Implementation of Direct torque control of an induction motor with High Gain Observer on FPGA

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Abstract—The Conventional Direct Torque Control (CDTC) of Induction Motor (IM) with an open loop estimator of stator flux suffers from problem like the correction of measurement errors of the stator current and estimation errors of stator flux and the problem of integration particularly at low speeds. To solve these problems, we use the high gain observer (HGO). This observer is used to estimate the stator flux, the stator currents, the load torque and the mechanical speed. In this paper, we investigate the hardware implementation method of Direct Torque control with High Gain Observer on FPGA using a Xilinx System Generator tool that generates synthesizable VHDL (VHSIC Hardware Description Language) code. The advantages of this method are the rapid time to market, real time and portability. The model of the CDTC with HGO has been designed and simulated using Xilinx System Generator blocks, synthesized with Xilinx ISE 12.4 tool and implemented on Xilinx Virtex-V FPGA.

Keywords— Direct Torque Control, High Gain Observer, Xilinx System Generator (XSG), FPGA, Real Time.

I. INTRODUCTION

For the real time implementation of the control algorithms of industrial systems two main families are used, such as the software solutions (Microcontroller, Digital Signal Processor “DSP (dsPACE”) and hardware solutions (FPGA). The main limitation of the software solutions is the sequential processing that affects the processing speed and increase the execution time [1] [2] [3]. To overcome the software solutions limitations, an alternative family based in the Field Programmable Gate Arrays (FPGAs). The use of these hardware solutions FPGA makes it possible to find some analog performance while keeping the advantages of digital solutions [4]. The inherent parallelism in these new digital solutions and their large computing capacity make the computation time are negligible despite the complexity of algorithms to implement. In recent years, many researches use the FPGA for the implementation of control algorithms of electrical system [5] [6] [7] [8] [9] [10] [11]. Most of them use the VHDL code. In this work we use Xilinx System Generator (XSG) approach to implement a design on the FPGA. The advantages of this approach are the rapid time to market, real time and portability.

Induction machines are currently the most used machines in the industrial field thanks to their low cost, robustness, good performance and simple implementation [12]. The CDTC is based on the orientation of the stator flux by a direct action on the states of the switches of the inverter [13] [14] [15] [16] [17] [18].

In recent years, several researchers have explored the development of sensorless control algorithms of induction motor using online estimation algorithms for the construction of the stator flux, stator current and rotor speed. Many algorithms are used, like the Model Reference Adaptive System (MRAS) [19] [20] [21] but the drawback of this observer is the sensitivity to the machine parameters such as the stator resistance in the Direct Torque Control, the Luenberger Observer is used [22] for state estimation, Extended Kalman Filter (EKF) algorithm is used in [23] [24], the limitation of this observer that the load torque dynamics is not known. In this work, we use a powerful observer named High Gain Observer (HGO), it can estimate simultaneously the stator flux, stator current, load torque, rotor speed and machine parameters. The open loop estimator of stator flux method used in the conventional DTC of IMs suffers from the problems of integration especially at low speed; it is so replaced by the High Gain Observer. This observer is used to estimate the stator currents, the stator flux and the rotation speed.

This paper is organized as follows. In section II, a brief introduction of design methodology for implementation on FPGA is presented. The state model of an induction motor, the principle and the model of the DTC control are presented in section III. In section IV, The mathematical model of the high gain observer and the hardware implementation on FPGA using Xilinx System Generator are presented. In Section V simulation results are presented to test the performance of the high gain observer. Finally, we conclude in section VI.

II. DESIGN METHODOLOGY FOR IMPLEMENTATION ON FPGA

The XSG software is a toolbox developed by Xilinx; it is integrated into Matlab/Simulink environment and lets the user to create parallel systems for FPGAs. The created models are displayed as blocks, and can be connected to other blocks and other toolbox of Matlab/Simulink-like (Sim-Power-Systems) SPS. Once the system is developed from Xilinx System Generator, we can generate the VHDL (VHSIC Hardware Description Language) code exactly reproduces the behavior observed in Matlab. The design flow from System Generator is given in Figure 1.
III. DIRECT TORQUE CONTROL OF INDUCTION MOTOR

A. State Model of Induction Motor

The state model of induction motor in the stationary reference linked to the stator is given as:

\[
\begin{align*}
\frac{dX}{dt} &= [A]X + [B]U \\
X &= \begin{pmatrix}
i_{sa} \\ i_{sb} \\ \varphi_{\alpha a} \\ \varphi_{\beta a}
\end{pmatrix}, \\
U &= \begin{pmatrix}
V_{\alpha a} \\ V_{\beta a}
\end{pmatrix}
\end{align*}
\]

Where \(X, U, A\) and \(B\) are the state vector, control vector, the dynamic and the control matrices respectively.

\[
[A] = \begin{pmatrix}
a & a_6 & a_2 & a_3 \\
a_6 & a_1 & a_3 & a_2 \\
a_4 & 0 & 0 & 0 \\
0 & a_4 & 0 & 0
\end{pmatrix}, \\
[B] = \begin{pmatrix}
b_1 & 0 \\
0 & b_1 \\
1 & 0 \\
0 & 1
\end{pmatrix}
\]

where \(a = \frac{R_s}{L_s} + \frac{R_r}{L_r}, a_2 = \frac{R_r}{\sigma L_s}, a_3 = \frac{\omega}{\sigma L_s}, a_4 = -R_s, a_6 = -\omega, b_1 = \frac{1}{\sigma L_s}\).

The electromagnetic torque is expressed by the following equation; this equation is used by Direct Torque Control.

\[
C_e = \frac{3}{2} p(\varphi_{\alpha a} i_{\beta a} - \varphi_{\beta a} i_{\alpha a})
\]

B. Principle of Direct Torque Control

Direct torque control of an induction machine is based on the direct determination of the control sequence applied to the switches of a voltage inverter. The choice of sequences is based on the two hysteresis comparators of the stator flux and electromagnetic torque [25]. The voltage vector \(V_s\) is the output of a three-phase voltage inverter whose the state of the inverter switches are controlled by three Boolean variables \(S_j\) (\(j = a, b, c\)). The voltage vector can be written as:

\[
\begin{align*}
V_s &= \sqrt{\frac{2}{3}} U(S_a + S_b e^{j\frac{2\pi}{3}} + S_c e^{j\frac{4\pi}{3}}) \\
v_s &= v_{sa} + jv_{sb}
\end{align*}
\]

The components of the stator voltage vector \(V_s(V_{sa}, V_{sb})\) and the stator flux vector \(\varphi_s(\varphi_{\alpha a}, \varphi_{\beta a})\) in Concordia reference are given by equations (4) and (5). The calculation of the position and module of the stator flux are based on the use of components \((\varphi_{\alpha a}, \varphi_{\beta a})\). The module of the stator flux and its position are given by equations (6).

\[
\begin{align*}
\varphi_{\alpha a} &= \int_0^t (V_{sa} - R_s i_{sa}) dt \\
\varphi_{\beta a} &= \int_0^t (V_{sb} - R_s i_{sb}) dt \\
V_{sa} &= \frac{2}{3} U(S_a - \frac{1}{2} (S_b + S_c)) \\
V_{sb} &= \frac{2}{3} U(S_b - S_c)
\end{align*}
\]

\[
\begin{align*}
\varphi_s &= \sqrt{\varphi_{\alpha a}^2 + \varphi_{\beta a}^2} \\
\theta_s &= \arctg \frac{\varphi_{\beta a}}{\varphi_{\alpha a}}
\end{align*}
\]

The electromagnetic torque is expressed in terms of the components of stator flux vector and the components of stator current vector as:

\[
C_e = \frac{3}{2} p(\varphi_{\alpha a} i_{\beta a} - \varphi_{\beta a} i_{\alpha a})
\]

The estimated values of the stator flux and electromagnetic torque are compared with their reference values \(\varphi_{\text{ref}}, T_{em}^*\) respectively. Switching states are selected by the switching table, where \(E_C\) is the error of
electromagnetic torque after hysteresis block and $E_{\phi}$ is the error of the stator flux after hysteresis block, $S_i (i = 1 \ldots 6)$ means the sector (Table 1) [26]:

Table 1. Switching table for direct torque control

<table>
<thead>
<tr>
<th>$E_{\phi}$</th>
<th>$E_{c}$</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>V2</td>
<td>V3</td>
<td>V4</td>
<td>V5</td>
<td>V6</td>
<td>V1</td>
</tr>
<tr>
<td>0</td>
<td>V7</td>
<td>V0</td>
<td>V7</td>
<td>V0</td>
<td>V7</td>
<td>V0</td>
<td>V7</td>
</tr>
<tr>
<td>-1</td>
<td>V6</td>
<td>V1</td>
<td>V2</td>
<td>V3</td>
<td>V4</td>
<td>V5</td>
<td>V6</td>
</tr>
<tr>
<td>0</td>
<td>V0</td>
<td>V7</td>
<td>V0</td>
<td>V7</td>
<td>V0</td>
<td>V7</td>
<td>V0</td>
</tr>
<tr>
<td>-1</td>
<td>V5</td>
<td>V6</td>
<td>V1</td>
<td>V2</td>
<td>V3</td>
<td>V4</td>
<td>V5</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>V2</td>
<td>V3</td>
<td>V4</td>
<td>V5</td>
<td>V6</td>
<td>V1</td>
</tr>
</tbody>
</table>

Figure 2: Schematic of Conventional direct Torque control

IV. DIRECT TORQUE CONTROL OF AN INDUCTION MOTOR WITH HIGH GAIN OBSERVER

A. Principle of High Gain Observer

The objective of the high gain observer is to estimate the stator flux, stator current and rotor speed. The Direct Torque Control with high gain observer of induction motor given in figure 3.

The dynamic model in the canonical form of the induction machine can be represented in the $(\alpha, \beta)$ frame as:

$$\begin{bmatrix}
\dot{i}_x \\
\dot{\varphi}_S \\
\dot{\omega}
\end{bmatrix} =
\begin{bmatrix}
A_i & kF(\omega) & BV_S \\
0 & 0 & -R_S \\
\frac{3}{2j} & \frac{1}{j} & \frac{1}{j} \\
\end{bmatrix}
\begin{bmatrix}
i_S \\
\varphi_S \\
\omega
\end{bmatrix}
$$

where:

- $F(\omega) = \begin{bmatrix}
\frac{1}{T_r} & \omega \\
-\omega & \frac{1}{T_r}
\end{bmatrix}$

The observation model of the induction motor can be represented in the $(\alpha, \beta)$ frame as:

$$\begin{bmatrix}
\hat{x} \\
\hat{v}(e_m)
\end{bmatrix} =
A_i \begin{bmatrix}
\hat{x} \\
\hat{v}(e_m)
\end{bmatrix} + B u + v(e_m)
$$

where:

- $v(e_m) = \theta_i \Delta^{-1}(x) \Delta_{\hat{\theta}}^{-1} S^{-1} C^T e_m$

and modeling error on the measured state.

- $\Delta_{\hat{\theta}} = diag(I_2, \frac{1}{k} I_2)$ and $\theta_i = 0$ \Rightarrow $\Delta_{\hat{\theta}}^{-1} = diag(I_2, \theta_i I_2)$
- $\Lambda = diag(I_2, kF(\omega))$ and $\theta_i = 0$ \Rightarrow $\Lambda^{-1} = diag(I_2, \frac{1}{k} F^{-1}(\omega))$
- $S$ is the unique symmetric positive definite solution of the algebraic Lyapunov equation which is a powerful tool for studying the stability of the observer.

$$S + A^T S + SA = C^T C \Rightarrow S^{-1} C^T = \begin{bmatrix}
C_1 I_2 \\
C_2 I_2
\end{bmatrix} \Rightarrow \begin{bmatrix}
2I_2 \\
I_2
\end{bmatrix}
$$

This gives:

$$v(e_m) = \begin{bmatrix}
\frac{27}{k} F^{-1}(\omega) e_m \\
\frac{1}{k} F^{-1}(\omega) e_m
\end{bmatrix}
$$

Replacing $v(e_m)$ by its expression, we obtain the following state model of the observer:
It is the synthesis parameter to adjust, if need be large enough to weaken the nonlinearity of the system. But to choose this value must ensure a compromise between stability and rapidity of convergence of the observer.

\[
\begin{aligned}
\frac{d \hat{i}_{Sa}}{dt} &= a_i i_{Sa} + a_p i_{Sp} + a_2 \phi_{Sa} + a_3 \phi_{Sp} \\
+ b_1 v_{Sa} + 2\theta_i (i_{Sa} - \hat{i}_{Sa}) \\
\frac{d \hat{i}_{Sp}}{dt} &= -a_i i_{Sa} + a_i i_{Sp} - a_3 \phi_{Sa} + a_2 \phi_{Sp} \\
+ b_1 v_{Sp} + 2\theta_i (i_{Sp} - \hat{i}_{Sp}) \\
\frac{d \hat{\phi}_{Sa}}{dt} &= a_4 i_{Sa} + v_{Sp} + \theta_i \frac{a_2}{a_2 + a_3} (i_{Sa} - \hat{i}_{Sa}) \\
+ \theta_i \frac{a_3}{a_2 + a_3} (i_{Sp} - \hat{i}_{Sp}) \\
\frac{d \hat{\phi}_{Sp}}{dt} &= a_4 i_{Sp} + v_{Sp} + \theta_i \frac{a_2}{a_2 + a_3} (i_{Sa} - \hat{i}_{Sa}) \\
+ \theta_i \frac{a_3}{a_2 + a_3} (i_{Sp} - \hat{i}_{Sp}) \\
\dot{\omega} &= \frac{3}{2} p^2 (\phi_{Sa} \hat{i}_{Sp} - \phi_{Sp} i_{Sa}) - p C_r - fp \Omega
\end{aligned}
\]

The Observer of load torque is given by the following system.

\[
\begin{aligned}
\dot{\Omega} &= \frac{3}{2} p (\phi_{Sa} \hat{i}_{Sp} - \phi_{Sp} i_{Sa}) \\
- \frac{1}{J} \dot{T}_L + 3\theta_2 (\Omega - \Omega) \\
\dot{T}_L &= \dot{T}_{lp} + 3\theta_2 J (\Omega - \Omega) \\
\dot{T}_{lp} &= \theta_2 J (\Omega - \Omega)
\end{aligned}
\]

The Observer of Rotor speed is given by the following system.

\[
\begin{aligned}
\dot{\theta}_r &= \frac{1}{T_r} (V_s - R_s \hat{i}_s) + pJ_s (V_s - R_s \hat{i}_s) \omega \\
- J_2 \omega - \frac{3}{k} \theta_3 (i_s - \hat{i}_s) \\
\dot{\Omega} &= \frac{3}{2J} \omega \hat{i}_s J F^{-1}(\omega) r - \frac{1}{J} T_L - \frac{1}{J} F \dot{\Omega}
\end{aligned}
\]

B. Design of the CDTC with High Gain Observer using Xilinx System Generator (XSG)

The design of the Direct Torque Control with High Gain Observer of induction motor using XSG is based on mathematical models obtained previously. So, we present some examples of mathematical design from Xilinx System Generator like the observed Stator current $\hat{i}_{Sa}$, observed Stator flux $\hat{\phi}_{Sa}$ and sector calculation, given by figure 4, figure 5 and figure 6 respectively.

![Figure 4: Design of observed Stator current $\hat{i}_{Sa}$](image)
V. SIMULATION RESULTS

A. Conventional DTC with speed sensor

The simulation of Conventional DTC with an open loop estimator using Xilinx System Generator. In this simulation; the reference of electromagnetic torque is presented by the output of PI controller of speed, the reference of the stator flux is 0.91 wb. The period of the system is 50 µs. The Motor started without load torque to achieve a certain steady-state speed, at t=0.5 sec, a 10 Nm of load torque was applied. We obtain in Figure 7 (a), Figure 7 (b), Figure 8 (a), Figure 8 (b), the evolution of real rotor speed, the variation of electromagnetic torque, the evolution of stator flux and the evolution of stator flux vector trajectory respectively.

The stator flux is quickly reaches its reference value of 0.91 Wb. It is noted that the speed reaches the reference speed without overshoot. It is important to note that the control system shows a good prosecution.

A. Conventional DTC with High Gain Observer (without sensor)

The simulation of CDTC with high gain Observer is achieved using Xilinx System Generator. In this simulation; the reference of electromagnetic torque is presented by the output of PI controller of speed, the reference of the stator flux is 0.91 wb. The period of the system is 50 µs. The Motor started without load torque to achieve a certain steady-state speed. At t=0.2 sec, t=0.5 sec and t=0.8 sec a load torque was applied. We obtain in Figure 9, Figure 10, Figure 11, Figure 12 and the variation of real and load torque, evolution of real and observed speed, variation of electromagnetic torque, evolution of stator flux and evolution of stator flux trajectory, respectively. Figure 13 present the evolution of the rotor speed and load torque with variation of speed (150 rad/s to 104 rad/s).

1. Test at constant speed and variable load torque

Figure 9: Evolution of the real and observed load torque
Figure 9 illustrates the observed load torque issued by High Gain Observer. We can see that the observed load torque converges to the real load torque.

![Figure 9: Evolution of the real and observed load torque](image)

2. Test with speed variation and no load torque

Figure (13) shows the simulation results during the change of operating mode by reducing the speed of 150 rad/s to 104 rad/s.

![Figure 13: Evolution of real and observed rotor speed](image)

In figure 13, the speed is almost similar with the real speed motor. It is important to note that the control system demonstrates good prosecution even in severe conditions like the speed variation.

Once the simulation is completed and gives the desired results, we can generate the VHDL code and synthesized the hardware block. The Table I show the implementation results for the Direct Torque Control with High Gain Observer using the FPGA Virtex-5 Device. The RTL schematic of the CDTC with high gain observer is given by figure 14.

![Figure 14: RTL schematic of the CDTC with high gain observe](image)

<table>
<thead>
<tr>
<th>Resources</th>
<th>DTC with High Gain Observer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of bonded I/O</td>
<td>68</td>
</tr>
<tr>
<td>Number of Slice LUs</td>
<td>1987</td>
</tr>
<tr>
<td>Number of Slices Registers</td>
<td>478</td>
</tr>
<tr>
<td>Nombre de BUFG/BUFGCTRLs</td>
<td>1</td>
</tr>
<tr>
<td>Number of DSP48Es</td>
<td>6</td>
</tr>
</tbody>
</table>

The performance of the FPGA Virtex V in term of execution time is given by the table II, with $t_{DTC-HGO}$ and $t_{ADC}$ is the execution time and analogue to digital conversion time respectively.

![Table I: RESOURCES UTILISATION](image)

In figure 10, illustrates the observed speed (rad / s) issued by High Gain Observer. The speed is almost similar with the real speed motor.

![Figure 10: Evolution of the real and observed rotor speed](image)

In figure 11, shows the estimated electromagnetic torque using the High Gain Observer. We can see that the estimated electromagnetic torque converges to the real electromagnetic torque.

![Figure 11: Evolution of real and estimated electromagnetic torque](image)

In figure 12, we can see that the observed flux converges to real stator flux.

![Figure 12: Evolution of the Real and observed Stator flux](image)
TABLE II. PERFORMANCE OF THE FPGA IN TERMS OF EXECUTION TIME

<table>
<thead>
<tr>
<th>Module</th>
<th>Execution Time (µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimator electromagnetic torque, stator flux module and angle calculation $\Theta_s$</td>
<td>0.32</td>
</tr>
<tr>
<td>High Gain Observer</td>
<td>0.34</td>
</tr>
<tr>
<td>Switching Table</td>
<td>0.16</td>
</tr>
<tr>
<td>Sector and hysteresis controller</td>
<td>0.32</td>
</tr>
<tr>
<td>$T_{DTC+HGO}$ = 0.12 + 0.32 + 0.32 = 1.26 µs</td>
<td></td>
</tr>
<tr>
<td>$+0.34 + 0.16 + 0.32 = 1.26$ µs</td>
<td></td>
</tr>
<tr>
<td>Total Time: $T_{total} = T_{DTC+HGO} + T_{ADC}$</td>
<td></td>
</tr>
</tbody>
</table>

VI. CONCLUSION

This work presents an improvement of CDTC using High Gain Observer. It aims to overcome the limitation of the open loop estimator and to avoid the disadvantages of mechanical sensor. The Direct Torque Control with High Gain Observer is designed to be implemented on the FPGA using Xilinx System Generator which presents an interesting approach. So, the obtained design can be translated automatically into VHDL programming language and can be embedded into the Xilinx FPGA. The simulation results have shown that the performance of the Direct Torque Control with high gain observer are successfully demonstrated. In the future work, we will try to validate experimentally the proposed scheme it and see the experimental results.

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