Performance Evaluation of Mobile Multistatic Search Operations via Simulation

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ABSTRACT

Multistatic sonar networks (MSNs) utilize non-co-located sources and receivers. Although there are many advantages of MSNs, the complex and unusual geometry of their detection regions brings additional analytic challenges, especially in measuring the performance of a multistatic search operation. Furthermore, the challenge becomes harder when mobile sources are included in the network. Previous work has determined a closed form analytic expression for the equivalent sweep width of a MSN that includes a mobile source and stationary receivers, as well as the coverage achieved by parallel sweeps conducted by mobile sources in a field of stationary receivers. These formulae were derived using particular assumptions that are not always met in practice. In this paper, we conduct Monte Carlo simulations to investigate the accuracy of these analytic results under more realistic circumstances.

Author Keywords

Multistatic sonar, parallel search, mobile sensor.

ACM Classification Keywords

Algorithms, performance.

1. INTRODUCTION

The basic operating concept of a sonar is to emit sound energy from a source into the water and listen for the reflected echoes using a receiver. Using this concept, sonar operators have been able to detect, localize, and track targets of interest for decades. In a monostatic sonar system, the source and receiver are integrated into a single device. In a multistatic sonar network (MSN), they are separated by a distance large enough to be comparable to the distance to the potential target [18]. In other words, a MSN is a generalization of the traditional monostatic active sonar to the case where the source and receiver are not co-located. The source of energy can be a ship with a hull-mounted sonar, a helicopter with a dipping sonar, an explosive charge dropped by an aircraft or an active sonobuoy. The receiver can be a passive sonobuoy or a hydrophone system [18].

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Multistatic systems have a number of advantages over monostatic systems. Perhaps the most important one is the covertness of the receiver platforms, which makes taking countermeasures difficult for the target. As Cox [5] states, "*countermeasure tactics are greatly complicated if the target does not know the position of the receivers*." Additionally, multistatic systems enable multi-platform operations (such as a surface ship source and airplane deployed receivers), which brings a flexibility to the force structure. Multiple receivers also enable multi-angle observations and can improve target tracking accuracy. Along these lines, [2], [3], [12] and [13] study data fusion techniques in multistatic systems where multiple measurements collected by different sensors are converted into a single track estimate that is more precise and eliminates some of the false alarms that occur on monostatic sonar systems. [9] and [17] discuss the cost effectiveness of MSNs and argue that deploying more receivers than sources might significantly reduce costs without sacrificing performance.

On the other hand, the performance of a MSN is significantly more difficult to measure than a monostatic sonar system, mainly because of the differences of the geometry of both systems. In a monostatic sonar system the detection probability of a target is a function of the distance between the sonar and target. This relationship is more complicated in a MSN, where the detection probability is a function of the product of the source-target and target-receiver distances [15]. Assuming a definite range ("cookie cutter") sensing model, the detection region of a monostatic sonar in a 2D environment is a disk with a radius equal to the sensor's detection range. For a multistatic source-receiver couple, the detection region is bounded by a Cassini oval, which is the set of all points whose distance from the source, multiplied by the distance from the receiver, equals the *equivalent monostatic range* [18], i.e., the range at which a monostatic sensor would detect the target. Figure 1 shows a family of Cassini ovals for various separation distances between the source and receiver. More sophisticated sensing models also consider a phenomenon known as the *direct blast effect* [5, 8]; we neglect this phenomenon in our study.

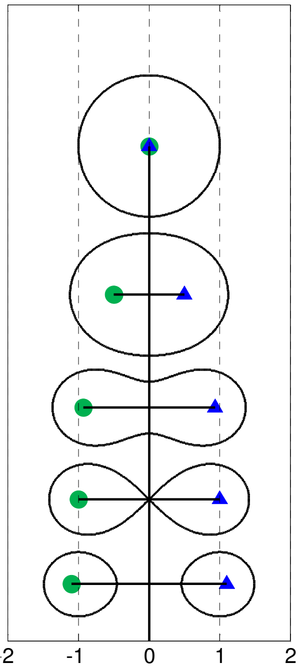

Figure 1. A family of Cassini ovals for various separation distances between the source and receiver (from [8]). Receivers and sources are denoted by ▲ and ●, respectively. Under a definite range sensor model, a target is detected if it lies within the detection region (Cassini oval) for some source and receiver, and otherwise it is not detected.

A variety prior works has attempted to quantify the performance of multistatic systems with stationary sources and receivers. For example, [1] develops the Multistatic Performance Prediction Methodology (MPPM), which evaluates detection probability as a function of source and receiver densities. [16] computes the expected probability of detection for a given target track in a MSN as a function of the number of sources and receivers, their location distribution functions, and the location and orientation of the track. Similarly, [6] derives the detection performance of a MSN based on the performance of a field of similar monostatic sonars. [18] considers a field of randomly-deployed multistatic sensors and develops an analytic theory for predicting the coverage of the network.

All of the abovementioned studies assume that both sources and receivers remain stationary. On the other hand, the coverage and tracking performance of a MSN can further be improved by utilizing a mobile source such as a surface ship with a hull-mounted sonar [14]. [4] proposes the use of a continuously emitting surface ship (possibly the towed-array low-frequency active sonar (LFAS)) with stationary receivers to search for objects such as mines, wrecks, or hostile submarines. However, the problem of performance measurements gets harder with a mobile source included in the network. Thus, a metric to predict the performance is essential.

One of the simplest metrics for quantifying the performance of a sensor configuration is its *sweep width*. The sweep width characterizes the average sensing capability ability of the configuration in a given search situation under a set of environmental conditions [10]; this concept will be defined more concretely in Section 2.1. Washburn and Karatas [17] derive an expression for the sweep width of a MSN with a mobile source. The authors also derive an expression for the coverage of parallel sweeps of mobile sources in an infinite Poisson field of stationary receivers. In this study, we use Monte Carlo simulations to investigate the accuracy of these analytic results.

The organization of the paper is as follows. Basic search theory concepts related to our problem, including the notion of lateral range curves, monostatic and multistatic sweep width, and parallel search, are briefly introduced in Section 2. Details of our simulation model are explained in Section 3. We present the numerical results of our experiments in Section 4, and we summarize our main results in Section 5.

2. PRELIMINARIES

2.1. Monostatic Sweep Width

A monostatic sensor's capabilities are of described by its lateral range curve, which is the plot of its lateral range function, $l(x)$. The lateral range function is defined as the cumulative detection probability of a target whose closest point of approach to the sensor occurs at range x. For a definite range monostatic sensor with a detection range of ρ, the lateral range function is expressed in Equation (1).

$$l(\mathrm{x}) = \begin{cases} 1 & \text{for} \quad -\rho \le x \le \rho \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

A sensor's sweep width, W, is a simple "detectability index" [10]. Mathematically, it is equal to the area under the lateral range curve, and it has units of distance. For the monostatic sensor with a detection range of ρ (as in Equation (1)), the sweep width is 2ρ. Note many different sensors can have the same sweep width; thus, this metric provides only a basic indication of a sensor's capabilities. Nonetheless, sweep width is a simple and widely-used measure of performance for search sensors.

2.2. Multistatic Equivalent Sweep Width

[17] considers a MSN consisting of a source moving on a straight line through a two-dimensional infinite Poisson field of stationary receivers of density h. Figure 2 shows a portion of the region covered by such a moving source. The authors derive the equivalent monostatic sweep width W_e as

$$W_e = 2\pi\rho^2\sqrt{h} \tag{2}$$

where ρ is the equivalent monostatic detection range (i.e., the detection range when source and receiver are co-located as a monostatic sonar).

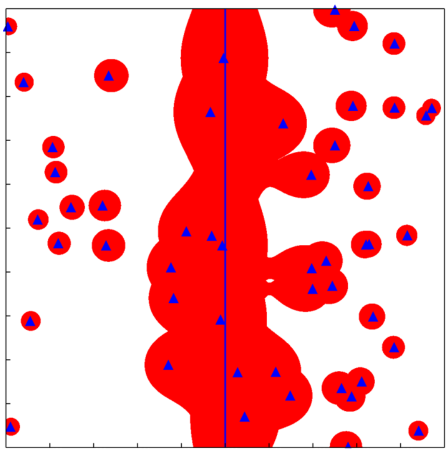

Figure 2. Coverage of a moving multistatic source in a field of stationary receivers of density *h*=0.05. Receivers are denoted by ▲, and a solid blue line represents the source path. The shaded region is the covered part of the search area.

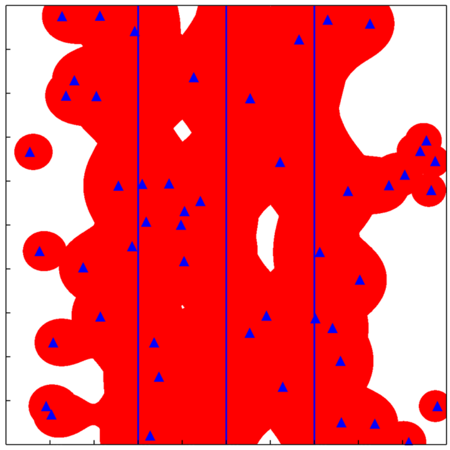

Figure 3. Coverage of a parallel search tracks of multistatic sources in a field of stationary receivers of density *h*=0.05. Receivers are denoted by ▲, and solid blue lines represent the source paths. The shaded region is the covered part of the search area.

2.3. Parallel Sweeps with Monostatic Sensors

When a region is swept by definite range monostatic sensors with parallel tracks separated by *W*, the entire region is covered and detection is assured. However, this is not the case if we adopt another useful and more realistic detection model, Koopman's inverse cube law of detection [11]. It models the detection probability of a target, *f*, as in Equation (3).

$$f(r) = \frac{k}{\gamma^3} \tag{3}$$

In this equation, γ is the distance between the searcher and target, and *k* is a constant (determined by experiment data) representing the environmental conditions and the target. This formula has been used in a wide variety of settings, most notably to model the performance of visual search [7]. For sensors that are characterized by such an inverse cube law of detection, the coverage of parallel search with a track spacing of *G* is computed as [19]

$$P_{mono}(G) = erf\left(\frac{\sqrt{\pi}}{2}\frac{W}{G}\right) \tag{4}$$

where *erf*() is the error function. This formula is of interest in sonar applications, as sonar devices typically do have decreasing detection probability with distance, when direct-path returns are considered. For the particular case of multistatic sonar, detection probability clearly decreases with distance, as shown in Figure 2. This is true even when a definite range law is used. We now turn our attention to the problem of conducting parallel sweeps with multistatic sources.

2.4. Parallel Sweeps with Multistatic Sources

Figure 3 shows the coverage achieved by parallel sweeps of mobile sources in an infinite Poisson field of stationary receivers. The coverage of a given sensor configuration can be thought of as the probability that a target placed uniform-ly at random within the area of interest will be detected; this definition accommodates sensor models other than the definite range model. Note that Equation (4) assumes that all parallel sweeps are independent, which is not true in the multistatic case. In a MSN, all receivers are used by each source; this Poisson field of receivers is the same for all sweeps.

Based on Koopman's inverse cube law sensor theory [11], the authors in [17] derive the probability of detection P_{multi} of parallel sweeps separated by distance *G* as

$$P_{multi} = 1 - e^{\left(-y^2/\pi\right)} + y\left(1 - \text{Gamma}\left(y^2/\pi\right)\right) \tag{5}$$

where $y \equiv W/S$ and Gamma() is the gamma distribution.

3. SIMULATION OF MOBILE MULTISTATIC SEARCH

We now describe our simulation model, which we use to perform computational experiments with two main goals:

- To investigate the accuracy of multistatic equivalent sweep width formula given in Equation (2).
- To test the parallel search performance of dependent tracks in a MSN and compare our results to those predicted by Equation (5).

Our simulation model assumes a definite range detection model in which receivers and targets are stationary and each source moves along a straight line at speed *v*. The pseudocode for this model appears in Figure 4. After initializing the input parameters, we create target and receiver locations distributed in the region.

For the case in which we wish to determine the sweep width for a single source, the initial location of the source is the midpoint of the uppermost boundary of the region. In each time step, the source moves distance $v\Delta t$ toward the bottom of the region. Targets and receivers are distributed uniformly at random in *A*.

Algorithm MSN_SEARCH

1. Inputs:
   - Source set S
     (*For Case 1:* $|S| = 1$)
     (*For Case 2:* $|S| = 2$)
   - Target set T
   - Receiver density h
   - Square search region with area A
   - Receiver set R ($|R| = hA$)
   - Equivalent monostatic detection range ρ
   - Source speed v
   - Time step length Δt
   - Separation distance G
2. Create entities:
   - *For Case 1:* Generate $|T|$ target locations (x_t) and $|R|$ receiver locations (x_r) uniformly at random in region A.
   - *For Case 2:* Generate $|T|$ target locations (x_t) uniformly between two source paths and $|R|$ receiver locations (x_r) uniformly at random in region A.
   - Generate source locations (x_s) as appropriate for the experiment.
3. Compute target-receiver distances
   $d_{t,r} = \|x_t - x_r\|, \forall t \in T, r \in R$
4. Initialize track length $l = 0$ and set of targets detected $\bar{T} = \varnothing$
5. **while** $l < \sqrt{A}$ **do**
   6. Compute distances:
      - Source-target distances:
      $d_{s,t} = \|x_s - x_t\|, \quad \forall s \in S, t \in T$
      - Source-receiver distances:
      $d_{s,r} = \|x_s - x_r\|, \quad \forall s \in S, r \in R$
   7. Determine detected targets:
      for all $t \in T$, $s \in S$, and $r \in R$ **do**
        if $d_{s,t} d_{t,r} \le \rho^2$
          $\bar{T} = \bar{T} \cup \{t\}$
        end if
      end for
   8. Move source(s) to their next location(s).
   9. Set $l = l + v\Delta t$.
10. **end while**
11. **return** coverage percentage = $|\bar{T}| / |T|$

    sweep width = $\dfrac{|\bar{T}|}{|T|} \cdot l$

Figure 4. The MSN_SEARCH pseudocode for our simulations.

To examine the parallel search performance of multiple dependent tracks, we initialize the locations two sources along the uppermost boundary of the region, separated from each other by distance G (see Figure 5). Again, each source moves distance $v\Delta t$ toward the bottom of the region in each time step. Receivers are distributed uniformly at random within A, while targets are distributed uniformly at random between the two search tracks. Note that in this case our main ambition is to compute the coverage performance of a mobile multistatic source repeatedly sweeps a Poisson field of stationary receivers with tracks spaced G. We will compa

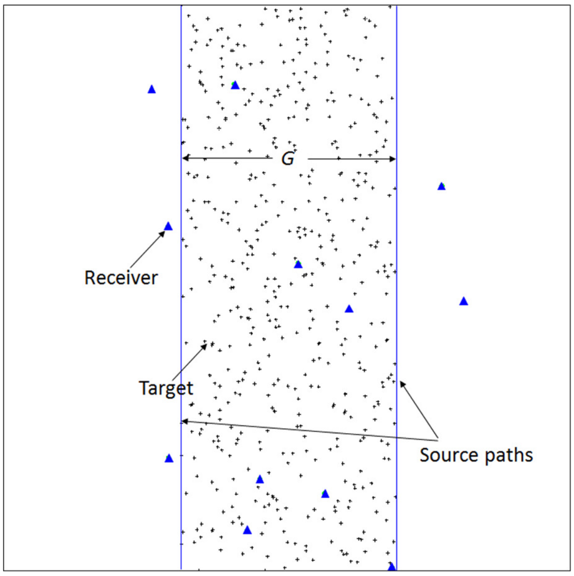

Figure 5. An example simulation setup for parallel search tracks of multistatic sources in a field of stationary receivers. Receivers are located uniformly at random in region A. Targets (denoted by + sign) are generated between the search paths of sources.

-re our simulation results with equation (5) which is also a function of the multistatic sweep width in equation (2).

Each time step, computations are performed to determine which targets are detected, and this information is ultimately used to compute the sweep width and parallel search performance, as appropriate. The coverage percentage is computed by the ratio of the number of detected targets to the total number of targets, while the sweep width is the product of coverage percentage and the width of the area of interest.

4. NUMERICAL RESULTS

For our computational experiments, we fix the area of the search region to A=10^3 nm^2, and we let $v\Delta t$=0.01 nm. To explore the accuracy of Equations (2) and (5) over a range of conditions, we vary two factors: h and ρ^2. We perform 30 replications for each combination of $h \in \{0.01, 0.02, \ldots, 0.05\}$ and $\rho^2 \in \{1, 2, \ldots, 10\}$. Thus, we perform a total of 1500 simulation runs for each of our test cases. We consider problem instances with 10^6 targets. For parallel sweep simulation we choose G=10 nm.

4.1. Test Case 1: Multistatic Equivalent Sweep Width

Figures 6 and 7 show the results of our first test case, in which a single source crosses the region and we wish to determine its sweep width. Figure 6 compares the sweep width measured by the simulation model (left) and computed with the analytic formula (right). As expected, equivalent sweep width increases with both the density of receivers h and equivalent monostatic detection range ρ. Although both the simulation and analytic results show a similar trend, the theoretical sweep width appears to be optimistic relative to the simulation.

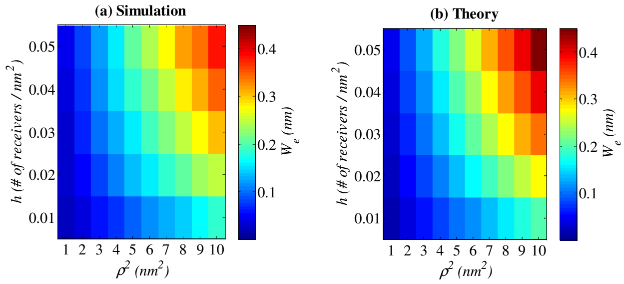

Figure 6: Comparison of the sweep width determined by (a) simulation and (b) Equation (2).

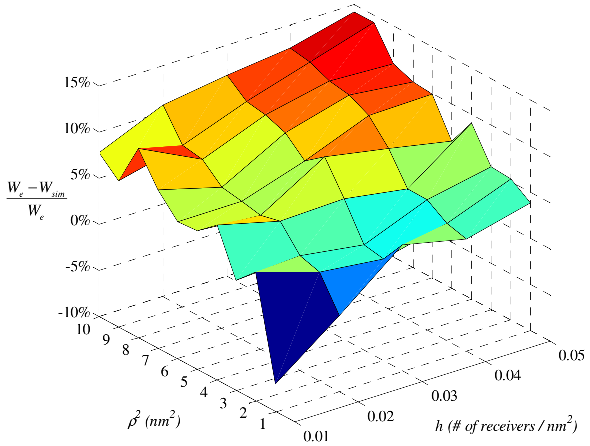

Figure 7: Relative difference in sweep width between simulation and analytic results.

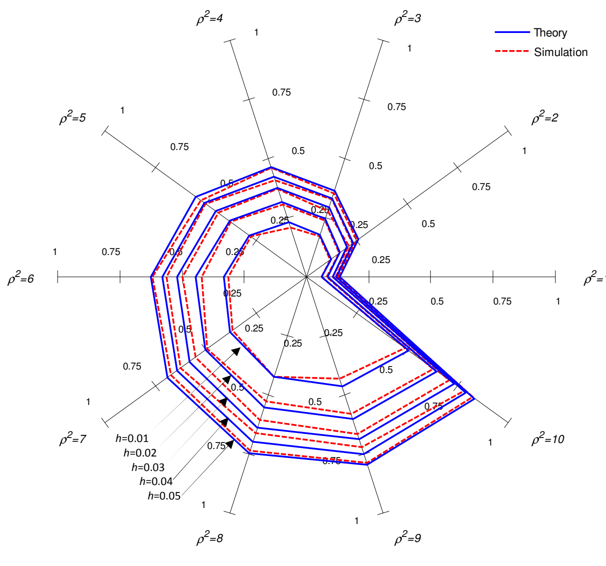

Figure 8: Comparison of probability of detection determined by simulation with analytic equation.

Figure 7 shows the ratio $(W_e - W_{sim})/W_e$, where W_{sim} is the sweep width calculated by the simulation, and W_e is the sweep width predicted by Equation (2). Figure 6 indicates that the discrepancy between the two sweep width values is not constant over all settings; rather, it is larger in conditions where a higher sweep width value is predicted. Recall that Equation (2) assumes an infinite Poisson field of receivers with density h. In our simulation, we restrict the search area to A and compute the coverage within that finite region, which we then use to calculate a sweep width. When sensing performance is poor (i.e., low h and ρ^2), most detection occurs close to the source and within our finite area of interest. For situations in which sensing performance is expected to be good (i.e., high h and high ρ^2), a significant amount of area can be covered outside of our area of interest. Thus, for practical applications in which the area of interest is not infinite, it is important to recall that the analytic sweep width formula may be optimistic, particularly when it predicts very large sweep widths.

4.2. Test Case 2: Coverage of Multistatic Parallel Search

Figure 8 shows the results of our second test case, in which we study the coverage achieved by multiple sources performing parallel sweeps. The figure compares the coverage measured by the simulation model to that predicted by Equation (5).

The analytical prediction exhibits a small degree of optimism (around 1%) compared to the simulation results. This level of optimism is approximately constant over all conditions. As expected higher coverage ratio is achieved for high h and ρ^2 values. We also see that the introduction of multiple tracks brings about an overall reduction in the discrepancy between the two results, due to the fact that it is now impossible for any part of the region to be "far from" all sources, as was the case in our first scenario.

5. DISCUSSION AND CONCLUSIONS

Sensor systems can be very costly, and appropriate analysis can ensure that time and money are well spent when a new system is deployed. Innovative and complicated new technologies such as multistatic sonar can benefit from both analytical and simulation-based analyses prior to deployment.

In this study, we compare the results of those predicted by analytical formulas for two scenarios involving mobile sources. In the first case, we consider the sweep width of single mobile source in a field of multiple stationary receivers. In the second case, we examine the coverage achieved by multiple sources in a parallel search setting. Numerical results for the first case show that although our simulation model produces results that are qualitatively similar to the analytical models, differences do occur. We believe that these differences are due mainly to the assumption of an infinite field made by the analytic models; this assumption may have important ramifications in a practical setting. In particular, the analytical formula for sweep width studied in this paper may produce optimistic results, particularly when sensing performance is predicted to be especially good. For the second case our numerical results match nicely with the analytic model, hence the parallel search formula can be used to approximate the coverage of a moving multistatic source repeatedly sweeps a Poisson field of stationary receivers.

Future work may study the sensitivity of our results to changes in other factors, such as the size of the area of interest. Future work may also extend the simulation model with new features, such as sea current and wind conditions, for which analytical study is intractable. Another future study may compare the performance of a traditional monostatic sensor network with an equivalent MSN in terms of coverage.

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