Quantitative Analysis of Mixer-Type Rheometers using the Couette Analogy

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Rheological characterization of complex fluids may in certain circumstances be a challenging task when conventional rheometers are used. Some food materials, for example, have microstructures with characteristic dimensions in the order of magnitude of the gap available for flow in the conventional rheometers. Placing a sample of such fluid in these geometries may result in a partial destruction of the internal structure. In other situations, phase separation of the basic constituents of the fluid to characterize occurs while the measurement is in progress. Mixer-type rheometry, consisting of a mixing device with a more or less complex geometry rotating in a fluid contained in a cylindrical tank, provides an alternative solution to such rheological characterization problems (Choplin and Marchal, 1996). Mixing devices with large local spacing can indeed be used to handle fluids with relatively large heterogeneities while providing continuous mixing. Monitoring torque and rotational speed during the mixing process can provide a rheological signature of the material under evaluation, i.e. torque/speed curves that are similar in shape to those obtained with conventional geometries. However, a more detailed analysis is required to translate this information into a complete rheological characterization, i.e. viscosity/shear-rate data in Couette virtual rheometers, where either flow kinematics or flow dynamics are controlled, are used.

From another standpoint, the market demand for sophisticated products with multifunctional characteristics is steadily increasing. Complex formulations are generally needed to meet stringent requirements of some applications. Such formulations are based on in-house know-how and experienced individuals. Efforts to integrate most of the aspects of such formulation recipes have recently been made and the qualitative results are quite interesting in terms of process follow-up and monitoring using rheological measurements (Choplin and Marchal, 1999). The approach allowed a study of complex and evolving systems with the help of rheometry using the so-called rheo-reactors, which can be operated under batch and/or semi-batch conditions. In these rheo-reactors, conventional geometries generally used in commercial rheometers are replaced by more complex geometries, e.g. impeller-vessel or any other cup-mixing device assembly.

In this work, we suggest a quantitative analysis of rotational speed/torque data of different types of mixing devices to extract the rheological parameters of interest. The analysis is based on a Couette analogy for the mixing device under consideration.

Theory
To quantitatively analyze torque-rotor speed data of mixer-type rheometers, we shall follow the approach used by Bousmina et al. (1999) to

A procedure based on a Couette analogy, to quantitatively analyze torque/rotor speed data in order to extract viscosity/shear-rate curves using non-conventional geometries is presented. It is first validated using a relatively simple geometry for which the equivalent internal radius used in the analogy can be analytically obtained. The results showed that the equivalent internal radius depends only slightly on the nature of the fluid and that there is an optimal radial position \( r^* \) in the analog Couette gap where the calculations can be easily performed for computing the viscosity/shear-rate data from torque/rotational speed data. The experimental results with complex geometries and complex fluids are found to coincide, within experimental errors, with those obtained using standard geometries over a wide range of shear rates. The approach is also found to be very useful to evaluate shear-rate and viscosity data in Couette viscometers when large gaps are used with non-Newtonian fluids.

Nous présentons une procédure basée sur une analogie Couette, permettant d’analyser de façon quantitative des données couple-vitesse angulaire de rotor afin d’extraire des rhéogrammes dans des géométries non conventionnelles. Cette procédure est tout d’abord validée à l’aide d’une géométrie simple pour laquelle un calcul analytique du rayon interne équivalent est possible. Les résultats montrent que ce dernier dépend peu de la nature du fluide et qu’il existe une position radiale optimale, \( r^* \), dans l’entrefer Couette virtuel permettant de calculer simplement les valeurs de viscosité-vitesse de cisaillement à partir des données de couple-vitesse angulaire de rotor. Les résultats expérimentaux obtenus à l’aide de géométries complexes et des fluides non-newtoniens complexes coïncident, aux erreurs expérimentales près, avec ceux obtenus à l’aide de géométries conventionnelles, sur une large place de vitesses de cisaillement. Cette procédure s’est avérée particulièrement utile pour des viscosimètres de Couette à entrefer large.

Keywords: viscometry, mixing devices, Couette analogy.

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analyze batch mixer devices used in the plastic industry. In their approach, the authors used the Couette analogy to model a dual mixing device in which each mixing chamber was replaced by a cylindrical bob rotating in a cylindrical chamber. The Couette analogy was also used in other situations, e.g. to correlate experimental data of complex geometry mixers (see the work of Chavan and Ulbrecht, 1973), who developed a correlation for helical ribbon agitators using Couette analogy, as well as the more recent work of Carreau et al., 1993, who used similar ideas in agitated vessels. The expression of the internal radius, \( R_i \) of the mixing device is obtained by solving hydrodynamic equations in the particular case of a generalized Newtonian fluid obeying a power law model (Bird et al., 1987) to obtain (Bousmina et al., 1999):

\[
\frac{1}{1 + \frac{4\pi}{2\pi} \left( 2 \frac{1 + \frac{1}{2}}{\Gamma} \right)} \left[ \frac{1}{2} \frac{1 + \frac{1}{2}}{\Gamma} \right]^{1/2}
\]

where \( R_i \) and \( R_e \) are the equivalent radius of the mixing element and the radius of the cylindrical mixing chamber respectively, \( L \) the length of the mixing chamber, \( \Gamma \) the gear ratio in the case of dual mixing chambers, \( g \) the measured torque, \( N \) the imposed rotational speed, \( \Delta \) the consistency and the flow index of the power law model, respectively, as given by the following equation:

\[
\eta(\dot{\gamma}) = \dot{\gamma}^{(\frac{-1}{n})}
\]

in which \( \eta \) is the viscosity and \( \dot{\gamma} = \frac{1}{2} \gamma : \dot{\gamma} \) is the amplitude of the rate-of-strain tensor, \( \gamma \). All the parameters in Equations (1) and (2) are in consistent dimensions.

Bousmina et al. (1999) found that the equivalent internal radius \( R_i \) given by Equation (1) varies only slightly with the power law index \( n \). The authors concluded that a calibration procedure can be performed using, for example, a Newtonian fluid or any other well-characterized power law fluid. Indeed, knowing the rheological characteristics of the fluid and monitoring the torque and the rotational speed, one can use Equation (1) and readily obtain an experimental value of \( R_i \) for the considered geometry. This value can then be used for any type of fluid. This is one of the requirements for the Couette analogy to be applicable.

Before we proceed with the use of this analogy for complex geometries, we shall first validate this requirement and verify how the equivalent internal radius \( R_i \) is sensitive to the nature of the fluid. For this purpose, we use a relatively simple geometry, namely a double-walled Couette geometry. The geometrical characteristics of this device are shown in Figure 1a. It consists of a hollow cylindrical bob rotating in a cup. If we equate the torque calculated for a power-law fluid in this geometry to the one calculated with the same fluid in the case of a single cylindrical bob rotating in a simple cylindrical cup at the same rotational speed (Couette analogy), we obtain the following analytical expression for the equivalent internal radius \( R_i \):

\[
\frac{1}{1 + \frac{4}{2^2} \left( \left( \frac{2}{3} \right)^{1/2} \right)^{1/2}} \left[ \frac{1}{2} \frac{2}{3} \left( \frac{2}{3} \right)^{1/2} \left( \frac{2}{3} \right)^{1/2} \right]^{1/2}
\]

where \( R_k \) \( k \in \{1, 4\} \), are defined in Figure 1a. In this case, the external radius of the cup in the Couette analogy is \( R_e = R_4 \). For the particular dimensions given in Figure 1a, the internal equivalent radius \( R_i \) varies from 16.44 mm to 16.97 mm for a power law index ranging from 1 to 0.15 which are typical values encountered for polymer solutions and melts. These variations are relatively small and if one takes the arithmetic mean value for the internal equivalent radius, the error in the evaluation of the viscosity (see below) is smaller than 2.5%.
which is below the maximum tolerated for the measurement of viscosity using conventional rheometers (below 5%). Variations of as \( R_i \) given by Equation (3) for the double-walled Couette of Figure 1a are shown in Figure 2. These results will be compared to those obtained experimentally using different types of fluids.

In order to verify the limitations of the Couette analogy, the expression of the equivalent radius given by Equation (3) has been used for unbalanced double-walled Couette geometries. The results of Figure 2 are based on the geometrical dimensions of Figure 1a. Equation (3) gives the internal radius based on the external radius of the double-walled Couette device. What if one of the gaps in this geometry is much larger (or much smaller) than the other one (unbalanced Couette geometry)? How would the Couette analogy apply in this case? For simplicity, we varied the gap (\( R_4 - R_3 \)) and left the other one [i.e. \((R_2 - R_1)\)] unchanged. The equivalent internal radius was then calculated using Equation (3) and the results of \( R_i \) as a function of \( R_4 \) for different values of the power-law index are shown in Figure 3a. In this case, the Couette analogy can still be used as far as \( R_4 \) is lower than a given value (say 25 mm for \( R_1 \), \( R_2 \) and \( R_3 \) given in Figure 1a). Above this critical value, \( R_i \) based on \( R_4 \) strongly depends on the power-law index, i.e. on the nature of the fluid, and the Couette analogy is not valid anymore. What is not valid is the way this analogy has been used. Indeed, increasing \( R_4 \) means that the contribution of the second gap of the double-walled Couette to the total torque is not important compared to that of the small gap. \( R_4 \) [or more precisely \((R_4 - R_3)\)] is not a characteristic length of the device anymore, and Equation (3) is expected to fail for \( R_4 \) values greater than a certain critical value. The same calculation has been done, but this time \( R_i \) is calculated on the basis of \( R_4 \) as an external radius. The results are shown in Figure 3b as a function of \( R_4 \) for different values of \( n \). As can be seen, the variations are very small and the Couette analogy can still be used. In other words, before one can proceed with the use of the Couette analogy for complex geometries, characteristic dimensions that control the process must be carefully identified.

Another important aspect in this approach that might limit the use of the Couette analogy is the effect of elasticity on the measured torque, particularly when mixing devices with complex geometries are used. This is a controversial subject and experimental results from different groups led to the opposite conclusion about the effect of elasticity on the measured torque. The elasticity is found to have no appreciable effect in the laminar region. However, for high rotational speed or Reynolds number, some authors found that elasticity increases the torque (Collias and Prud’homme, 1985; Brito De La Fuente et al., 1991), while others found just the opposite (Kelkar et al., 1972; Rieger and Novak, 1974; Oliver et al., 1984; Skelland, 1983; Ulbrecht and Carreau, 1985). This contradiction has been attributed to the indirect effect of elasticity on flow patterns which in turn may affect energy consumption in one direction or the other (Ulbrecht and Carreau, 1985). We shall see in the forthcoming sections that this may indeed happen in some cases. The Couette analogy, then, may not be applicable.

Now we suppose that all conditions to safely use Couette analogy are satisfied. To obtain viscosity/shear-rate data from torque/rotational speed data in a simple Couette geometry, we need, for an imposed rotational speed, the shear rate and the shear stress values at a given position in the gap. The shear stress is obtained directly from the \( \theta \)-component of the
momentum equation and is related to the measured torque, \( \tau \), by:

\[
\tau_{\text{ext}} = \frac{\Gamma}{2\pi z^2} \tag{4}
\]

To relate the shear rate to the imposed rotational speed, \( N \), the rheology of the fluid must be known, which is not the objective of the operation since we want to characterize the fluid under mixing. However, if \( |\gamma_c - \gamma| \) where \( \gamma_c = \gamma_{c\theta}(R_c) \) is relatively small at each value of \( N \), then the rheological behavior of the fluid can be approximated by a power-law model. Under these conditions, the shear rate at a position \( r \) (in the simple Couette geometry) is given by:

\[
\dot{\gamma}(r) = \frac{4\pi}{2} \left( \left( \frac{-s}{2} \right)^2 - 1 \right)
\tag{5}
\]

where \( s \) is the local power law exponent of the fluid. The torque at a given rotational speed for a Couette geometry with a power-law fluid is given by:

\[
\Gamma = 2(2 + s)n( + 1)
\tag{6}
\]

The parameter \( s \) is then readily extracted from the slope of the experimental data of \( \log(\Gamma) \) versus \( \log(N) \).

The viscosity of the fluid can then be obtained using the definition of this material function in combination with Equations (4), (5) and (6):

\[
\eta(\dot{\gamma}) = \frac{\tau_{\text{ext}}(r)}{\dot{\gamma}(r)} = \frac{\Gamma}{2\pi z^2 \dot{\gamma}(r)} \tag{7}
\]

where \( \dot{\gamma} \) is the amplitude of the rate-of-strain tensor \( \dot{\gamma} = \dot{\gamma}_{\theta}(r) \) for a simple shear flow and \( \tau_{\text{ext}} \) is the shear stress. In the case of small gaps \( [(R_c - R)/(R_c) << 1] \), Bousmina et al. (1999) showed that when evaluated at \( r = R_{1/2} = (R_c + R)/2 \), the shear rate \( \dot{\gamma}_{\theta}(R_{1/2}) = \dot{\gamma}_{1/2} \) is independent of the nature of the fluid, i.e. independent of the local power law index \( s \). In this case, the above developments can be applied to a fluid with unknown rheological behavior and the shear rate at \( R_{1/2} \) can be approximated using the expression for a Newtonian fluid, i.e. by setting \( s = 1 \) and \( r = R_{1/2} \) in Equation (5). This is a powerful result that has been discussed in details in Bousmina et al. (1999) and which indicate that, finally, a Couette viscometer can be used to obtain single measurement viscosity data without having to perform multiple experiments and complex calculations to correct the experimental data, as far as all the calculations (shear rate and shear stress) are done away from the Couette walls. In the case of large gaps, we can show that an optimal position where the rate-of-strain depends only slightly on the nature of the fluid exists. Indeed, if we plot \( K_{\gamma} = \gamma \Omega \), where \( \Omega = 2\pi N \), as a function of \( x = (R_c + R)/r \) for several \( n \) values, we find that there is a narrow region of \( x \) for which \( K_{\gamma} \) is only slightly dependent on \( n \) (see Figures 6a and 6b). This region can be determined either graphically or analytically (see Appendix A). The resulting expression of the optimal position \( r^* \) is the following one:

\[
x^* = \left[ \frac{\gamma}{\gamma_{1/2}} \right]^{2/1} \left( \frac{\gamma_{1/2}}{\gamma_{1/2} - 1} \right) \left( \frac{1}{\gamma} \right)
\tag{8}
\]

where \( n \) and \( n'(n \neq n') \) are two flow behavior indices representative of the range of shear-thinning fluids to be studied. Variation of \( x^* = (R_c + R)/r^* \) as a function of \( R_c/R_e \) for different values of \( n \) with \( n' = 1 \), are shown in Figure 4. The results

**Figure 4.** Variation of the optimal position \( x^* = (R_c + R)/r^* \) as a function of \( R_c/R_e \) for different values of \( n \) with \( n' = 1 \) (see Equation 8).

**Figure 5.** Effect of power law index on the Newtonian approximation of shear rate for different ratios of \( R_c/R_e \).
The shear-rate at the

The optimal position is, as expected, in the middle of the 

gaps with non-Newtonian fluids, the shear-rate at one of the 

limits where \( \dot{\gamma} \) is a function of the power law index can be identified. This will be 

same position for a Newtonian fluid as a function of 

where \( \dot{\gamma} \) is the rate-of-strain at 

extensive analysis of torque/rotational speed data (Krieger , 

be applied. It is worth mentioning here that in the case of large 

which the Couette analogy and the Newtonian approach can 

clearly shown later for a specific geometr y . Variations of \( \dot{\gamma} \) for 

lower than 0.7, an area where the rate-of-strain is a weak 

indicate that, indeed, when the gap is small enough \((R_i/R_e \geq 0.7)\), the optimal position is, as expected, in the middle of the 

an approach to avoid the complexity of these procedures using the Tikhonov regularization approach to solve the inverse and ill-posed problem of Couette viscometry with an extension to fluids with yield stress. Our approach avoids both the heavy handling of experimental data (Krieger, 1968; Schowalter, 1978) and the heavy mathematics of the Tikhonov regularization approach. It provides a mean to obtain viscosity/shear-rate data with a single measurement with only a small error. Fluids with yield stress are however not included in the evaluation.

It should be mentioned at this stage that, once \( r^* \) has been determined for a given system, the coefficient of proportionality \( K \) between \( \dot{\gamma} \) and \( \Omega \) is nothing but a Metzner-Otto constant \( K_{\Omega} \) (Metzner and Otto, 1957). Our method presents the advantage of allowing its analytical calculation, in place of a time-consuming experimental evaluation.

Furthermore, the above analysis indicates the limitations of the Couette analogy and how it can be used to safely quantify experimental torque/rotational speed data of complex geometry devices. In the next section, we shall show how the theoretical results obtained in the preceding sections are verified experimentally for a double-walled Couette geometry and for more complex geometries (see Figure 1). Different kinds of fluids are used to assess the approach.

**Experimental**

To experimentally verify the validity of the approach described above and to extend it to complex materials and complex geometries, we used three model fluids: a water/glycerol 10 wt%/90 wt% (Newtonian fluid); a 0.1 wt% polyacrylamide solution in 59.4 wt% glycerol and 40.5 wt% water (a viscoelastic fluid); and a 0.5 wt% xanthan solution in water (a power-law fluid). For comparison purposes, these fluids were first characterized using a double wall Couette geometry (see Figure 1a) with a Rheometric Scientific RFS-II rheometer. These fluids are used to obtain the internal radius of the equivalent Couette geometry for a variety of complex geometries. The geometries used here to illustrate the approach are: double-walled Couette (DC), anchor-in-cup (A); vane-in-cup (V) double helical ribbon-in-cup with hemispheric bottom (DHR); helical ribbon-in-cup (HR). These geometries are illustrated in Figure 1. For each geometry, the internal radius of the equivalent Couette is calculated using the different fluids and their rheological characteristics obtained from the cone-and-plate measurements (obtained with the RFS-II). All the results are reported in Table 1. Also given in this table are the stress and shear rate constants to introduce in the rheometer's software to calculate the rheological data of the tested fluids. These two constants are given by: stress constant \( K_\tau (\text{Pa}\cdot\text{N}^{-1} \cdot \text{m}^{-1}) \); shear rate constant, \( K_\gamma (\text{rad}^{-1}) \): \( K_\gamma = \dot{\gamma} / \Omega \).

The results indicate that for all the geometries presented in this study, the Couette analogy works correctly as far as the internal radius of the equivalent Couette geometry is concerned. Indeed, for all these mixer geometries, \( R_i \) is found to be almost independent of the fluid used to evaluate it. In terms of optimal position, \( r^* \), calculated with the average value of \( R_i \) and \( n = 1 \) in Equation (8), the results of Table 1 indicate that, except for the vane-in-cup and the helical ribbon-in-cup assemblies, this position is situated in the middle of the gap. The results of \( x^* = (R_i + R_e) / r^* \) for the vane-in-cup and the double-walled Couette assemblies using Equation (8) for different values of the power law index \( n \) are shown in Figures 6a and 6b respectively. These results illustrate that, indeed, when \( R_i / R_e \)
and temperature is used (see Table 1). The discrepancy between the Xanthan solution, and according to the results of Figure 5, is worth noting that, except for the vane-in-cup case with the vane mixer (for comparison with the experimental data of Brito De La Fuente et al., 1991), we obtained

\[ \frac{R_i}{R_e} \approx 0.25 \]

is close to 1, the optimal position is in the middle of the gap (see Figure 6a). For smaller values of \( \frac{R_i}{R_e} \), there is no single position but a region where, indeed, \( \frac{dy}{dn} \) is small, supporting the assumption made to derive Equation (8) (see Figure 6b). It is worth mentioning here that, in the case of water/glycerol-polyacrylamide solution, the viscosity/shear rate results are compared to those of the conventional one, the largest deviations are observed for the parameter \( n \). Viscosity-shear rate data for the different fluids are shown in Figures B1 to B3 presented in Appendix B.

It is worth mentioning here that, in the case of water/glycerol-polyacrylamide solution, the viscosity/shear rate results extracted from the raw torque/rotational speed data differ from one geometry to the other at a certain shear rate (around 10 s\(^{-1}\) in the case of Figure B3). This might be due to the effect of elasticity. Indeed, using the following definition for the Reynolds number (Brito De La Fuente et al., 1991):

\[ Re = \frac{\rho ND^2}{\mu} \]

(9)

where \( \rho \) is the density of the fluid, \( \mu \) its viscosity and \( D \) the mixing device diameter, and calculating \( Re \) with the necessary parameters extracted from Figure B3 and Table 1 for the helical ribbon mixer (for comparison with the experimental data of Brito De La Fuente et al., 1991), we obtained \( Re = 4.83 \). This value is much higher than the critical Reynolds number found.

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Equivalent Couette geometrical characteristics} & \text{Rheological characteristics} \\
\text{} & \text{} & \text{K} & \text{K} & \text{M} & \text{M} \\
\text{Double Couette (DC)} & \text{Double Couette} & \text{Mixer geometry} & \text{M} & \text{n} & \text{n} \\
\text{water/glycerol (25°C)} & 16.44 & 16.70 & 2.002 & 1.78 \times 10^4 & 29.9 & 0.110 \times 1 \times 10^{-1} & 1 \times 10^{-1} \\
polyacrylamide (27°C) & 16.65 & 16.82 & 2.001 & 1.76 \times 10^4 & 48.1 & 0.229 \times 0.518 & 0.230 \times 0.518 \\
salad dressing (26°C) & 16.90 & 16.95 & 2.000 & 1.73 \times 10^4 & 169 & 7.746 \times 0.220 & 7.795 \times 0.220 \\
xanthan (24°C) & 16.97 & 16.99 & 2.000 & 1.72 \times 10^4 & 564 & 2.843 \times 1.375 & 2.905 \times 1.375 \\
\hline
\end{array}
\]
by Brito De La Fuente et al. (1991), \((Re \approx 0.1)\), where the effect of elasticity becomes important. It is, however, in the range of values reported in the literature to define the laminar region (where no effect of elasticity can be detected) for close clearance mixing devices \((Re \geq 10)\). This value might however depend on geometrical ratios (different in our case from those of Brito De La Fuente et al., 1991).

Three other fluids were tested with both conventional and mixer-type geometries. These fluids are a Heinz tomato ketchup, a salad dressing preparation with no oil content (Maille) and a 2 wt% caboxymethylcellulose (CMC) solution in water. We remind readers here that for the tomato ketchup it was not possible to use conventional geometries to obtain its rheological behavior. Indeed, this fluid is supposed to be composed of flocs or aggregates of solid particles in a serum. The floc or aggregate dimensions are of the order of magnitude of the available gap generally encountered in conventional geometries (cone-and-plate, single or double wall Couette, parallel plates). Sample loading under these conditions resulted in structure damaging of the fluid (loss of interstitial fluid), thereby making reliable rheological data very difficult to obtain (Choplin and Marchal, 1997). The CMC solution has been chosen to evaluate the approach with a fluid having a smooth transition from the Newtonian plateau to the shear thinning region and to assess the local power law assumption used in the derivation of the equations.

The results for the salad dressing are shown in Figure 7. As for the model fluids, the experimental results with the different geometries considered here coincide perfectly over almost five decades of shear rates and four decades of viscosity.

The results for the tomato ketchup are shown in Figure 8. For the reasons mentioned earlier, the data are available only with the helical ribbon geometry. Moreover, the results shown in Figure 8 are obtained by starting the measurement with the highest shear rate and then decreasing progressively down to the lowest shear rate attainable. Here, we take advantage of the mixing capability of the helical ribbon to avoid phase separation and flocculation during measurement. The results are obtained with two different experiments using fresh samples. As can be seen, the results are reproducible over the whole range of shear rates. However, care must be taken with this type of fluids since preliminary experiments showed that when the sample is loaded for characterization, the initial structure changes from one experiment to the other. The characterization starting from the lowest shear rate differs from one sample to the other, mainly in the low shear rate region. This is why we decided to start taking data from the highest shear-rate.

The results for the CMC solution are shown in Figure 9. Again, the experimental data show a very good agreement between the different geometries. The results with this particular fluid indicate that the use of a local power-law approach (Equation 5) is valid in this case.

It is worth noting here that for all the fluids tested in this work, the shear rate covered is slightly different from one geometry to the other. In general, lower shear rates can be attained with the helical ribbon geometry used in this study. The range depends on the geometrical characteristics of the mixing device.

**Conclusions**

The procedure used here to quantify torque-rotor speed data of mixer type rheometers is found to be very useful. The approach was satisfactorily tested with model fluids and conventional geometries before it was extended to more complex geometries.
and fluids. The viscosity/shear-rate curves obtained using a helical ribbon mixer are found to perfectly coincide, over a large range of shear rates, with those obtained using a cone-and-plate and a double wall Couette geometries. The viscosity/shear rate curves for a tomato ketchup fluid could not be obtained using conventional geometries. Only the helical ribbon geometry was able to provide reproducible data for this complex fluid.

The approach is also found to be very useful in the evaluation of shear-rate and viscosity data in the Couette viscometer when large gaps are used with non-Newtonian fluids. Complex calculations and graphical derivation of torque/rotational speed data can be avoided and single point viscosity evaluation can be performed when the shear rate and the shear stress are evaluated away from the walls. When the viscometer gap is small the optimal position is in the middle of the gap and when the gap is relatively large, a region where the shear-rate is a weak function of the nature of the flow can be identified. Shear-rate and viscosity can be obtained in this region using the Newtonian approximation.

Appendix A: Calculation of the Optimal Position in the Couette Geometry

The expression of the shear rate at any position is given by:

\[ \dot{\gamma}(x) = \frac{4\pi}{R_e^2} \left( \frac{1}{1 - \left( \frac{R_i}{R_e} \right)^n} \right)^{\frac{2}{n}} \]  

(A1)

We seek an optimal position, \( r^* \), where \( \dot{\gamma} \) is almost independent of \( n \). If we plot \( K_r = \dot{\gamma}/\Omega \), where \( \Omega = 2\pi N \), as a function of \( x = (R_i + R_e)/r \) for several \( n \) values, we find that there is a narrow region of \( x \) for which \( K_r \) is only slightly dependent of \( n \) (see Figures 6a and 6b). This region can be determined analytically by noticing the fact that \( x^* \) point, where \( K_r \) is the less sensitive to the flow index \( n \), coincides with the crossover point of the two curves corresponding to the lower and the higher flow indices. Noting these indices \( n \) and \( n' \), \( x^* \) and \( r^* \) can be obtained via the resolution of the following equation:

\[ K_r(n) = K_r(n') \]  

(A2)

Then, combining Equations (A1) and (A2), we get:

\[ r^* = \frac{\left( \frac{R_e}{R_i} \right)^{2/n'} - 1}{\left( \frac{R_e}{R_i} \right)^{2/n} - 1} \times \frac{1}{\left( \frac{R_e}{R_i} \right)^{2/n} - 1} \]  

\[ = \frac{n}{n'} \times \frac{\left( \frac{R_e}{R_i} \right)^{2/n'} - 1}{\left( \frac{R_e}{R_i} \right)^{2/n} - 1} \times R_e \left( \frac{\left( \frac{R_e}{R_i} \right)^{2/n} - 1}{\left( \frac{R_e}{R_i} \right)^{2/n'} - 1} \right)^{2/(n-2/n')} \]  

(A3)

In order to cover a large range of shear-thinning behavior, we may select quite different values of \( n \) and \( n' \) as, for instance, \( n = 0.15 \) and \( n' = 1 \). However, the closer \( n \) and \( n' \) are, the more accurate is the determination of \( r^* \). Plots of \( x^* = (R_i + R_e)/r^* \) as a function of \( R_e/R_i \) are shown in Figure 4. They indeed indicate that for \( R_e/R_i \) higher than 0.9, the optimal position is in the middle of the gap (\( x^* = 2 \)). The ratio \( \dot{\gamma}/\dot{\gamma}_N \), calculated using a mean value of \( r^* \), is given by:

\[ \frac{\dot{\gamma}}{\dot{\gamma}_N} = \frac{\left( \frac{R_e}{r^*} \right)^{2/n} - 1}{\left( \frac{R_e}{R_i} \right)^{2/n} - 1} \times \frac{1}{\left( \frac{R_e}{R_i} \right)^{2/n} - 1} \]  

(A4)

where \( r^* \) is given by Equation (A3) for \( n' = 1 \). The results of \( \dot{\gamma}/\dot{\gamma}_N \) as a function of \( n \) for different \( R_e/R_i \) values are shown in Figure 5.

Appendix B

![Figure B1. Viscosity data for a water/glycerol solution.](image1)

![Figure B2. Viscosity/shear rate data for xanthan solution.](image2)

continued on p. 1174
Figure B3. Viscosity data for a water/glycerol-polyacrylamide solution.

Acknowledgements
The authors acknowledge the financial support provided by the Natural Sciences and Engineering Research Council of Canada (NSERC) and the Fonds pour la Formation de chercheurs et l’aide à la recherche (FCAR) of the province of Quebec. One of the authors (A. Aït-Kadi) acknowledges the financial support by Elf-Atochem for his sabbatical leave in France.

Nomenclature
\[ D \] mixing device diameter, (m)
\[ g \] gear ratio
\[ K \] stress constant, (Pa·N^{-1}·m^{-1})
\[ \gamma \] shear rate constant
\[ l \] length of the mixing chamber, (m)
\[ M \] power law consistency, (Pa·s)
\[ n \] power law flow index
\[ n' \] power law flow index
\[ Re \] Reynolds number, (Equation 9)
\[ R_e \] external radius, (m)
\[ R_i \] equivalent internal radius, (m)
\[ R_e \] radii in the double-walled Couette geometry, (m)
\[ r \] radial coordinate, (m)
\[ r^* \] optimal position in the Couette gap, (m)
\[ s \] local power law flow index
\[ x \] dimensionless radial position in the Couette gap
\[ x^* \] dimensionless optimal position in the Couette gap

Greek Symbols
\[ \dot{\gamma} \] shear rate, (s^{-1})
\[ \dot{\gamma}_e \] shear rate evaluated at \( R_e \), (s^{-1})
\[ \dot{\gamma}_i \] shear rate evaluated at \( R_i \), (s^{-1})
\[ \gamma_{\theta} \] \( \theta \)-component of the rate of deformation tensor, s^{-1}
\[ \Gamma \] torque, (N·m)
\[ \eta \] non-Newtonian viscosity, (Pa·s)
\[ \mu \] viscosity, (Pa·s)
\[ \rho \] liquid density, (kg/m^3)
\[ \tau_{\theta} \] \( \theta \)-component of the stress tensor, (Pa)
\[ \Omega \] angular velocity, (s^{-1})

References


Manuscript received April 15, 2002; revised manuscript received October 17, 2002; accepted for publication November 8, 2002.