COOPERATIVE SENSOR LOCALISATION IN DISTRIBUTED FUSION NETWORKS BY EXPLOITING NON-COOPERATIVE TARGETS

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Abstract

We consider geographically dispersed and networked sensors collecting measurements from multiple targets in a surveillance region. Each sensor node filters the set of cluttered, noisy target measurements it collects in a sensor centric coordinate system and with imperfect detection rates. The filtered multi-target information is then communicated to the nearest neighbours. We are interested in network self-localisation in scenarios in which the network is restricted to use only the multi-target information shared. We propose an online distributed sensor localisation scheme based on a pairwise Markov Random Field model of the problem. We first introduce parameter likelihoods for pairs of sensors – equivalently, edge potentials – which can be computed using only the incoming multi-target information and local measurements. Then, we use belief propagation with the associated posterior model which is Markov with respect to the underlying communication topology. We demonstrate the efficacy of our algorithm for cooperative sensor localisation through an example with complex measurement models.

Index Terms— cooperative localisation, multi-target tracking, sensor networks, graphical models, Monte Carlo algorithms

1. Introduction

We consider fusion networks comprised of a moderate number of sensor platforms positioned at different geographical locations for surveillance applications in which the trajectories of multiple targets are estimated. Each platform has moderate sensing, computational and communication capabilities and collects noisy measurements from targets with given probability of detection in an environment of false alarms from clutter and surroundings. In order to exploit the target information from multiple sensors, the information shared needs to be registered in a common coordinate system.

In this work, we consider self-localisation of sensor nodes in such networks. We are interested in scenarios in which a global positioning system (GPS) is not available (e.g., underwater sensor networks) or reliable (e.g., presence of a jammer) and the network is restricted to use only the measurements from the targets for the purpose of self-localisation.

A centralised processing approach in which a processing centre collects all sensor measurements is often not viable given the communication bandwidth and computational resource limitations [1]. We resort to a distributed paradigm in which each sensor performs local filtering of the measurements it collects in its sensor centric coordinate system (SCCS). The filtered multi-target information is transmitted to the neighbouring nodes and the incoming information from neighbours is translated to the local SCCS using the respective locations before they are fused to improve upon the myopic accuracy [2]. Therefore, sensor localisation, which is a sensor registration and calibration problem, needs to be solved for enabling distributed fusion.

Sensor registration and calibration has been studied in the context of target tracking in centralised settings considering conventional multi-target trackers (e.g., [3]) and Random Finite Sets (RFS) based multi-target filtering [4]. Common to these approaches is the use of a predictive parameter likelihood (see, e.g., [5, Sec. IV]) based on the measurements from the (hidden) target process. The computation of this multi-sensor likelihood, however, requires the network wide collected measurements to be filtered after they are transmitted to a designated centre (centralised operation), or, to all platforms for joint filtering in a distributed operation [6], creating a highly undesirable communication cost in either case.

We propose a self-localisation scheme which avoids centralised or joint filtering. It operates based solely on the locally filtered multi-target information already being exchanged for distributed fusion thereby conforming with the communication constraints. We facilitate distributed operation by, first, considering a pairwise Markov Random Field (MRF) model for the localisation posterior which is Markov with respect to the underlying communication topology. Such models have proved useful for distributed fusion [9] and target tracking [10] applications in wireless sensor network as well as self-localisation using noisy distance measurements [11] through the use of message passing algorithms for distributed inference such as Belief Propagation (BP) [7] and its efficient particle realisations [8].

Second, we introduce edge potentials which can be computed locally based on the multi-target information exchanged between sensor pairs: The edge potentials of the pairwise MRF model of our problem setting are predictive likelihoods for the sensor pairs. Therefore, they have a time-recursive structure – unlike the self-localisation model in [11] accommodating measurements directly related to the sensor locations (see, e.g., [12]) – and, their computation requires transmission of sensor measurements between sensor pairs. We remove this need for local centralisations by introducing a set of assumptions on the equivalence of the individual measurement histories and their pairs and obtain node-wise separable potentials.

In the resulting algorithm, upon exchanging multi-target posterior (for the purpose of fusion), each pair of neighbouring nodes updates the time-recursive likelihood of the respective location – equivalently, their edge potential – based on the incoming posterior and local target measurements. Then, all the nodes iterate BP message passings with these potentials leading to decentralised and
collaborative estimation of sensor locations. We benefit from the rich source of multi-target information using RFS models [14]. This enables us to handle multi-target filtering with imperfect detection rates by building upon local multi-target filtering and avoid having to explicitly find target-measurement associations.

We provide the problem statement and review the centralised solution in Section 2. We introduce node-wise separable likelihoods, and the collaborative localisation scheme in Sections 4 and 3, respectively. We demonstrate our algorithm through an example in Section 5, and conclude in Section 6.

2. PROBLEM STATEMENT

2.1. Network Coordinate System

We consider the sensor platforms \( V = \{1, ..., N\} \). The communication links available between sensor pairs constitute the edge set \( E = \{(i, j)\mid i \text{ and } j \text{ are linked}\} \). We assume bidirectional links and hence the network is represented by the undirected graph \( G = (V, E) \).

We are interested in estimating the sensor locations in a network coordinate system: Three nodes are selected as anchors specifying the origin, first axis and the first axis of the coordinate system. The first node is located at the origin and denoted by \( \theta_1 = [0, 0]^T \). The position of the second node is on the \( x \)-axis and the third node is located in the first quadrant, i.e., \( \theta_2 = [a, 0]^T \) and \( \theta_3 = [b, c]^T \) where \( a, b \) and \( c \) are positive random variables [11]. The concatenation of all unknown location variables is denoted by \( \theta \) and is the random vector subject to estimation based on the measurements induced by the target process which we discuss next.

2.2. Multi-target process and the sensor measurement model

Each sensor node receives multi-object posteriors from its neighbours. We assume that a Poisson model for the multi-target scene can be obtained from any of these posteriors, without loss of generality. Therefore, an unknown number of \( M \) targets each corresponding to a point in some state space \( X \) are represented by a Poisson RFS \( X_k = \{x_1, ..., x_M\} \) which is characterised by the expected number of targets \( \lambda_k \), a spatial density \( s_k(x) \) over \( X \), and a probability density given by [14]

\[
f(X_k) = \exp(-\lambda_k)\lambda_k^M \prod_{x \in X_k} s_k(x).
\]

The right-hand side (RHS) of the equality above will be denoted by \( \text{Pois}(X_k; \lambda_k, s_k(x)) \) throughout.

A target with state \( x \) generates a measurement \( z \) in a measurement space \( Z \) according to the likelihood model \( l(z|x) \) with probability of detection \( P_D(x) \). Let the set of such measurements be denoted by \( Z_k \), then

\[
\bar{Z}_k = \bigcup_{x \in X_k} z(x)
\]

and \( z(x) = \{z\} \) with prob. \( P_D(x) \)

\[
\emptyset, \text{ with prob. } 1 - P_D(x)
\]

and \( z \) is a random vector with density \( l(z|x) \) [14].

We model the false alarms with a Poisson RFS, as well, with rate \( \lambda_C \) and point density \( s_C(z) \) over \( Z \). Therefore, the false alarms are modelled with a random set \( C_k \sim \text{Pois}(.; \lambda_C, s_C(z)) \).

The set of measurements collected by a sensor at time \( k \) and denoted by \( Z_k \) is, then, given by

\[
Z_k = \bar{Z}_k \cup C_k
\]

2.3. Sensor localisation problem and the centralised solution

Let us denote the measurement history of sensor \( i \in V \) up to time \( k \) by \( Z_{1:i}^k \). We assume that the sensor locations \( \theta \) (Sec. 2.1) is a random vector with a prior distribution \( p_0(\theta) \). Without any further consideration on the network resource constraints, the sensor localisation problem can be treated as finding the posterior

\[
p(\theta|Z_{1:k}^1, ..., Z_{1:k}^N) \hat{F}_0(\theta|Z_{1:k}^1, ..., Z_{1:k}^N) \Delta \theta
\]

where \( \hat{F}_0(\theta|Z_{1:k}^1, ..., Z_{1:k}^N) \) is the parameter likelihood. The sensor locations can be found, then, by using the minimum mean squared error (MMSE) or the maximum a posteriori (MAP) estimation rules.

The parameter likelihood \( l(Z_{1:k}^1, ..., Z_{1:k}^N|\theta) \) based on all the measurement histories across the network is given by [5, Sec.IV]

\[
l \left(Z_{1:k}^1, ..., Z_{1:k}^N|\theta\right) = \prod_{i=0}^{k-1} p(Z_{t+1}^1, ..., Z_{t+1}^N|Z_{1:t}^1, ..., Z_{1:t}^1, \theta)
\]

where the factorisation follows from the chain rule. Each factor on the RHS is an instantaneous likelihood at time step \( t \) and admits the interpretation of being due to an independent observation of \( \theta \). Furthermore, the current sensor measurements and the recent history are conditionally independent given the current value of the target process. Let us denote this by \( Z_{1:t}^1 \perp \perp Z_{1:t}^i | X_{t+1} \). As a result, the instantaneous likelihood at time step \( t \) is given by [5]

\[
p(Z_{t+1}^1, ..., Z_{t+1}^N | Z_{1:t}^1, ..., Z_{1:t}^N, \theta) = \int p(Z_{t+1}^1, ..., Z_{t+1}^N | X_{t+1}, \theta)p(X_{t+1} | Z_{1:t}^1, ..., Z_{1:t}^N, \theta)dX_{t+1},
\]

where the first term inside the integral is the measurement likelihood and the second term is a prediction distribution for the target process at time \( t + 1 \), based on the observation histories of all the nodes in the network until \( t \). This distribution is output by the prediction stage of a “centralised” Bayesian recursive filter, which is not a feasible processing strategy in a distributed setting.

For a Poisson RFS \( X_{t+1} \), the integral in (5) becomes a set integral (see, e.g., [14, Eq.(11.96)]) and a closed form expression is given in [13, Eq.(116)] which will be introduced later in Section 3.

3. A MRF MODEL BASED ON TARGET MEASUREMENTS

The nearest neighbour information exchanges in our distributed paradigm motivates the use of likelihoods local to the neighbouring pairs of nodes. In other words, we approximate to the parameter posterior (3) using a pairwise MRF which is Markov with respect to the communication topology \( G \):

\[
\bar{p}(\theta|Z_{1:k}^1, ..., Z_{1:k}^N) \propto \prod_{i=0}^k \psi_i(\theta_i) \prod_{(i,j) \in E} \psi_{ij}(\theta_i, \theta_j),
\]

where the decomposition of the prior distribution follows from the assumption that the local variables (sensor locations, for the case) are independent. The edge potential for the pair \((i,j) \in E\) is a pairwise predictive likelihood and has a time-recursive structure (different from the model in [11]):

\[
\psi_{ij}(\theta_i, \theta_j) = \prod_{t=0}^{k-1} p(Z_{t+1}^1, Z_{t+1}^j | Z_{1:t}^1, Z_{1:t}^j, \theta_i, \theta_j)
\]

and

\[
\psi_{ij}^{k-1}(\theta_i, \theta_j) = p(Z_{k}^1, Z_{k}^j | Z_{1:k-1}^1, Z_{1:k-1}^j, \theta_i, \theta_j).
\]
3.1. Node-wise separable edge potentials

The edge potential in (7) requires pairwise centralisation as its computation needs access to the measurement histories of both nodes. We introduce a set of assumptions under which the instantaneous likelihood terms in the RHS of (7) can be computed using only the multi-target posteriors already received from the neighbours for distributed fusion and local measurements:

**Assumption 1.** The measurements of sensors \(i\) and \(j\) are conditionally independent given the unknown parameters \(\theta_i, \theta_j\) and their measurement histories, i.e., \(Z_{1:t} \mid Z_{1:t}^j, \theta_i, \theta_j\).

**Assumption 2.** The current observation of sensor \(i\) and its measurement history are conditionally independent given the unknown parameters and the measurement history of its neighbour \(j\), i.e., \(Z_{1:t} \mid Z_{1:t}^j, \theta_i, \theta_j\).

**Assumption 3.** Assumption 2 symmetrically holds for both nodes, and, hence, \(Z_{1:t}^j \mid Z_{1:t}^i, \theta_i, \theta_j\).

Under Assumptions 1–3, it can easily be shown that the instantaneous likelihoods are multiplicatively separable into node-wise terms each depending on a single sensor measurement history, and, the edge potential in (7) becomes

\[
\psi^j_i(\theta_i, \theta_j) = l_i^j(\theta_i, \theta_j)l_j^i(\theta_i, \theta_j) \tag{9}
\]

\[
l_i^j(\theta_i, \theta_j) = \prod_{k=1}^{\lambda_i} p(\theta_i, \theta_j)
\]

\[
l_j^i(\theta_i, \theta_j) = \prod_{k=1}^{\lambda_j} p(\theta_i, \theta_j)
\]

where the node-wise terms also have the time-recursive structure

\[
l_i^j(\theta_i, \theta_j) = l_i^{k-1}(\theta_i, \theta_j) p(Z_k^i | Z_{1:k-1}^i, \theta_i, \theta_j). \tag{11}
\]

Assumptions 1–3 imply that, given the correct values for \(\theta_i\) and \(\theta_j\), the predictability of the local measurements do not change with the selection of the individual measurement history or the pair to condition on. The connection with the principle of maximising the mutual information of \(Z_k^i\) and \(Z_k^j\) is beyond the scope of this work.

4. THE COOPERATIVE SELF-LOCALISATION SCHEME

In this section, we first detail the computation of the update term in the RHS of (11) for localisation, and, then describe the proposed algorithm. Under the assumption that \(X_k\) is generated from \(Pois(\lambda_{k-1} s_{k-1} | x(k-1))\), the update term can be found as

\[
p(Z_k^i | Z_{1:k-1}^i, \theta_i, \theta_j) \propto \exp \left\{ - \sum_{z \in Z_k^i} \left[ \lambda_{C} s_{C}(z) + \int P_D(x)l_i(z|x) \lambda_{k-1} s_{k-1}(x; \theta_i, \theta_j) dx \right] \right\}
\]

where \(P_D(x)\) is the detection probability of a target with state \(x\), and, \(\lambda_{C}\) and \(s_{C}(z)\) characterise the Poisson clutter (see, e.g., [13], Eq.(116) and [4]), considering the difference that our likelihoods evaluate measurements of the \(i\)th sensor at \(k\) given the measurement history of the \(j\)th sensor until \(k-1\). This term lead by the Poisson assumption features linear complexity with the number of target measurements. The prediction distribution \(Pois(\lambda_{k-1} s_{k-1}(x))\) is based on the posterior from sensor \(j\) at time \(k-1\), and the density of targets that first appear at time \(k\) [13]. We ignore target births at \(k\) and modify the original formulae accordingly:

\[
s_{k-1}(x) \propto \int P_d(x') \pi_{k-1}(x|x') s_{k-1}(x') dx'
\]

\[
\lambda_{k-1} = \lambda_{k-1-1} \int P_d(x') \pi_{k-1}(x|x') dx'
\]

where \(\pi_{k-1}(x|x')\) is the state transition density and \(P_d(x')\) is the probability that a target with state \(x'\) at \(k-1\) continues to exist at \(k\).

For the localisation problem, the likelihood term \(l_i(z|x)\) in (12) takes the target measurement argument \(z\) in the SCCS of sensor \(i\). Sensor \(i\) receives from sensor \(j\), however, \(s_{k-1}(x)\) that takes its state variable argument \(x\) in the SCCS of sensor \(j\). Let us denote the state \(x\) in the \(j\)th SCCS by \([x]_{j}\). Then, \([x]_{j}\) can be found by

\[
T([x]_j; \theta_i, \theta_j) \triangleq [x]_j - \theta_j + \theta_i = [x]_j. \tag{13}
\]

Therefore, before computing the node-wise update of the edge potential (12) at node \(i\), for its location respective to sensor \(j\), sensor \(i\) constructs \(s_{k-1}(\cdot; \theta_i, \theta_j)\) in (12) in its local SCCS through

\[
s_{k-1}(\cdot; \theta_i, \theta_j) = \int P_d([x]_{j-1}) \pi_{k-1}([x]_{j-1}; [x]_{j}) dx_{j-1}. \tag{14}
\]

Using the node-wise term update given by (11)–(14), the nodes update the edge potentials (9) of the pairwise MRF model (6) using only the incoming fusion information from the neighbours and local target measurements without any need for transmitting target measurements and with a linear computational complexity with the number of measurements. This model enables us to use Belief Propagation (BP) [7] message passings for finding the marginal distributions of (6). The location of the \(i\)th sensor can, then, be estimated using, e.g., the MMSE rule with the \(i\)th posterior marginal.

BP is an iterative algorithm in each step of which nodes pass messages to their neighbours and update their states – which are estimates for the marginals of the associated variables– based on the messages they receive:

\[
\tilde{p}_i(\theta_i) \propto \psi_i(\theta_i) \prod_{\theta_j \in \text{nei}(i)} m_{ij}(\theta_i), \tag{15}
\]

\[
m_{ij}(\theta_i) \propto \psi_{ij}(\theta_i, \theta_j) \prod_{\theta_{ij} \in \text{nei}(j)} m_{kj}(\theta_j)d\theta_j. \tag{16}
\]

As (6) is Markov with respect to the underlying network \(G\), the BP messages directly map to communications over the links providing a collaborative estimation mechanism.

Our self-localisation algorithm starts with non-informative location priors \(\tilde{p}_0(\theta_i)\) in the network coordinate system. The nodes exchange their location distributions with their neighbours and start filtering target measurements using, e.g., a PHD filter. They exchange the filtered distributions at each time step and for a time window of length \(L\), and, each node computes its instantaneous update terms using (12)–(14) for all the edges it shares with its neighbours. At the \(L\)th step, nodes exchange the node-wise components \(l_i^j(\theta_i, \theta_j)\) defined in (10) with their neighbours and compute the edge potentials using (9). Then, BP message passings are iterated using (16) and (15) for \(S\) steps. At the end of the \(S\)th message passings, the posterior marginals are estimated as the updated node states which are used to estimate the sensor locations using, e.g., the MMSE rule. The same procedure is repeated after node states are evolved with a random walk-like dynamic [5] and exchanged with the neighbours.

We realise our self-localisation scheme using particle representations for the distributions involved and Monte Carlo (MC) computations. We compute BP messages in (16) and sample from the posterior marginals (15) using Nonparametric Belief Propagation [8]. We estimate the likelihoods in (12) using MC integration methods [16] given particles representing the filtered distributions. It is worth noting that, for the case, the transform in (13) becomes simply the translation of the particles.

5. EXAMPLE

We demonstrate our cooperative self-localisation scheme by an example in which five range-bearing sensors collect measurements from two targets as illustrated in Fig. 1(a). Target states are given...
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by their position and velocity \([x, y, \dot{x}, \dot{y}]^T\). Targets are initiated at \(k = 0\) and \(k = 25\), and, at \([1100.0, 1000.0, 0.0, 10.0]^T\) and \([150.0, 1600.0, 9.0, -2.0]^T\), respectively, and, move with a constant velocity model with slight white Gaussian process noise.

The sensors can communicate with their first degree neighbours in the network graph \(G = (\mathcal{V}, \mathcal{E})\) (blue line segments) in Fig. 1(a).

The probability of detecting each target is \(P_D = 0.97\) and the standard deviations in range and bearing are selected as 5m and 1°, respectively, for all sensors. The clutter is uniformly distributed over the surveillance region with Poisson rate \(\lambda_C = 3\). Each sensor collects its measurements in its SCCS with the sensor at the origin and \(x\) and \(y\) axes aligned with North and West, respectively. Local filtering of these measurements are performed using a Sequential MC realisation of the PHD filter described in [15], from which target states can be estimated using clustering techniques.

We use the proposed self-localisation algorithm described in Section 4 alongside local filtering of target measurements and nearest neighbour posterior exchanges. We use 300 particles for each sensor location and start with noninformative priors. We compute the node-wise separable edge potentials for \(L = 5\) time steps followed by \(S = 5\) iterations of BP messaging. Therefore, once in every five steps of filtering, we update location estimates and repeat this until \(k = 150\). Equivalently, we update the edge potentials of BP once in every five messaging iterations. In Fig. 1(b), we present the localisation particles at \(k = 10, 20, 30, 150\) and 150 for a typical run.\(^2\) We repeat this example with 200 Monte Carlo realisations of the sensor measurement sequences and present a box-plot of the maximum (max.) estimation errors obtained for BP iterations in Fig. (c). Note that convergence occurs in less than 100 time steps. The highest of the max. errors between \(k = 100\) and \(k = 150\) over all runs (peak of the upper envelope drawn by the whiskers in Fig. (c)) is 24.1m which is a reasonably small error bound. The average for the maximum error at \(k = 150\) is 7.8m which is less than 0.02 of the 430.1m distance between the closest sensors in \(G\), and, close in value to the uncertainties related to the target measurements.

6. CONCLUSION

We proposed a cooperative self-localisation scheme for networks of sensors tracking targets. For sensor localisation based on target measurements in distributed fusion, we introduced an MRF model with node-wise separable edge potentials which can be computed locally. The proposed scheme consecutively updates these time recursive potentials and uses BP for decentralised estimation. It is capable of incorporating information from multiple targets and handle cluttered noisy measurements with a given detection probability. We demonstrated the efficacy of our algorithm through an example.

\(^2\)Results comparing the tracking performances of individual and fused target estimates are not presented due to lack of space. Obtaining the latter is made possible by the use of sensor location estimates from our algorithm.

7. REFERENCES


