c)). Another soft material that has caught the attention of researchers for buckling-induced applications is ionic polymer–metal composites (IPMC). The features of IPMCs are their light weight, their ability to sustain large deformation, and their ability to be used in wet conditions. Typical IPMC materials include Nafion and Flemion [169]. Figure 12(d) presents a double-clamped buckled beam under the electrical actuation of an IPMC [208]. Cellini *et al* [54] have explored the feasibility of using the snap buckling of two IPMC sliding cranks induced by a steady fluid flow for energy harvesting purposes (figure 12(e)). Beyond snap-through buckling of beam-like IPMC samples, a recent study by Shen *et al* [16] showcased a prototype, along with a novel fabrication method, for sensors based on the buckling of axially compressed IPMC pipes (figure 12(f)). A comprehensive review of IPMC can be found in [209], whereas studies [123] also indicate that there is plenty of room left for

material science researchers to develop artificial materials to achieve elastic instabilities.

## 6. Summary

This topical review presented state-of-the-art developments regarding the harnessing one of the elastic instabilities (namely, buckling and postbuckling behavior) for smart purposes. The buckling response in slender structures and systems has been an area of little relevance due to the significant loss of load-carrying capacity and large deformations resulting from such an event. However, the increasing interest in various disciplines for developing smart applications over the last decade has motivated creative researchers to transform buckling behavior into a promising phenomenon for the design of novel materials and mechanical systems. Harnessing buckling and other elastic instabilities is a promising technique that is yet to be fully explored. Four major aspects include application purposes, application scales, structural prototypes, and material prototypes. The following lessons learned can be drawn from the presented review:

- Existing endeavors regarding buckling-induced applications can be generally divided into two main categories: energy-related and motion-related. Energy-related applications can be further categorized into energy production and energy dissipation. Energy released from snap-through buckling can be used in the design of energy harvesters, sensors, actuators, and micro-electromechanical systems (MEMS), whereas the energy dissipated from buckling events can be used for the design of energy dissipaters, dampers, stabilizers, isolators, etc. For motion-related applications, snap-buckling instabilities have been proved to be useful in the design of morphing, adaptive, and deployable structures.
- Compiled research efforts have shown that buckling and other elastic instabilities can be found across scales: from nanostructured materials to macro-scale structural elements. Energy-related and motion-related applications using snapping and buckling have been explored primarily at the microscale level. However, an emerging trend is to extend the basic principles as a platform for nanoscale applications. Prototypes of EI-induced sensors and actuators are also gaining momentum for the monitoring of large structures.
- The key step for using a specific unstable event is to identify a desirable structural prototype. The studies highlighted in this review have shown that laterally loaded beams and bistable plates are the most frequently used structural forms for the purpose of energy harvesting, whereas cylinder-type structures have been heavily studied for energy dissipation purposes. Three-dimensional forms, by their nature, are more favorable for motion-based applications with morphing or deployable

features. Extending beyond the bistability featured by many existing forms, a research trend is emerging to explore the structural systems that can feature multistable events by varying material properties, using hybrid structural/mechanical systems, using hybrid materials, and adding constraints. The search for structural forms to achieve desirable snap-through instabilities is also being assisted by optimization methods, such as topology optimization.

- Another key step for using a specific unstable event is to select a suitable material. Two major features in the selection of a material for EI-induced purposes are the capability of shape change and the capability of shape memory. A material may have these features separately or simultaneously. Four major reported materials with the capability of shape change are metals (including metalloid), fiber-reinforced composites, polymers, and piezoelectric materials. Shape-memory alloys (SMAs) and shape-memory polymers (SMPs) are well-known candidates among many smart materials. An ongoing trend is using soft and superelastic materials, such as dielectric elastomers (DEs), graphene, and ionic polymer-metal composites (IPMCs), to trigger elastic instabilities.

This review has covered diverse studies under the overarching concept of harnessing buckling and elastic instabilities. However, it should be noted that snap buckling is only one type of instability phenomenon. This narrower treatment follows from the fact that it is not easy to classify or unify all kinds of instabilities into a single category. Nonetheless, the research trend is toward the exploration of many kinds of instabilities. Having said this, we apologize to those researchers who are, or have, worked on the study of elastic instabilities if their works were not covered by this review. Furthermore, this review featured a great body of experimental works on the subject and focused on conceptual and physical aspects, but it did not highlight aspects of fundamental analysis. Research studies have shown that many EI-induced responses can be captured by using numerical methods, many of them commercially available. Finally, continuous advances in materials processing and fabrication methods are likely to motivate the creativity of researchers seeking to harness EI-induced mechanisms, indicating increasing growth and development in this research field for several years to come.

## Acknowledgments

The authors acknowledge inspiration for this review from an on-line discussion forum on elastic instabilities led by Dr Douglas Holmes in the February 2012 Journal Club on the mechanics website iMechanica (<http://imechanica.org/node/11812>).



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