Determination and compensation of the “reference surface” from redundant sets of surface measurements

François Polack*, Muriel Thomasset

Synchrotron SOLEIL, L’Orme des Merisiers, Saint-Aubin, BP48, 91192 Gif-sur-Yvette Cedex, France

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ABSTRACT

When trying to measure an optical surface at utmost absolute precision, the problem of the missing or unknown “reference surface” is often encountered. It is obvious with Fizeau and Michelson’s interferometry, where the height difference between the surface under test (SUT) and a reference surface is measured. It is also true from slope measurements in long trace profilers (LTP), where due to small construction errors, the response to a perfectly flat ideal surface can be considered as an unknown reference to be subtracted from the measurement data. As no “perfect artifact” can exist, these references cannot be directly determined. The addition of the unknown reference can severely bias the reconstructed surface when field stitching is applied.

The results of ptychography have proved that when a measurement is a function of a unique object function with a translated but unique response function, the redundancy of a large set of data allows accurate reconstructions of the object and response function despite the presence of measurement noise. In the case of LTP and interferometry, the basic problem is linear and can be solved by linear algebra rather than iteratively. The method has been already applied to SOLEIL and ESRF LTPs and is successfully used on a regular base. We show here that the method can be also applied to interferometry and improve stitching results.

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1. Introduction

Quality of an optical surface is evaluated from the departure of the surface from an ideal optimal shape. Progress of optical surface polishing techniques is such that local correction of less than 1 nm in height can be reliably applied. However asserting a shape error with an accuracy below 1 nm is not an easy task because shape measurements are never absolute.

Two different kinds of instruments are used for measuring the shape of optical surfaces, interferometers and slope profilers. Interferometry is relative by nature, since the measured quantity is the height difference between the surface under test (SUT) and a reference surface. Slope profilers are measuring the angular deviation of a probe beam which is scanned on the SUT along a line. Different schemes are used, but the angle measuring part is typically an autocollimator head. At the microradian level, the linearity of the angle response can no longer be assumed, because inhomogeneities in the glass of the autocollimator lenses produce small spurious deviation depending on the position of the returning beam. Moreover, as the beam return path depends on the travel distance from the autocollimator to the SUT this calibration is valid for a defined travel distance.

In order to recover the “true surface”, the reference surface, or the slope calibration curve, need to be determined with the same accuracy. This determination is difficult because straightforward methods, such as measuring a perfectly known surface or tilting a surface with perfectly known angles, is not possible. When using an interferometer, a classical way to turn around the difficulty is to use repeated overlapped measurements which contains information of both the SUT and the reference (or SUT slope profile and calibration curve). For profiler type measurement we already showed in a previous paper [2] that it was possible to recover both the surface slope and the calibration curve. More examples on this are given here.

We also show that the application of the same redundancy principle to interferometric measurements allows recovering both the SUT shape and the reference.
2. Solving the reference problem from redundant overlapped measurements

Let us assume that we have a measuring procedure whose result, \( \mathbf{M}(x_i) \), recorded on a pixelated detector, is the sum of the true surface signal, \( \mathbf{S}(x_i + x_0) \), a reference signal \( \mathbf{R}(x_i) \) which depends on the position \( x \). The measurement can be repeated while changing the (known) offset \( x_0 \) between the object and the reference. So, datasets \( \mathbf{M}(x_i) \) are being built where the point to point coupling between the signal and the reference is each time different. Neglecting measurement errors, we have

\[
\mathbf{M}_k(x_i) = \mathbf{S}(x_i + x_0) + \mathbf{R}(x_i).
\]

Since the two vectors \( \mathbf{S} \) and \( \mathbf{R} \) are unique we are getting a redundant information on them. In more mathematical terms, we are building a large set of overdefined linear equations which can be only solved in a least square sense. In other words we can look are building a large set of overdefined linear equations which can also determined together with \( \mathbf{R} \). The residuals, \( \mathbf{M}_k(x_i) - \mathbf{S}(x_i + x_0) - \mathbf{R}(x_i) \), are the statistical errors inherent to measurements mixed with other sources of deviation not included in the model. When one of these deviation sources is the total error \( E \) coming from the change of the return path being negligible with respect to the slope induced correction \( C(s) \). For solving the problem, the linearity correction needs to be evaluated on a grid of tabulated slope values \( s_p \) and therefore the correction for a given measured slope \( s = \mathbf{M}_k(x_i) \) must be interpolated on this base in a form such as

\[
C(s) = \sum B_p(s)C_p.
\]

As mentioned before, the interpolation should be local to preserve the sparseness of the equation matrix. In our implementation we use a cubic B-spline interpolation for which the at most 5 coefficients \( B_p \) are non zero for any \( s \). With this interpolation, the system of Eq. (2) can be rewritten as a set of linear equations of the unknown vectors \( \mathbf{S}(x), \mathbf{C} \), and \( \mathbf{T} \). And, since this system is overdefined, it can be solved in a least square sense as stated before. The LTP correction problem is one-dimensional and therefore remains small enough to be solved on a personal computer.

At SOLEIL, the method is applied to all surfaces for which stitching is required and when the utmost accuracy is needed. It was also successfully applied to ESRF LTP measurements. The spherical test wavefront \( \mathbf{R} = 9.3113 \text{ m} \) the shape error of which is given in Fig. 1 is a typical example. This strongly curved mirror has been circulating between synchrotron laboratories for mutual comparison of their measuring instruments. Fig. 1 is plotted with the height errors of this mirror as measured by the Hemoltz Zentrum Berlin (HZB)/Bessy II NOM [5], by ESRF LTP without application of the LEEP method and by SOLEIL LTP with application of the LEEP method. The agreement between SOLEIL LTP and

3. Application of redundant overlapped measurements to LTP

As said in Section 1, the response of the long trace profiler (LTP) of SOLEIL presents slight (few microrads) nonlinearities over its 8 mrad measuring range, and hence, as any instrument of its kind, it needs to be calibrated. Moreover, since the distance between the SUT and the slope measuring head can have large variation, this calibration may substantially differ from one measurement to another. The overlapped redundant measurement method, allowing to simultaneously obtain the SUT profile and the linearity error, has been quite systematically used at SOLEIL since it was developed a few years ago under the name LEEP (linearity error elimination procedure) [2]. The method is usually applied as a stitching method for strongly curved surfaces when the slope range exceeds the 8 mrad measuring range of the LTP. The surface is placed on the LTP bench and tilted in such a way that one extremity of the mirror is measured at the center of the LTP measuring range and a profile measurement is made on the part of the mirror which can be measured. Then the mirror is tilted step by step until the other end of the mirror is also measured at the center of the measuring range. Profiles are recorded at each step on the measurable part of the surface. Tilts steps are chosen so that any position \( x \) of the SUT is measured with enough redundancy (20 times whenever possible).

Denoting the linearity correction for the measured slope, \( s \), by \( C(s) \) and the tilts by \( T_k \), the recorded slope data can be described as

\[
\mathbf{M}_k(x_i) = \mathbf{S}(x_i + x_0) + (\mathbf{M}_k(x_i)) + T_k
\]

where it is assumed that the SUT is short enough for the error coming from the change of the return path being negligible with respect to the slope induced correction \( C(s) \). For solving the problem, the linearity correction needs to be evaluated on a grid of tabulated slope values \( s_p \) and therefore the correction for a given measured slope \( s = \mathbf{M}_k(x_i) \) must be interpolated on this base in a form such as

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NOM, which is also corrected, but once for all, from systematic deviation to linearity, is 1.8 nm PV and 0.4 nm RMS. A larger difference is found between SOLEIL and ESRF LTPs. LEEP was not applied to ESRF data in this case, but we know from earlier results that applying LEEP to both LTPs brings measurements to the same level of agreement [3].

The method does not only provide a corrected slope profile of the SUT but also gives an estimate of the linearity error $C(s)$ of the LTP. The linearity error is an intrinsic property of the optical system and therefore should not depend on the optical surface being measured. However it is sensitive to path changes in the LTP optics, namely, it has to be checked again after optics realignment and it significantly depends on the position of the SUT along the bench due to the path length change. This is why the full application of the LEEP method is actually preferred to the 3 component where $F(x,y)$, $G(x,y)$ and $H(x,y)$ are integer numbers of pixels, there is an exact number of pixels was not found critical experimentally. An error of one or two pixels per stage step (144 pixels) produces a slight blur of the tiniest details (scratches and digs of the surface), and of course a small change of the reconstructed image width, but no significant distortion of the shape.

The amount of data collected in a stitching experiment can be quite large. A strip of 150 mm long by 12 mm is typically recorded with a 3 mm step (144 pixels) in order to get a redundancy factor of about 5. Hence, the 45 recorded image frames generate around 20 M data points for about 4 M independent variables. Because of this large size, the numerical solution of the problem was implemented under Matlab on SOLEIL mainframe computer. A stitching problem of the above size is typically solved in 10–15 min by applying straightforward methods. However the convergence speed and the quality of the solution were found to depend significantly on the way the problem is expressed. In particular, a better convergence was found when subtracting beforehand an estimate of the reference surface from all measurements, rather than using this estimate as a starting point of the computation algorithm. This certainly due to the fact that, with 170 nm PV, the reference surface of our interferometer is presently much poorer than the surfaces we tried to measure. It is therefore believed that better conditioning the problem by exploiting its particular structure could result in much faster and less memory demanding algorithms.

4. Application of redundant overlapped measurements to stitching interferometry

4.1. The stitching procedure

Stitching interferometry is a procedure for measuring an optical surface the size of which exceeds the field of view of the interferometer. It consists in recording a sequence of overlapped interferometer fields, usually by translating the SUT under the instrument and reconstructing the height map of the SUT from this set of data. The reconstruction problem is nearly identical to the basic reference problem treated in Section 2, with the exception that we have to consider that the translation stage which carries the SUT is not perfect and induces unknown piston and tilt displacements. Hence we need to add one term to Eq. (1), which becomes

$$M_k(x_j) = S(x_j + x_k) + R(x_j) + F(x_j)T_k$$

where $F(x_j)$ is the simple 3 line matrix which relate the height change at any point of the interferometer field to the 3 component tilt tilt vector $T_k$ of the translation stage at position $k$.

If the displacements $x_k$ are integer numbers of pixels, there is no need for interpolating the data and therefore each measurement point is related to only 5 unknowns, the measured point on the surface, the corresponding point on the reference and 3 values of the tip tilt vector of the image frame $k$. The matrix of the equation system is therefore extremely sparse which makes it suitable to be solved by iterative least square resolution routines such as those provided by Matlab [6].

The method was applied to stitching data acquired with the new phase shift microinterferometer of SOLEIL. This instrument, called Nanopro, has been constructed on SOLEIL specifications and is based on use of a telecentric objective with a Michelson interferometer [4]. It has therefore a low image distortion and a great insensitivity to image defocusing. The recorded object field of view is $16 \times 12$ mm$^2$, 780 × 580 pixels, with an object pixel size of 20.8 μm. Positions of the interferometer relative to the SUT can be changed by means of two motorized translation stages. The X stage carries the SUT. The Y stage is set on a transverse beam over the X stage and holds the interferometer. The stages have a position accuracy of 1 μm. Only X translation was used in the first stitching experiments reported here. Making the displacements $x_k$ an exact number of pixels was not found critical experimentally. An error of one or two pixels per stage step (144 pixels) produces a slight blur of the tiniest details (scratches and digs of the surface), and of course a small change of the reconstructed image width, but no significant distortion of the shape.

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4.2. Stitching experiments

The first experiment was done on a circular Zerodur flat surface (0200 mm) which has been used as a reference for the LTP for more than 10 years and is therefore well known. Its topography presents circular structures about 35 nm PV left by polishing process. A sequence of 30 topographic images was acquired along a diameter [7] of this surface with an X step of 3 mm. The set of images covers a 12 mm wide and 103 mm long strip centered on the surface.

This recorded area was stitched with the Optophia software available with our phase shift interferometer, and with the MLR procedure of Section 4.1. The stitched images are compared in Fig. 3. Panel (a) presents the surface reconstructed with our MLR method and panel (b) presents the surface reconstructed by Optophia. Beside the obvious difference of curvature along the X-direction, two images display very similar features. A more careful observation reveals that Optophia image is slightly twisted from one end of the stripe to the other while MLR image is not. This twist is not consistent with the known circular symmetry of the piece and should be an artifact of stitching procedure. Fig. 4 shows height profiles taken from these images by averaging 11 lines (0.2 mm) around the position $Y=5$ mm.
These profiles are compared to the height profile obtained by integrating LTP data recorded on the whole same diameter. Panel (a) shows the profiles as computed from the images and illustrates that none of the stitching methods recovers correctly the curvature along the stitching axis. This is expected because, since we remove a tilt from each topographic image, the curvature term of the SUT cannot be distinguished from a curvature term attributed to the reference. Therefore the reconstructed curvature depends very much on the initial guessed shape given to the reference. It is clear from panel (b) that both curves can be made very similar to the LTP profile by adding an appropriate curvature. Recovering the absolute curvature would require measuring the stage tilt with a very high accuracy (~10 nrad).

A second stitching experiment was made on a flat surface of higher quality, which has been produced by SESO by ion beam deterministic surface correction in the framework of a research project named AXOC [8]. This surface has been made on a 200 mm long, 30 mm wide and 60 mm thick silicon blank. It was first polished by conventional means and measured. Then it was processed by ion beam polishing in order to correct a ~3 mm wide and 120 mm long stripe for ultimate possible flatness.

This surface has been measured with the Nanopro. A sequence of 45 topographic images spaced by 3 mm steps has been recorded, which covers an area of $12 \times 144 \text{ mm}^2$ around the corrected stripe. The data have been stitched with our MLR procedure after removing a first estimate of the reference surface, which was simply the average of all the 45 measurements. A topographic map of the achieved reconstruction is shown in Fig. 5. A reconstruction was also attempted with Optophia but without success. Fig. 6 shows two profiles of the stitched topographic map around the center of the corrected strip, marked by a dotted line in Fig. 5. These profiles are unaveraged (1 pixel wide) and they are separated by a distance of 210 $\mu$m across the surface. The high signal to noise ratio denotes the quality of the Nanopro and of the tested optical surface as well. The flatness correction of the surface is remarkable on a length of 50 mm, from $X=65$ to $X=115$ mm, where the height variations are 1 nm PV. The other half shows larger fluctuations. We are wondering if it could be an indication of surface drift during ion beam processing.

![Fig. 3](image1)

Fig. 3. Stitching reconstructions of the central area of the 0200 reference mirror from a sequence of 30 topographic images. x and y scales are in millimeters, the height scales on the right are in nanometers:

Panel (a) reconstruction with the MLR method and Panel (b) reconstruction with the Optophia software.

![Fig. 4](image2)

Fig. 4. Profiles at position $Y=5$ mm of the two reconstruction shown in Fig. 3 compared to a LTP profile of the same diameter. Stitched image profiles are plotted as measured on panel (a). In panel (b), they have been added a curvature to best fit to the LTP data Image profile are averaged on 11 rows (200 $\mu$m), while the LTP data are measured with a spot size of ~1.5 mm diameter.

![Fig. 5](image3)

Fig. 5. Stitching reconstruction of the AXOC mirror computed with the MLR method. The height scale on the right is in nanometers. The dotted line marks the center of the ion beam corrected stripe. The short vertical line, which is visible on the bottom right of the image, is one of the fiducial marks etched in the mirror for accurate registration of the surface.

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5. Conclusion

We described the underlying principles of solving the reference problem of non-absolute measurements from a redundant set of overlapped data in terms of a maximum likelihood regression (MLR) and showed how akin the problem is to the stitching one. Two applications case were given. The first one is stitching LTP measurements and, in the same operation, correcting them from systematic linearity errors. Cross checks with other LTPs show a very nice convergence of measurements carried out by this method, with results obtained with a carefully calibrated instrument such as HZB NOM. The implementation of this one-dimensional problem is rather light. However, as the linearity correction depends on the measured slope, a linear interpolation of the correction must be implemented in order solve the problem.

We also show that the same underlying principle can be applied to eliminate the unknown reference influence in reconstructing the topography of an optical surface by stitching multiple interferometric height maps. The problem is simpler because the reference adds to the measurement in a direct point to point relationship. But the problem is much heavier than the previous one because the volume of recorded data and the size of the reconstructed images can be very large. Nevertheless we were able to obtain good quality reconstruction of flat surfaces from sets of topographic images recorded with the Nanopro, our new phase shift interferometer, despite the poor quality of its reference surface. The reconstruction of a test flat of medium quality, 35 nm PV is in quite good agreement with profiles obtained from LTP measurements. Another experiment was made on a high quality surface and gave high signal to noise height maps and profiles.

This second application looks quite promising for controlling the mirrors needed by nanofocusing X-ray beamlines. More development is still needed. Clearly, the brute and direct method applied here can be improved by algorithms which would take advantage of particular features of the data structure. The convergence is also limited by several sources of errors from the Nanopro interferometer, such as phase shift nonlinearities or residual image distortion. They need to be either corrected or included in the regression model.

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References

[7] The measured diameter is defined by precise fiducial marks etched in the surface.
[8] AXOC is a research project for producing Advanced X-ray Optical Components, funded by the French Agence Nationale de la Recherche (ANR). It federates the works of two laboratories, SOLEIL and Laboratoire des Matériaux Avancés, CNRS, Lyon (France) and two private companies, SESO, Aix-en-Provence (France) and ISP System, Vic-en-Bigorre (France).