Production Planning of Reconfigurable Manufacturing Systems with Stochastic Demands Using Tabu Search

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Abstract: In the new competitive dynamic market, manufacturing success and survival are becoming more and more difficult to ensure. In other words, getting the right product with low cost and high quality is not the only key to success. New requirements such as production responsiveness and flexibility should be considered. Reconfigurable Manufacturing System (RMS) is a new paradigm that enables manufacturing systems to achieve rapid response to market demand. The effectiveness of an RMS depends on implementing key characteristics and capabilities of RMS in system design stage and benefiting from them in utilization stage. In this paper, we introduced a methodology to adjust rapidly and productively scalable production capacities and the functionality of system to market demands. It is supposed that arrival orders follow Poisson distribution and they are missed, if they are not available. According to these assumptions, a Mixed Integer Non-linear Programming (MINLP) model is developed to determine optimum sequence of production tasks, corresponding configurations and batch sizes. A Tabu search based procedure is used to solve the model. Finally a numerical example is used to illustrate the procedure.

Keywords: Reconfigurable Manufacturing System; production planning; Tabu search; stochastic demands.


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1 Introduction

With the emergence of the new business era embracing “change” as one of its major characteristics, it would be increasingly difficult for manufacturing companies to survive. In the current competitive markets, getting the right product with low cost and high quality is not sufficient to success, but new requirements such as production responsiveness and flexibility should be considered (Koren et al., 1999). Therefore, the manufacturers need to adopt some new approaches leading to achieve more adaptability to market changes. Reconfigurable Manufacturing System (RMS) is a new paradigm that enables manufacturing systems to rapidly react to market demand.

RMS is a system, designed from the outset, for rapid changes in both hardware and software components, in order to quickly adjust production capacity and functionality within a part family in response to sudden market changes (Koren et al., 1999). These new systems provide exactly the functionality that is needed at the right time (Mehrabi et al., 2000). An RMS is designed and built to allow for convenient reconfiguration. The configuration of system can easily be changed by rearrangement of equipment, reallocation of workers, retooling of machines etc. Having the reconfiguration capability, the manufacturing system can produce more responsibly and productively than a non-reconfigurable manufacturing system (Lacksonen and Hung, 1998; Yang and Peters, 1998; Kochhar and Heragu, 1999).

RMSs must be designed with certain qualitative and quantitative characteristics to achieve exact flexibility in response to demand fluctuations. In this manner, RMSs are described by five key characteristics (Mehrabi et al., 2000):

1. Modularity: All system components, both software and hardware are designed to be modular.
2. Integrability: Systems and components are designed for both ready integration and future introduction of new technology.
3. Convertibility: Quick changeover between existing products and quick system adaptability for future products are allowed.
4. Diagnosability: The sources of quality and reliability problems that occur in large systems are defined quickly.
5. Customization: The system capability and flexibility (hardware and controls) are designed to match the application (product family).

Reconfigurable manufacturing systems are designed and operated according to a set of basic principles given below (Koren et al., 1999).

1. The RMS contains adjustable production resources to respond to imminent market needs.
   • The RMS capacity is rapidly scalable in small, optimal increments.
   • The RMS functionality is rapidly adaptable to the production of new products.
2. The RMS is designed around a part/product family, with just enough customized flexibility needed to produce all members of that family.
3. To enhance the responsiveness of a manufacturing system, RMS key characteristics should be embedded in the whole system as well as in its components (mechanical, communications and controls).
4. The RMS contains an economical mix of flexible and reconfigurable equipment with customized flexibility, such as reconfigurable machines whose functionality and productivity can be readily changed when needed.
5. In general, systems with a large number of alternative routes to producing a part are more reconfigurable, but they require higher investment cost in tooling and in material-handling systems.
6. The RMS possesses hardware and software capabilities to respond cost effectively to unpredictable events (market changes and machine failure).
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The first three principles are the core principles that define a reconfigurable system. The others are secondary principles that assist in designing a cost-effective RMS.

The effectiveness of an RMS depends on implementing these principles in system design as well as operation stages. This paper focuses on the operation stage of an RMS and introduces a mathematical model to manage and evaluate effectiveness of RMS. This model considers the key characteristics and capabilities of RMS to adjust scalable production capacities and the functionality of the system to respond rapidly to market demands and fulfill productivity.

2. Literature review

In RMS, the required products are classified into several product families, each of which is a set of similar products (Zhao et al., 2000a). Corresponding to each product family, there are several feasible configurations (Lacksonen and Hung, 1998; Yang and Peters, 1998; Kochhar and Heragu, 1999). These feasible configurations possess different production speeds, production costs and changeover costs. Any time the manufacturer selects a product family as a production task, RMS produces a number of products belonging to the selected family in a selected configuration. On finishing a product, a reward is earned. In the completion of a production task, the manufacturer must select a family as the subsequent production task, and so on. A changeover cost is incurred when the configuration changes from one to another (Zhao et al., 2000a; Liles and Huff, 1990; Makino and Arai, 1994; Rheault et al., 1995; Aronson, 1997; Lee, 1997; Koren et al., 1999).

Previously, a few tasks on modelling of RMS have been published (Lee, 1997; Koren et al., 1999; Zhao et al., 2000a, 2000b, 2001a and 2001b). Zhao et al. have developed the first stochastic model of an RMS that gave a thorough insight in modelling RMS. Their particular research concerns with the optimal configurations in the design stage, optimal selection policy in the utilization stage and increasing the performance measures of the system in the improvement. These are three important factors in a successful RMS implementation. In their first paper, a stochastic model of RMS is proposed and two case studies are evaluated (Zhao et al., 2000a). Their second paper proposed an algorithm to choose the optimal configuration for production of a product family in order to maximize the average profit in the infinite horizon (Zhao et al., 2000b). Their third paper focused on the optimal selection policy (Zhao et al., 2001a). The last one focused on the system’s performance measures, allowing the manufacturer to optimize the maximum number of orders that can be accepted where there is a high fluctuation in the market demand (Zhao et al., 2001b).

Considering the modelling approach i.e. stochastic modelling, Zhao et al.’s model faces with following shortcomings:

- According to the Zhao’s model (Zhao et al., 2000b), in the design stage, only one of the feasible configuration of a product family is selected and fixed as optimum configuration. In other words, no other configurations can be selected in the utilization stage. That means the full capabilities of RMS such as alternative routes, scalability and flexibility have not been considered properly. Ohiro et al. directly extended the Zhao’s model to remove this limitation (Ohiro et al., 2006).
- The model is concerned about overproduction and sets it to zero. According to model, RMS dose not produce any product unless it is required. Considering the production time, most of orders are delayed. It is not suitable in competitive market in which responsiveness is in the main key issue.
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- The required time for changing the configuration of system from one to another is ignored.
- Complexity of analyzing the model highly depends on number of product families and their upper bounds for accepting orders. As these values are increased, the space state dimension is increased exponentially.

In this paper considering competitive markets conditions, overproduction is allowed and it is supposed that arrival orders are missed where they can not be met or fulfilled immediately. Therefore the manufacturer must anticipate market demands to cover arrival orders as many as possible with rational further costs. Moreover, all of the feasible configurations of a product family are taken into account in utilization stage. In other words, the manufacturer can select every feasible configurations of a product family according to current system condition and configurations’ parameters such as production speed, production cost and changeover cost and time. According to RMS concepts, any RMS could involve five important issues as follows:

- Optimum length of time horizon to plan the production tasks
- Optimum number of production tasks to be done through the optimum time horizon
- Optimum sequence of production tasks
- Optimum configuration of the selected product family at each production task
- Optimum batch sizes at each production task.

In this paper, a mixed integer non-linear programming (MINLP) model is introduced to determine above mentioned issues to maximize the earned profits minus production costs. Production costs are the related costs such as operation costs, inventory holding costs and changeover costs. This model makes a compromise between the earned profit from market demands coverage and production costs.

In following sections, an RMS modelling procedure is introduced. In this modelling procedure, firstly the affects of stochastic arrival orders on in-hand inventory levels, inventory holding costs and sales are estimated. Then, using the estimating equations and considering the system’s parameters a mathematical model is developed. To solve the model, a Tabu search based procedure is introduced. Finally a numerical example is presented.

3. Modelling of RMS

3.1. Problem description

In this paper following conditions and notations are presented for modelling of RMS:

- Products are classified into several product families, each of which is a set of similar products \( M = \{1, 2, \ldots, m\} \).
- Sale price of a finished product belonging to product family \( i(\in M) \) is equal to \( S_i \).
- Inventory holding cost for a product belonging to product family \( i(\in M) \) is equal to \( h_i \).
- Arrival rate of orders belonging to product family \( i(\in M) \) is assumed to follow Poisson distribution and is denoted by \( \lambda_i \). Each order is a single product belonging to a particular product family.
- The arrival orders must be responded immediately. In the other words, if there isn’t any finished product to satisfy arrived order, it will be rejected or missed.
- There is a set of feasible configurations corresponding to each product family \( i(\in M) \) which is defined by \( C_i = \{c_{i,1}, c_{i,2}, \ldots, c_{i,n_i}\} \) and \( n_i \) denotes the number of feasible
production configurations of product family \( i(\in M) \), i.e. any product belonging to product family \( i \) can be produced by all configurations specified in the \( C_i \).

- The production rate of \( j^{th} \) configuration of product family \( i \) is equal to \( CR_{i,j} \cdot i(\in M) \) and \( j=\{1,...,n_i\} \).
- The production cost of \( j^{th} \) configuration of product family \( i \) is equal to \( CC_{i,j} \cdot i(\in M) \) and \( j=\{1,...,n_i\} \).
- Changing from \( j^{th} \) configuration of product family \( i \) to \( l^{th} \) configuration of product family \( k \) causes a reconfiguration cost that is denoted by \( GC_{i,j,k,l} \cdot i,k(\in M) \), \( j=\{1,...,n_i\} \) and \( l=\{1,...,n_k\} \).
- Changing from \( j^{th} \) configuration of product family \( i \) to \( l^{th} \) configuration of product family \( k \) requires a time that is denoted by \( GT_{i,j,k,l} \cdot i,k(\in M) \), \( j=\{1,...,n_i\} \) and \( l=\{1,...,n_k\} \).
- There is no restriction in supplying raw materials.

Also following definitions are used throughout this paper.

- **Run**: Completing a production task in a specified configuration and reconfiguring it to the next production task.
- **Arrangement**: An arrangement consists of a number of successive runs; when the final production task is completed, the system configuration should be changed to the first production task’s configuration. An arrangement is defined by \( i \) the number of its runs, \( ii \) the sequence of selected product families, \( iii \) the selected configurations and \( iv \) the batch sizes. Figure 1 illustrates an arrangement with \( R \) runs.
- **Length of an arrangement**: A period of time during which the arrangement is completed.

The goal of the model is to determine optimum arrangement that maximize total profit subject to the system conditions. Total profit is equal to earned selling prices minus production costs. Production costs consist of operating costs, changeover costs and inventory holding costs. Optimum arrangement concerns with optimum number of runs, optimum sequence of production tasks and optimum batch sizes. The optimum arrangement adjusts production outputs to arrival orders in such a way to cover maximum orders in minimum costs.

### 3.2. Proposed modeling approach

Considering the conditions discussed in section 3.1, development of a stochastic model to determine the optimum arrangement and to analyze the system’s behaviour is too complex and difficult. Therefore, in this paper to achieve a good solution through reasonable time, a mathematical model is developed.

It is not possible to use stochastic parameters directly in mathematical models. In this paper the in-hand inventory levels, inventory holding costs and sales are evaluated and estimated according to stochastic orders. These estimated values are used in a Mixed Integer Non Linear Program (MINLP) to determine optimum or near to optimum arrangements.

### 3.2.1. Estimating effects of stochastic arrival orders on system’s conditions

Considering stochastic orders, following theorems are presented to evaluate behaviours of in-hand inventory levels, inventory holding costs and sales.
Theorem 1: If i) Arrival rate of orders belonging to product family \( i \) (\( \in M \)) follows Poisson distribution with a rate of \( \lambda_i \), ii) Arrival orders are missed, if they are not fulfilled immediately and iii) at each run only one product family is selected as current production task, Then expected in-hand inventory level of product family \( i \) (\( \in M \)) at time \( t \) (\( EI'_i \)) is obtained by following equations:

a. If product family \( i \) (\( \in M \)) is not selected as \( r \)-th production task:

\[
EI'_i = \sum_{j=0}^{I'_i} \left( I'_i - x'_i \right) \frac{\left( \int_{t'=0}^{1} e^{-\lambda_i (t-T'_i) \lambda_i} dt \right) e^{-\lambda_i (t-T'_i)}}{x'_i !} \text{ for } T'_i \leq t \leq T'_F
\]  

(1)

b. If product family \( i \) (\( \in M \)) is selected as \( r \)-th production task:

\[
EI'_i = \sum_{j=0}^{I'_i} \left( I'_i - x'_i \right) \frac{\left( \int_{t'=0}^{1} e^{-\lambda_i (t-T'_i) \lambda_i} dt \right) e^{-\lambda_i (t-T'_i)}}{x'_i !} \text{ for } T'_i \leq t \leq T'_F
\]  

(2)

Where i) \( r \)-th run starts at \( T'_S \) and finishes at \( T'_F \), ii) \( I'_i \) is in-hand inventory level of product family \( i \) at the start of \( r \)-th run, iii) \( x'_i \) is accumulated arrived orders from the start of \( r \)-th run to \( t \) \((T'_i \leq t \leq T'_F)\), iv) \( D'_i \) is production rate of product family \( i \) in \( r \)-th run and v) \( Q_i' \) is the number of products belonging to family \( i \) which are produced in \( r \)-th run.

Proof: Since arrival orders belonging to product family \( i \) (\( \in M \)) follows Poisson distribution with a rate of \( \lambda_i \), \( x'_i \) follows Poisson distribution with a rate of \( \lambda_i (t-T'_i) \) (Gross and Harris, 1985). Considering the possibility of missing orders, in-hand inventory level always is positive \((I'_i \geq 0)\). If product family \( i \) (\( \in M \)) is not selected as \( r \)-th production task, then \( D'_i = Q'_i = 0 \). Therefore \( EI'_i \) is equal to:

\[
EI'_i = E[I'_i - x'_i | x'_i \leq I'_i] \text{ for } T'_S \leq t \leq T'_F
\]  

(3)

Considering the statistical distribution of \( x'_i \), Eq. (1) is concluded.

Now, suppose that product family \( i \) (\( \in M \)) and \( j \)-th configuration of it is selected as \( r \)-th production task, then \( D'_i = CR_i \) and \( Q'_i = 0 \). It is obvious that \( r \)-th production task is finished at \( T'_S + Q'_i / D'_i \). Theoretically, during \([T'_S, T'_S + Q'_i / D'_i]\), product family \( i \) is being produced with rate \( D'_i \). Therefore, \( EI'_i \) is equal to:

\[
EI'_i = E[I'_i + D'_i (t-T'_i) - x'_i | x'_i \leq I'_i + D'_i (t-T'_i)] \text{ for } T'_S \leq t \leq T'_S + Q'_i / D'_i
\]  

(4)

Changeover is started at \( T'_S + Q'_i / D'_i \) and finished at \( T'_F \). During changeover time, production is stopped. Therefore, \( EI'_i \) is equal to:

\[
EI'_i = E[I'_i + Q'_i - x'_i | x'_i \leq I'_i + Q'_i] \text{ for } T'_S + Q'_i / D'_i < t \leq T'_F
\]  

(5)

Similarly, since \( x'_i \) follows Poisson distribution with rate \( \lambda_i(t-T'_i) \), then Eq. (2) is concluded □.

Theorem 2: Considering the mentioned assumptions in theorem 1, expected inventory holding cost of product family \( i \) at time \( t \) (\( EIH'_i \)) where \( T'_S \leq t \leq T'_F \), can be obtained by following equations.

a. If product family \( i \) (\( \in M \)) is not selected as \( r \)-th production task:

\[
EIH'_i = h_i \sum_{j=0}^{I'_i} \left( I'_i - x'_i \right) \frac{\left( \int_{t'=0}^{1} e^{-\lambda_i (t-T'_i) \lambda_i} dt \right) e^{-\lambda_i (t-T'_i)}}{x'_i !} \text{ for } T'_S \leq t \leq T'_F
\]  

(6)

b. If product family \( i \) (\( \in M \)) is selected as \( r \)-th production task:
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\[
EI_i = \begin{cases} 
  h^* \sum_{T'_{r-1}}^{T'_{r+1}} \left( I'_{r} + DI'_{r}(t-T_{r}) - x'_{r} \right) \left( \frac{\lambda I'_{r}(t-T_{r})^e e^{-\lambda(t-T_{r})}}{x_{r}!} \right) dt & \text{for } T_{r} \leq t \leq T_{r} + \mathcal{Q} / DI'_{r} \\
  h^* \sum_{T'_{r-1}}^{T'_{r+1}} \left( I'_{r} + DI'_{r}(t-T_{r}) - x'_{r} \right) \left( \frac{\lambda I'_{r}(t-T_{r})^e e^{-\lambda(t-T_{r})}}{x_{r}!} \right) dt + h^* \sum_{T'_{r-1}}^{T'_{r+1}} \left( I'_{r} + \mathcal{Q} - x'_{r} \right) \left( \frac{\lambda I'_{r}(t-T_{r})^e e^{-\lambda(t-T_{r})}}{x_{r}!} \right) dt & \text{for } T_{r} + \mathcal{Q} / DI'_{r} < t \leq T_{r+1}
\end{cases}
\]

**Proof:** multiplying \( EI_i \) by the coefficient of inventory holding cost \((h_i)\) and \(dt\), expected inventory holding cost during \(t \pm \varepsilon\) is obtained. By integrating \( h^* EI_i^* dt \), expected inventory holding cost during \(r^th\) run is obtained \(\Box\).

**Theorem 3:** Considering the mentioned assumptions in theorem 1, If \(r^th\) run has a length of \(T' = T'_{r} - T'_{r-1}\), then expected sold product belonging to product family \(i\) during run \(r\) (\(ES'_{r} \)) can be obtained by following equations.

**a.** If product family \(i\) \((i \in M)\) is not selected as \(r^th\) production task:

\[
ES'_{r} = \sum_{x'_{r} = 0}^{I'_{r}} x'_{r} \left( \frac{\lambda I'_{r}^e e^{-\lambda I'_{r}}}{x'_{r}!} \right) + \sum_{x'_{r} - I'_{r} + Q'_{r}}^{x'_{r}} x'_{r} \left( \frac{\lambda I'_{r}^e e^{-\lambda I'_{r}}}{x'_{r}!} \right)
\]

**b.** If product family \(i\) \((i \in M)\) is selected as \(r^th\) production task:

\[
ES'_{r} = \sum_{x'_{r} = 0}^{I'_{r}} x'_{r} \left( \frac{\lambda I'_{r}^e e^{-\lambda I'_{r}}}{x'_{r}!} \right) + \sum_{x'_{r} - I'_{r} + Q'_{r}}^{x'_{r}} x'_{r} \left( \frac{\lambda I'_{r}^e e^{-\lambda I'_{r}}}{x'_{r}!} \right)
\]

**Proof:** Suppose that product family \(i\) \((i \in M)\) is not selected as \(r^th\) production task. If \(x'_{r} \leq I'_{r}\), then \(ES'_{r}\) is equal to \(x'_{r}\). Otherwise it is equal to \(I'_{r}\). Similarly when product family \(i\) \((i \in M)\) is selected as \(r^th\) production task, if \(x'_{r} \leq I'_{r} + Q'_{r}\), then \(ES'_{r}\) is equal to \(x'_{r}\). Otherwise it is equal to \(I'_{r} + Q'_{r}\). Multiplying these values by corresponding probabilities and summing the results, Eq. (8) and Eq. (9) are concluded \(\Box\).

Figure 2a shows graphical illustration of expected in-hand inventory level (Eq. (1)) and Figure 2b shows graphical illustration of expected inventory holding cost (Eq. (6)) where product family \(i\) is not selected as \(r^th\) production task. Calculations are done by *Mathematica* 5.1. Suppose that \(T'_{r}=0\) and \(I'_{r}=10\).

If product family \(i\) is not selected as \(r^th\) production task, then in-hand inventory level decreases gradually and approaches zero (Figure 2a). The slopes of graphs directly depend on value of \(\lambda / I'_{r}\). Expected inventory holding cost increases and eventually it will be approximately fixed where in-hand inventory level approaches to zero (Figure 2b). If value of \(\lambda / I'_{r}\) increases, then expected inventory holding cost during \(r^th\) run decreases.

Figure 3a shows graphical illustration of expected in-hand inventory level (Eq. (2)) and Figure 3b shows graphical illustration of expected inventory holding cost (Eq. (7)) where product family \(i\) is selected as \(r^th\) production task. Calculations are done by *Mathematica* 5.1. Suppose that \(T'_{r}=0\) and \(I'_{r}=10\).

If product family \(i\) is selected as \(r^th\) production task, then in-hand inventory level (\(EI_i^r\)) increases gradually. The slopes of graphs directly depend on the value of \(D'_{i}/\lambda_i\). When the production task is completed, reconfiguration will be performed. During changeover time, in-hand inventory level decreases (Figure 3a). Expected inventory holding cost (\(EI_{H_i}^r\)),
where product family $i$ is selected as $r^{th}$ production task, increases with a greater slope than where the product family $i$ is not selected as $r^{th}$ production task (Figures 2b and 3b). During the changeover time, the $\text{EIHi}_r$ slope is decreased (Figure 3b).

Above mentioned equations are very complex and it is not possible to use them directly in a mathematical model. Numerical experiences show that they have simple behaviour and they may be estimated with a good precision. Estimating equations are introduced as follows.

If product family $i$ is not selected as $r^{th}$ production task, the expected in-hand inventory level ($\text{EI}_i$; Eq. (1)) and the expected inventory holding cost ($\text{EHI}_i$; Eq. (6)) may be estimated by Eq. (10) and Eq. (11) respectively.

$$\text{EEI}_i' = \begin{cases} I'_r - \lambda_r * (t - T'_r) & \text{for} \quad T'_r < t < T'_r + \frac{I'_r}{\lambda_r} \\ 0 & \text{for} \quad T'_r + \frac{I'_r}{\lambda_r} < t \leq T'_r \end{cases} \quad (10)$$

$$\text{EEHI}_i' = \begin{cases} h_r * (U'_r * (t - T'_r) - \frac{\lambda_r}{2} * (t - T'_r)^2) & \text{for} \quad T'_r < t < T'_r + \frac{I'_r}{\lambda_r} \\ \frac{h_r * (U'_r)^2}{2 \lambda_r} & \text{for} \quad T'_r + \frac{I'_r}{\lambda_r} < t \leq T'_r \end{cases} \quad (11)$$

Figure 4a shows actual values of expected in-hand inventory levels ($\text{EI}_i$; Eq. (1)) in continuous curves and estimations of them ($\text{EHI}_i$; Eq. (6)) in dashed lines.

Maximum deviation occurs at $t = I'_r/\lambda_r$. Numerical experiences showed when $I'_r/\lambda_r$ increases, the maximum deviation increases very slowly, but the maximum deviation divided by $I'_r$ decreases. For example if $I'_r = 10$ and $\lambda_r = 5$, the maximum deviation occurs at $t = 2$ and it is equal to 1.25 and if $I'_r = 100$ and $\lambda_r = 10$, the maximum deviation occurs at $t = 10$ and it is equal to 3.99. That shows accuracy of estimations is reasonable.

Figure 4b shows actual values of expected inventory holding cost ($\text{EHI}_i$; Eq. (6)) in continuous curves and estimations of them ($\text{EEHI}_i$; Eq. (11)) in dashed curves.

Maximum deviation occurs when $t$ approaches $\infty$. Numerical experiences showed that when $I'_r/\lambda_r$ increases, the maximum deviation increases very slowly but the maximum deviation divided by $I'_r$ decreases. For example if $I'_r = 10$ and $\lambda_r = 5$, the maximum deviation occurs when approaches $\infty$ and it is equal to $h_r$ and if $I'_r = 100$ and $\lambda_r = 10$, the maximum deviation is equal to $5h_r$. This value is less than one percent of $\text{EIHi}_r$. That shows accuracy of estimations is reasonable.

If product family $i$ is selected as $r^{th}$ production task, $\text{EI}_i$ and $\text{EHI}_i$ can be estimated by following equations:

$$\text{EEI}_i' = \begin{cases} I'_r + (D'_r - \lambda_r) * (t - T'_r) & \text{for} \quad T'_r < t \leq T'_r + \frac{Q'_r}{D'_r} \\ I'_r + Q'_r - \lambda_r * (t - T'_r) & \text{for} \quad T'_r + \frac{Q'_r}{D'_r} < t \leq T'_r \end{cases} \quad (12)$$

$$\text{EEHI}_i' = \begin{cases} h_r * (U'_r * (t - T'_r) - \frac{\lambda_r}{2} * (t - T'_r)^2) & \text{for} \quad T'_r < t < T'_r + \frac{Q'_r}{D'_r} \\ \frac{h_r * (U'_r)^2}{2 \lambda_r} & \text{for} \quad T'_r + \frac{Q'_r}{D'_r} < t \leq T'_r \end{cases} \quad (13)$$

Actual equation of expected in-hand inventory level ($\text{EI}_i$; Eq. (2)) is estimated by Eq. (12) and actual equation of expected inventory holding cost ($\text{EHI}_i$; Eq. (7)) is estimated
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by Eq. (13). Numerical experiences showed that deviation of estimating equations and actual equations are very small and reasonable for all of values of $I'_i$, $\lambda_i$, and $D'_i$.

Expected sales for product family $i$ in $r^{th}$ run ($ES'_i$) can be estimated by following equations:

$$ES'_i = \begin{cases} \lambda_i * T' & \text{for } T' \leq \frac{I'_i}{\lambda_i} \text{ OR } D'_i > 0 \\ I'_i & \text{for } T' > \frac{I'_i}{\lambda_i} \end{cases}$$ (14)

If expected in-hand inventory level at the end of $r^{th}$ run ($EI'_i$) is positive, then expected sales during $r^{th}$ run ($ES'_i$) is equal to expected arrived orders ($\lambda_i*T'$). If $EI'_i$ is equal to zero, $ES'_i$ is equal to $I'_i$. Therefore, when the product family $i$ is selected as $r^{th}$ production task, $ES'_i$ is equal to $\lambda_i*T'$.

Several numerical examples are conducted and behaviour of actual equations is evaluated precisely. The results show the high efficiency of estimating equations. The estimating equations are summarized in Table 1.

Where $D'_i > 0$, the value of $EEHI'_i$ is transformed, in such a way to be used easily in the mathematical model. It is supposed that expected in-hand inventory level at the end of each run is equal to initial in-hand inventory level at the start of the next run. Also each run starts immediately after finishing previous run. Considering these facts, it is possible to apply the estimating equations to a set of successive runs of an arrangement. Using these equations, the model may be formulated as following.

### Table 1: Estimating equations

<table>
<thead>
<tr>
<th>Run $r$ with a length of $T'$</th>
<th>Product family $i$ is selected as $r^{th}$ production task ($D'_i &gt; 0$)</th>
<th>Product family $i$ is not selected as $r^{th}$ production task ($D'_i = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EI'_i$: Expect inventory level at the end of the $r^{th}$ run</td>
<td>$I'_i + Q'_i - \lambda_i * T'$</td>
<td>$I'_i - \lambda_i * T'$</td>
</tr>
<tr>
<td>$EEHI'_i$: Expected inventory holding cost during $r^{th}$ run</td>
<td>$h(D'_i - \lambda_i)(T')^2/2$</td>
<td>$h(I'_i * T' - \lambda_i * T')^2/2$</td>
</tr>
<tr>
<td>$EES'_i$: Expected sold products in $r^{th}$ run</td>
<td>$S_i * \lambda_i * T'$</td>
<td>$S_i * \lambda_i * T'$</td>
</tr>
</tbody>
</table>

3.2.2. Modelling Formulation

The model’s indices are as follows.

- $i$: Index for product families.
- $m$: Total number of product families.
- $j$: Index for feasible configurations for each family.
- $ni$: Total number of feasible configuration for each family.
- $r$: Index for production task.

The model’s parameters are as follows:

- $LPH$: Length of planning horizon.
Total number of production tasks to form optimum arrangement

$K_i^+$  Penalty coefficient of positive gap between in-hand inventory level of product family $i$ at the start and at the end of an arrangement.

$K_i^-$  Penalty coefficient of negative gap between in-hand inventory level of product family $i$ at the start and at the end of an arrangement.

$BigM$  Very big value which approaches infinitive.

The model’s decision variables are as follows:

$dev_i^+$  Positive gap between in-hand inventory levels of product family $i$ at the start and end of an arrangement.

$dev_i^-$  Negative gap between in-hand inventory levels of product family $i$ at the start and end of an arrangement.

$TC_r^+$  Required time to change system configuration in $r$th run.

$N_i^+$  \[ \begin{cases} 1 & \text{if in-hand inventory level for family } i \text{ at the end of } r \text{th run is equal to 0} \\ 0 & \text{otherwise} \end{cases} \]

$M_i^+$  \[ \begin{cases} 1 & \text{if product family } i \text{ is selected as } r \text{th production task} \\ 0 & \text{otherwise} \end{cases} \]

$P_{i,j}^+$  \[ \begin{cases} 1 & \text{if } j \text{th configuration of product family } i \text{ is selected in } r \text{th production task} \\ 0 & \text{otherwise} \end{cases} \]

The model is formulated as follows.

**Problem (P):**

**Maximize**

\[
\sum_{r=1}^{R} \sum_{i=1}^{n} N_i^+ (S_i^+ T_r - h_i^+ T_r^+ + \frac{M_i^+ D_r^+ - \lambda_i^+ T_r^+}{2} - D_r^+ T_r^+ / 2) + (1 - N_i^+)(S_i^+ T_r - \frac{I_i^+ h_i^+}{2 \lambda_i^+}) = \\
\sum_{r=1}^{R} \sum_{i=1}^{n} \sum_{j=1}^{m} CC_{i,j} + \sum_{r=1}^{R} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{m} P_{i,j}^+ P_{j,k}^+ G(c_{i,j}, c_{j,k}) - \sum_{r=1}^{R} \sum_{i=1}^{n} dev_i^+ K_i^+ - \\
\sum_{r=1}^{R} \sum_{i=1}^{n} dev_i^+ K_i^-
\]

**Subject to:**

\[
\sum_{i=1}^{n} M_i^+ = 1 \\
\forall r = 1, ..., R \quad (15)
\]

\[
Q_r^+ \leq M_i^+, \text{BigM} \\
\forall r = 1, ..., R ; \forall i (eM) \quad (16)
\]

\[
\sum_{j=1}^{m} P_{i,j}^+ = M_i^+ \\
\forall r = 1, ..., R ; \forall i (eM) \quad (17)
\]

\[
D_r^+ = \sum_{j=1}^{m} P_{i,j}^+ CR_{i,j} \\
\forall r = 1, ..., R ; \forall i (eM) \quad (18)
\]

\[
TC_r^+ = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{m} \sum_{l=1}^{m} P_{i,j}^+ P_{j,k}^+ G(c_{i,j}, c_{j,k}) \\
\forall r = 1, ..., R \quad (19)
\]
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\[ T^r = \sum_{i=1}^{R} \frac{Q^r}{D^r_i} + TC^r \]

\[ \forall r = 1, ..., R \] (20)

\[ T^r + N^r_i * \text{BigM} \geq \frac{Q^r_i + I^r_i}{\lambda_i} \]

\[ \forall r = 1, ..., R ; \forall i (\in M) \] (21)

\[ T^r \leq (1 - N^r_i) * \text{BigM} + \frac{Q^r_i + I^r_i}{\lambda_i} \]

\[ \forall r = 1, ..., R ; \forall i (\in M) \] (22)

\[ I^r_{i,s+1} = N^r_i (I^r_i + Q^r_i - \lambda_i T^r) \]

\[ \forall r = 1, ..., R ; \forall i (\in M) \] (23)

\[ I^r_i - I^r_{r+1} + \text{dev}^r_i - \text{dev}^r_i = 0 \]

\[ \forall i (\in M) \] (24)

\[ P^r_{i,j} = D^r_{i,j} \]

\[ \forall i, j (\in M) \] (25)

\[ \sum_{r=1}^{R} T^r \leq \text{LPH} \]

(26)

\[ Q^r_i \geq 0, \text{ Integer} \]

\[ \forall r = 1, ..., R ; \forall i (\in M) \] (27)

\[ D^r_i \geq 0 \]

\[ \forall r = 1, ..., R ; \forall i (\in M) \] (28)

\[ I^r_i \geq 0, \text{ Integer} \]

\[ \forall r = 1, ..., R ; \forall i (\in M) \] (29)

\[ TC^r \geq 0 \]

\[ \forall r = 1, ..., R \] (30)

\[ \text{dev}^r_i \geq 0, \text{ Integer} \]

\[ \forall i (\in M) \] (31)

\[ \text{dev}^r_i \geq 0, \text{ Integer} \]

\[ \forall i (\in M) \] (32)

\[ 0 \leq M^r_i \leq 1, \text{ Integer} \]

\[ \forall r = 1, ..., R ; \forall i (\in M) \] (33)

\[ 0 \leq N^r_i \leq 1, \text{ Integer} \]

\[ \forall r = 1, ..., R ; \forall i (\in M) \] (34)

\[ 0 \leq P^r_i \leq 1, \text{ Integer} \]

\[ \forall r = 1, ..., R ; \forall i (\in M) \] (35)

The goal is to maximize the total sales minus inventory holding costs, production costs and reconfiguration costs. The constraints are as follows. In each run, only one of product families can be selected as current production task (Constraint (15)). If product \( i \) is selected as \( r^\text{th} \) production task, one of its feasible configurations must be selected (Constraint (17)). Production rate and batch size for product family \( i \) in \( r^\text{th} \) run depend on whether or not that family is selected as \( r^\text{th} \) production task. If product family \( i \) is not selected as \( r^\text{th} \) production task, then \( Q^r_i = D^r_i = 0 \). Otherwise, it is equal to production rate of selected configuration (Constraints (16) and (18)).

The changeover time in each run depends on the selected configuration for the current production task and selected configuration for next production task. Constraint (19) determines this value in each run. Constraint (20) determines the length of \( r^\text{th} \) run. It is equal to production time plus changeover time. Production time is equal to batch size of \( r^\text{th} \) production task divided by its production rate.

Constraints (21) and (22) are used to determine the value of \( N^r_i \). These values determine whether the expected in-hand inventory level of product family \( i \) at the end of \( r^\text{th} \) run is positive or zero. Using these values and the estimated values in Table 1, following values are obtained:
1. The expected inventory levels of each product family at the start of \( r+1 \) run (Constraint (23)).
2. The expected sales of each product family in \( r \) run.
3. The expected inventory holding cost of each product family in \( r \) run.

Constraints (24) and (25) are considered to form a repeatable arrangement. For each product family, gap between in-hand inventory level at the start and end of an arrangement must be minimized as much as possible. To minimize these gaps, some penalties are charged in the objective function of the model. Constraint (24) determines these gaps. Constraint (25) imposes that the selected configuration for final production task must be changed to the selected configuration in the first production task.

Constraint (26) limits the length of arrangement to the length of planning horizon. Without this constraint, the solution may be unbounded. The goal is to find the optimum arrangement in a finite planning horizon. Other constraints (27-36) obtain feasibility of solutions.

3.3. Solution procedure

If the length of arrangement is not restricted, the objective function of the model is unbounded. In this model, the arrangement’s length is restricted by a planning horizon. The planning horizon must be effectively determined, such as one working day, one working week or etc. In the planning horizon, the goal is to determine the optimum arrangement having optimum number of runs, optimum sequence of production tasks and optimum batch sizes. Following procedure is suggested to determine the optimum or near to optimum arrangement.

Algorithm 1:

\begin{itemize}
  \item **Step A:** Define a finite planning horizon (LPH).
  \item **Step B:** Set the number of runs (R) to 1.
  \item **Step C:** Find the optimum or near to optimum arrangement with R runs.
  \item **Step D:** Set R to R+1
  \item **Step E:** Check the stopping criterion. If it is confirmed go to step F, otherwise go to step C
  \item **Step F:** Set another finite planning horizon
  \item **Step G:** Check the stopping criterion. If it is confirmed go to step H, otherwise go to step B
  \item **Step H:** Stop.
\end{itemize}

Since planning horizon is finite, the optimum number of runs is finite. In other words, if the number of runs is very large, RMS wastes a considerable portion of its production time due to changeovers. When the number of runs is low, RMS can not adapt itself to arrival orders effectively. Therefore, in the early iterations as the number of runs increases, the objective function may be improved. However, following some iterations, if the number of runs increases, the objective function may be decreased. A stopping criterion may be defined based on this fact. For example, if the objective function of the model dose not increases considerably for \( n \) successive iteration, then stopping criterion will be confirmed. Also the same stopping criterion may be defined for planning horizon.

If batch sizes and production configuration are not considered, the problem of the optimum arrangement will be converted to a Travelling Salesman Problem (TSP). TSP is considered as a \textit{NP-Complete} problem, therefore to find optimum or near optimum solutions, it is necessary to apply a heuristic method. In this paper, a Tabu search based method is proposed to determine near to optimum arrangement where the number of runs (R) is predefined and its length is restricted by predefined planning horizon (LPH). The sequence of selecting product families and one of their configurations are proposed by
the Tabu search algorithm. Using these values, the model will be simplified and can be solved by LINGO to determine the optimum batch size of each production task.

Tabu Search is a heuristic approach for solving combinatorial optimization problems by using a guided, local search procedure to explore the entire solution space without becoming easily trapped in local optimum [Glover 1990]. Tabu has shown to be effective for scheduling problems, Travelling Salesman Problems, Constraint Satisfaction Problems, general zero-one mixed integer programming problems as well as many engineering optimization problems [Dowsland 1998, Gendreau et al. 1998, Nonobe and Lbaraki 1998, Lokketangen and Glover 1998].

The Tabu search algorithm is applied to determine the near to optimum arrangement in this work and a numerical case study is demonstrated in section 4.

The basic elements of a Tabu search algorithm are starting solution, movement, neighbour solution, aspiration and stopping criteria [Glover 1990]. Since every sequence of arbitrary production tasks are feasible, the following changes can be considered as a movement.

1) **Horizontal movement**: To replace two production tasks with each other in the sequence.

2) **Vertical movement**: To replace the selected configuration of a production task with another feasible configuration.

3) **Diagonal Movement**: To replace one of the product families with another one.

The priority in the movements should be horizontal, vertical and diagonal movement respectively. Note that there are different Tabu lists, different Tabu sizes and different number of iterations for each type of movements.

### 4. Numerical experience

The optimum or near to optimum arrangement for an RMS having four product families is determined according to the presented heuristic algorithm. The design parameters of this RMS are shown in Table 2.

**Table 2**  The parameters of an RMS

<table>
<thead>
<tr>
<th>Families {1 , …, m}</th>
<th>{1, 2, 3, 4}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family (i)</td>
<td>1</td>
</tr>
<tr>
<td>Arrival rate (\lambda_i)</td>
<td>3.3</td>
</tr>
<tr>
<td>Selling price (S_i)</td>
<td>50</td>
</tr>
<tr>
<td>Inventory holding cost (h_i)</td>
<td>0.02</td>
</tr>
<tr>
<td>Feasible production configurations (C_i={c_{i,1},\ldots,c_{i,n_i}})</td>
<td>(C_1={c_{1,1},c_{1,2}})</td>
</tr>
<tr>
<td>Production configuration cost ((CC_{i,j}))</td>
<td>{19, 22}</td>
</tr>
<tr>
<td>Production configuration rate ((CR_{i,j}))</td>
<td>{6, 7}</td>
</tr>
</tbody>
</table>
Algorithm 1, step A:
At the first stage, one working day is suggested as the first planning horizon (i.e. 8*60=480 minutes). Therefore, the length of arrangements is limited to 480 (Constraint (26)).

Algorithm 1, step B:
The number of production runs (R) is sets to one. The arrangement judgment criterion is defined as follow:

\[ \text{Arrangement's Judgment criterion} = \frac{\text{the value of arrangement's objective function}}{\text{the length of the arrangement}} \]
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Note: the lengths of some arrangements are less or equal to 480 (Constraint (26)). Therefore the judgment criterion is defined as above to judge fairly the results.

Algorithm 1, step C:
As mentioned before, sequence of production tasks are proposed using Tabu search algorithm and the optimum batch sizes are obtained by optimizing the simplified model. The results of these steps are shown in Table 3.

<table>
<thead>
<tr>
<th>No.</th>
<th>Sequence of production tasks</th>
<th>Batch sizes ((V))</th>
<th>Length of arrangement (Minute)</th>
<th>Judgment criterion $$/min$$</th>
<th>The best solution $$/min$$</th>
<th>Movement</th>
<th>Movement type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{PT_{1,1}} (, (1584))</td>
<td>264 + 216 idle time</td>
<td>95.17</td>
<td>95.17</td>
<td>(PT_{1,1} ) to (PT_{1,2})</td>
<td>Vertical</td>
<td>Vertical</td>
</tr>
<tr>
<td>2</td>
<td>{PT_{1,1}} (, (1584))</td>
<td>226+ 254 idle time</td>
<td>84</td>
<td>95.17</td>
<td>(PT_{1,2} ) to (PT_{1,1})</td>
<td>Diagonal</td>
<td>Diagonal</td>
</tr>
<tr>
<td>3</td>
<td>{PT_{2,1}} (, (1296))</td>
<td>324+ 156 idle time</td>
<td>107</td>
<td>107</td>
<td>(PT_{2,1} ) to (PT_{2,2})</td>
<td>Vertical</td>
<td>Vertical</td>
</tr>
<tr>
<td>4</td>
<td>{PT_{2,1}} (, (1296))</td>
<td>259+ 221 idle time</td>
<td>109.8</td>
<td>109.8</td>
<td>(PT_{2,1} ) to (PT_{1,1})</td>
<td>Vertical</td>
<td>Vertical</td>
</tr>
<tr>
<td>5</td>
<td>{PT_{3,1}} (, (1296))</td>
<td>216 + 264 idle time</td>
<td>97.3</td>
<td>97.3</td>
<td>(PT_{3,1} ) to (PT_{3,2})</td>
<td>Vertical</td>
<td>Vertical</td>
</tr>
<tr>
<td>6</td>
<td>{PT_{4,1}} (, (1008))</td>
<td>252 + 228 idle time</td>
<td>100.2</td>
<td>100.2</td>
<td>(PT_{4,1} ) to (PT_{3,1})</td>
<td>Vertical</td>
<td>Vertical</td>
</tr>
<tr>
<td>7</td>
<td>{PT_{4,1}} (, (1008))</td>
<td>201 + 279 idle time</td>
<td>92.9</td>
<td>92.9</td>
<td>(PT_{4,1} ) to (PT_{4,2})</td>
<td>Vertical</td>
<td>Vertical</td>
</tr>
<tr>
<td>8</td>
<td>{PT_{4,1}} (, (912))</td>
<td>228 + 252 idle time</td>
<td>116.8</td>
<td>116.8</td>
<td>(PT_{4,2} ) to (PT_{4,1})</td>
<td>Vertical</td>
<td>Vertical</td>
</tr>
<tr>
<td>9</td>
<td>{PT_{4,1}} (, (912))</td>
<td>182 + 298 idle time</td>
<td>110.2</td>
<td>110.2</td>
<td>Stop</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Production of product family 1 in its first configuration (\(PT_{1,1}\)) is considered as starting solution. The optimum batch size for this production task is equal to 1584. Production time of this batch size is 264 minutes. RMS must be idle for remaining 216 minutes of planning horizon. Notice that the rate of arrival orders is less than production rates. Therefore RMS must be stopped for a while. The objective function of model is 45681.6 and the judgment criterion is equal to 95.17.

In this iteration, horizontal movement is not possible, the procedure starts with a vertical movement and the first configuration of product 1 is replaced with the second one. The objective function decreases in this movement. In the next iteration, there is no other vertical movement and a diagonal movement is applied. Therefore first configuration of product family 2 is selected. The optimum sequence with one run is \{\(PT_{2,1}\)\} and optimum batch size is 912. Its judgment criterion is 116.8 $$/min$$.

Algorithm 1, step B:
In next iteration, the number of runs \((R)\) is increased by one. Since \(PT_{4,1}\) is the optimum solution in previous stage, \{\(PT_{4,1}, PT_{2,2}\)\} is selected as starting solution.

Algorithm 1, step C:
In this step near to optimum arrangement is determined. The results are shown in Table 4.

<table>
<thead>
<tr>
<th>No.</th>
<th>Sequence of production tasks</th>
<th>Batch sizes ((V))</th>
<th>Length of arrangement (Minute)</th>
<th>Judgment criterion $$/min$$</th>
<th>The best solution $$/min$$</th>
<th>Movement</th>
<th>Movement type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{PT_{4,1}, PT_{2,2}} (, (857,1295))</td>
<td>480</td>
<td>345.1</td>
<td>345.1</td>
<td>(PT_{2,2} ) to (PT_{3,2})</td>
<td>Horizontal</td>
<td>Horizontal</td>
</tr>
<tr>
<td>2</td>
<td>{PT_{3,2}, PT_{1,2}} (, (1227,911))</td>
<td>480</td>
<td>353.9</td>
<td>353.9</td>
<td>(PT_{3,2} ) to (PT_{2,1})</td>
<td>Vertical</td>
<td>Vertical</td>
</tr>
<tr>
<td>3</td>
<td>{PT_{3,2}, PT_{1,2}} (, (981,912))</td>
<td>480</td>
<td>329.6</td>
<td>353.9</td>
<td>(PT_{3,2} ) to (PT_{2,1})</td>
<td>Horizontal</td>
<td>Horizontal</td>
</tr>
<tr>
<td>4</td>
<td>{PT_{3,2}, PT_{1,2}} (, (1296,597))</td>
<td>480</td>
<td>285.1</td>
<td>353.9</td>
<td>(PT_{2,2} ) to (PT_{3,2})</td>
<td>Vertical</td>
<td>Vertical</td>
</tr>
</tbody>
</table>


The result shows the optimum or near to optimum sequence where \( R=2 \) is \( \{PT_{1,1},PT_{4,2}\} \), batch sizes are \((302,174)\) respectively and its judgment criterion is \(388.5 \$/min\).

The number of runs is increased while the length of planning horizon is one working day (480 Minutes). The results are shown in Table 5.

<table>
<thead>
<tr>
<th>Table 5</th>
<th>The summary of computation results for one day planning horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planning time horizon: 480 min and ( R=2 )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No.</th>
<th>Sequence of production tasks</th>
<th>Batch sizes (V)</th>
<th>Length of arrangement (Minute)</th>
<th>Judgment criterion $/min</th>
<th>The best solution $/min</th>
<th>Movement</th>
<th>Movement type</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>( {PT_{4,1},PT_{1,1}} )</td>
<td>((597,1296))</td>
<td>480</td>
<td>285.1</td>
<td>353.9</td>
<td>(PT_{4,1}) to (PT_{1,1})</td>
<td>Horizontal</td>
</tr>
<tr>
<td>6</td>
<td>( {PT_{4,1},PT_{1,1}} )</td>
<td>((981,912))</td>
<td>480</td>
<td>329.6</td>
<td>353.9</td>
<td>(PT_{4,1}) to (PT_{1,1})</td>
<td>Vertical</td>
</tr>
<tr>
<td>7</td>
<td>( {PT_{2,3},PT_{4,1}} )</td>
<td>((302,213))</td>
<td>112</td>
<td>367</td>
<td>367</td>
<td>(PT_{2,3}) to (PT_{4,1})</td>
<td>Horizontal</td>
</tr>
<tr>
<td>8</td>
<td>( {PT_{4,1},PT_{2,3}} )</td>
<td>((213,302))</td>
<td>112</td>
<td>377.9</td>
<td>377.9</td>
<td>(PT_{4,1}) to (PT_{2,3})</td>
<td>Horizontal</td>
</tr>
<tr>
<td>9</td>
<td>( {PT_{2,3},PT_{4,1}} )</td>
<td>((212,133))</td>
<td>78.5</td>
<td>342.8</td>
<td>377.9</td>
<td>(PT_{2,3}) to (PT_{4,1})</td>
<td>Horizontal</td>
</tr>
<tr>
<td>10</td>
<td>( {PT_{4,1},PT_{2,3}} )</td>
<td>((133,212))</td>
<td>78.5</td>
<td>322.6</td>
<td>377.9</td>
<td>(PT_{4,1}) to (PT_{2,3})</td>
<td>Vertical</td>
</tr>
<tr>
<td>11</td>
<td>( {PT_{2,3},PT_{1,1}} )</td>
<td>((81,51))</td>
<td>30</td>
<td>328.8</td>
<td>377.9</td>
<td>(PT_{2,3}) to (PT_{1,1})</td>
<td>Horizontal</td>
</tr>
<tr>
<td>12</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((1259,1583))</td>
<td>480</td>
<td>352.5</td>
<td>377.9</td>
<td>(PT_{2,3}) to (PT_{1,2})</td>
<td>Vertical</td>
</tr>
<tr>
<td>13</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((255,312))</td>
<td>94.5</td>
<td>386.8</td>
<td>386.8</td>
<td>(PT_{2,3}) to (PT_{1,2})</td>
<td>Horizontal</td>
</tr>
<tr>
<td>14</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((100,195))</td>
<td>59</td>
<td>343.2</td>
<td>386.8</td>
<td>(PT_{2,3}) to (PT_{1,2})</td>
<td>Horizontal</td>
</tr>
<tr>
<td>15</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((195,100))</td>
<td>59</td>
<td>328.9</td>
<td>386.8</td>
<td>(PT_{2,3}) to (PT_{1,2})</td>
<td>Diagonal</td>
</tr>
<tr>
<td>16</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((174,302))</td>
<td>91.7</td>
<td>367.6</td>
<td>386.8</td>
<td>(PT_{2,3}) to (PT_{1,2})</td>
<td>Horizontal</td>
</tr>
<tr>
<td>17</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((302,174))</td>
<td>91.7</td>
<td>388.5</td>
<td>388.5</td>
<td>(PT_{2,3}) to (PT_{1,2})</td>
<td>Vertical</td>
</tr>
<tr>
<td>18</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((82,92))</td>
<td>48.2</td>
<td>324.5</td>
<td>388.5</td>
<td>(PT_{2,3}) to (PT_{1,2})</td>
<td>Horizontal</td>
</tr>
<tr>
<td>19</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((92,82))</td>
<td>48.2</td>
<td>315.5</td>
<td>388.5</td>
<td>(PT_{2,3}) to (PT_{1,2})</td>
<td>Vertical</td>
</tr>
<tr>
<td>20</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((82,92))</td>
<td>48.2</td>
<td>315.5</td>
<td>388.5</td>
<td>(PT_{2,3}) to (PT_{1,2})</td>
<td>Vertical</td>
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<td>21</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((92,82))</td>
<td>48.2</td>
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<td>(PT_{2,3}) to (PT_{1,2})</td>
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<td>22</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((82,92))</td>
<td>48.2</td>
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<td>23</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((92,82))</td>
<td>48.2</td>
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<td>(PT_{2,3}) to (PT_{1,2})</td>
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<td>24</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((82,92))</td>
<td>48.2</td>
<td>315.5</td>
<td>388.5</td>
<td>(PT_{2,3}) to (PT_{1,2})</td>
<td>Vertical</td>
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<tr>
<td>25</td>
<td>( {PT_{2,3},PT_{1,2}} )</td>
<td>((92,82))</td>
<td>48.2</td>
<td>315.5</td>
<td>388.5</td>
<td>(PT_{2,3}) to (PT_{1,2})</td>
<td>Vertical</td>
</tr>
</tbody>
</table>

As Figure 5 shows, the judgment criterion in the first four iterations is increased, in fifth to seventh iterations decreased and converged to 478.1. The solutions in the eighth iteration and subsequent iterations converge to the optimum arrangement found in the
Production Planning of Reconfigurable Manufacturing Systems with Stochastic Demands Using Tabu Search

fourth iteration. This arrangement is suggested as the optimum or near to optimum arrangement where the planning horizon has a length of 480 minutes.

As the length of planning horizon is increased, no better solution is found. Therefore above mentioned arrangement is suggested as near to the best arrangement. Indeed, this is a special case that each product family has relatively similar parameters such as arrival rates, net profits.

In this research, several numerical examples and experiments were conducted. The results show:

- If changeover costs and times approach to zero, optimum batch sizes approach to one.
- If changeover costs \( G(C_{i,j}, C_{k,l}) \) decrease in comparison with coefficient of inventory holding costs \( h_i \), then the greater number of runs is expected such as \( R = 8, 12, 16 \) and etc. Conversely, if \( G(C_{i,j}, C_{k,l}) \) increase in comparison with \( h_i \), then a fewer number of runs is expected.
- Similarly, if changeover time \( G(T(C_{i,j}, C_{k,l})) \) decrease in comparison with production rates \( CR_{i,j} \), then the greater number of runs is expected such as \( R = 8, 12, 16 \) and etc. Conversely, if \( G(T(C_{i,j}, C_{k,l})) \) increase in comparison with \( CR_{i,j} \), then a fewer number of runs is expected.

5. Conclusion remarks

In the new business era embracing “change” as one of its major characteristics, the manufacturers need to adopt approaches that can lead them to achieve more adaptability to market changes. RMS is a system designed, from the outset, for rapid changes in structure, both in hardware and software components, in order to quickly adjust production capacity and functionality within a part family in response to sudden market changes.

The effectiveness of an RMS depends on implementing these principles in system design as well as utilization stages. This paper focuses on the utilization stage of an RMS and introduces a mathematical model to manage and evaluate effectiveness of RMS. This model considers the key characteristics and capabilities of RMS to adjust scalable production capacities and the functionality of the system to respond rapidly to market demands and fulfil productivity. Therefore following issues are considered:

- Optimum length of time span to plan the production tasks
- Optimum number of production tasks to be done within the optimum time span
- Optimum sequence of production tasks
- Optimum configuration of the selected product family at each production task
- Optimum batch size at each production task.

The goal is to maximize the earned profits minus production costs. Production costs are the related costs such as operation costs, inventory holding costs and changeover costs. Therefore a mixed integer non-linear programming (MINLP) model is introduced to determine above mentioned issues. Since arrival orders follow Poisson distribution, the behaviour of expected in-hand inventory levels, inventory holding costs and sales within each run are estimated. Then a mixed integer non-linear programming model is proposed to determine the best arrangement/order of production tasks. This arrangement are a near to optimum solution in a finite planning horizon, and may be selected as a repeating arrangement, if there are no considerable changes in system’s condition such as the number of product families, configurations and rate of order arrivals.

To solve the MINLP model and find the near to optimum arrangement, a finite time horizon such as a working day or a week is considered, and the number of the production tasks performed within the mentioned time horizon is increased step by step until there is
M. Abbasi and M. Houshmand

no improvement in the objective function of model. At each step, considering the number of production tasks, the sequence of production tasks and corresponding configurations are suggested by a Tabu search based algorithm. According to the suggested sequence, the MINLP model could be simplified. Then the optimum batch size in each production task is determined by optimizing the simplified model using LINGO.

Numerical examples and experiments are conducted to illustrate the effectiveness of the proposed procedure. The results show that, if changeover costs and times approach to zero, then optimum batch size at each production task approaches to one. This situation is one of the ultimate goals of a Lean production system. Conversely, if changeover times and costs are increase where compared with inventory holding costs coefficient, then the bigger batch size at each production task is expected. Some advantages of proposed procedure are summarized as follows.

- It is supposed that the manufacturer should respond to arrival orders immediately.
- This model focuses on minimizing inventory holding costs and production costs where response time is set it to zero.
- According to this model, all of feasible configurations can be selected in utilization stage and a product family can be selected as two successive production tasks.
- In proposed model, the reconfiguration time is considered.

Following issues can be seen as future direction of this research:

- In some businesses environments, it is preferable not to miss orders and respond to the orders as soon as they get ready for delivery. Considering this business environments and shortage costs for delayed orders may be proposed as a future task.
- Although Tabu search is an effective algorithm to solve scheduling problems [Dowsland, 1998], the use of other heuristic methods can be examined in future tasks.

References


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![Figure 1](attachment://figure1.png)  
**Figure 1** Schematic presentation of Production task, Run and Arrangement

![Figure 2](attachment://figure2.png)  
**Figure 2** Graphic illustration of $EI_i^t$ and $EI_{hi}^t$ if product family $i$ is not selected as $r^{th}$ production task
Figure 3 Graphic illustration of $EI_i^t$ and $EHI_i^t$ if product family $i$ is selected as $r^{th}$ production task

Figure 4 Graphic illustration of $EI_i^t$ and $EHI_i^t$ and corresponding estimating equations, if product family $i$ is not selected as $r^{th}$ production task

Figure 5 The judgment criterion against the number of runs