Erratum


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In this paper, the study of ordering based on implications, given in (Information Sciences, 276 (2014) 377–386) is revised. The implication given in Example 1 is corrected and its right version is shown.

In [2], in Example 1 on page 384, we have introduced a function \( I : [0, 1]^2 \rightarrow [0, 1] \) given by

\[
I(x, y) = \begin{cases} 
1 & x = 0 \text{ or } y = 1, \\
1/2 & x < 1/2 \text{ and } y > 1/2, \\
\max(1 - x, y) & \text{otherwise}.
\end{cases}
\]

claiming that \( I \) is an implication. However, the correct version of \( I \) for this example should be now this new formula

\[
I(x, y) = \begin{cases} 
1 & x < 1/2 \text{ and } y > 1/2, \\
\max(1 - x, y) & \text{otherwise}.
\end{cases}
\]

The assertion mentioned by Example 1 is valid. According to the new formula for implication, the corrected version of Example 1 is as follows.

**Example 1.** Consider the t-norm \( T : [0, 1]^2 \rightarrow [0, 1] \) defined as

\[
T(x, y) = \begin{cases} 
0 & (x, y) \in (0, 1/2)^2, \\
\min(x, y) & \text{otherwise},
\end{cases}
\]

on \([0, 1]\). Let \( S \) be the dual t-conorm of \( T \) and take \( N(x) = N_c(x) = 1 - x \). Then, \( I(x, y) = S(N(x), y) \) is an implication satisfying the exchange principle (EP) and the contrapositive symmetry (CP) with respect to the strong natural negation \( N_i = N \) ([1], Prop. 2.4.3). The implication \( I : [0, 1]^2 \rightarrow [0, 1] \) is defined as follows:
Let us prove that $I(x, y) = y$. Assume that $I(x, y) = x$ for some $x < 1/2$ and $y > 1/2$, otherwise it would be $y = 1$, contradiction. Then, $x \geq 1/2$. Since $y = I(x, x) = \max(1 - x, x)$ and $x \neq y$, we obtain that $y = 1 - x$. According to our assumption $y > 1/2$, it must be $\ell < 1/2$, a contradiction again. This shows that $x$ and $y$ are not comparable w.r.t. $\leq_1$. If $1/2 < y < x$, then by similar way, it can be shown that $x$ and $y$ are not comparable w.r.t. $\leq_1$. Thus, if $x$ and $y$ are not comparable elements w.r.t. $\leq_1$, it must be $x, y > 1/2, x, y \neq 0.1$ and $x \neq y$. The elements not comparable with respect to $\leq_1$ can be depicted in Fig. 1.

Now, let us show that $x \wedge_1 y = 1/2$ if $x$ and $y$ are not comparable w.r.t. $\leq_1$. Since $x, y > 1/2$, $I(1 - x, 1/2) = \max(x, 1/2) = x$ and $I(1 - y, 1/2) = \max(y, 1/2) = y$ hold. So, $x \leq_1 1/2$ and $y \leq_1 1/2$, that is, $1/2 \in [x, y]_1$. Let $k \in [x, y]_1$ be arbitrary. Then, $x \leq_1 k$ and $y \leq_1 k$.

Then, there exist two element $\ell_1, \ell_2 \in [0, 1]$ such that $I(\ell_1, k) = x \neq 1$ and $I(\ell_2, k) = y \neq 1$.

Thus, it must be either $\ell_1 \geq 1/2$ or $\ell_2 \leq 1/2$, otherwise it is obtained that $x = 1$ which is a contradiction. Let $\ell_1 \geq 1/2$. Since $I(\ell_1, k) = \max(1 - \ell_1, k) = x$, either $x = k$ or $1 - \ell_1 = x$. If $x = k$, it would be $y \leq_1 x$ since $I(\ell_2, x) = I(\ell_2, k) = y$, which is a contradiction. So, it must be $1 - \ell_1 = x$. Since $x > 1/2$, it is obtained that $\ell_1 < 1/2$, this is a contradiction. Then, it must be $k \leq 1/2$. Since $I(1/2, k) = \max(1/2, k) = 1/2$, we obtain that $1/2 \leq_1 k$, that is, $1/2$ is the least of the upper bounds of the elements $x, y$ w.r.t. $\leq_1$, whence $x \wedge_1 y = 1/2$.

Let us prove that $x \wedge_1 y = k$. Suppose that $x \wedge_1 y = k$ and $k \neq 1$. Then, $k \leq_1 x$ and $k \leq_1 y$. By the definition of the order $\leq_1$, there exist two elements $\ell_1, \ell_2 \in [0, 1]$ such that $I(\ell_1, x) = k$ and $I(\ell_2, y) = k$.

By Proposition 2, it is clear that $1/2 < x \leq k$ since $k \leq_1 x$ and $x > 1/2$. Since $x > 1/2$, it is not possible $\ell_1 < 1/2$, otherwise we obtain that $k = 1$, a contradiction. So, $\ell_1 \geq 1/2$. Since $k = I(\ell_1, x) = \max(1 - \ell_1, x) = x$, we have that $x = k = I(\ell_2, y)$, contradicts that $x$ and $y$ are not comparable with respect to $\leq_1$. So, we have that $k = 1$, i.e., $x \wedge_1 y = 1$.

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