Optimal Pricing and Ordering Policies for a Two-Layer Supply Chain with Imperfect Quality Items under Two Inspection Scenarios

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This paper developed integrated production–inventory-marketing models for a two-stage supply chain consisting of one vendor and one buyer. The aim of the paper was to decrease the inconsistency between theory and practice of inventory management by relaxing some unrealistic assumptions such as constant demand and fully perfect items that make the models practically impossible for real-life problems. Two inspection scenarios were considered, and for each one, the supply chain’s total profits under independent and also joint optimization were maximized by determining the optimal values of selling price, order quantity and number of shipments. A numerical example, including sensitivity analysis for critical parameters, was given. The computational results revealed that cooperation between the buyer and the vendor, regardless of inspection scenario, is beneficial for the supply chain, especially for more price sensitive demands. In addition, specifying the best inspection scenario is dependent on the unit warranty and unit inspection costs.

Keywords: integrated inventory, vendor, buyer, pricing, imperfect quality, inspection

1. INTRODUCTION

In the past decade, supply chain inventory management has received considerable attention, and many researchers such as Goyal (1976), Banerjee (1986), Goyal (1988), Hill (1997), Lu (1995), Ouyang et al. (2004) and Rad et al. (2011) have investigated integrated inventory models. Recently, Ben-Daya et al. (2008) and Glock (2012) presented a comprehensive review of these models.

The main drawback of the aforementioned models is that the customer’s demand was unrealistically considered independent of the product’s selling price. Therefore, from a practical situation, some researchers have addressed models that can simultaneously optimize the inventory (operations) and pricing (marketing) decisions. The incentive is developed from the following observations in real life. On one hand, firms often use pricing to influence the demand and enhance their profits. On the other hand, firms also want to make an appropriate production and ordering decision to satisfy the demand at a cost as minimal as possible. These decisions must support each other in order to obtain maximal profit. The first inventory model with price dependent demand was suggested by Whitin (1955) who incorporated pricing into traditional EOQ model through a linear price dependent demand. Later, Porteus (1985) obtained an explicit solution for this model. Recently, Sajadieh and Jokar (2009) provided an integrated production-inventory-marketing model to maximize the joint total profit of both the vendor and the buyer. Some other researchers such as Ho et al. (2008), Chen and Kang (2010b), Ouyang et al. (2009) and Ho (2011) extended integrated inventory models that involve price sensitive demands. They considered flexible production rates, which are fixed ratios of the demand rates. Later, Chung and Liao (2011) simplified the solution algorithm described in Ho et al. (2008). However, none of them considered imperfect quality items in their models. In the literature, most of the inventory models developed based on the unrealistic assumption of completely perfect items. This assumption is not always valid because of the weak process control, deficient planned maintenance, inadequate work instructions and/or damage during transportation. The first studies which explicitly relaxed an impractical assumption of completely perfect product and integrated the effect of defective items into the inventory model was presented by Porteus (1986) and Rosenblatt and Lee (1986). Salameh and Jaber (2000) developed the traditional EOQ model by accounting for imperfect quality items. They
assumed that defective items are sold as a single batch at the end of the 100% screening process. Huang (2002) and Goyal et al. (2003) extended the model in Salameh and Jaber (2000) to present models for a vendor–buyer supply chains. Later, Ha and Kim (2011) identified two errors in the paper of Huang (2002) and provided a reformulation of the model. Huang (2004) reformulated the model in Goyal et al. (2003), but he used the work of Salameh and Jaber (2000) to model the annual cost of the buyer. Ouyang et al. (2006) presented inventory models in a two-stage supply chain for crisp and fuzzy defective rates. Sana (2011) developed an integrated production-inventory model for supplier, manufacturer and retailer supply chain, considering perfect and imperfect quality items. Some other studies which considered imperfect quality items in integrated inventory models in order to develop more practical models are Yang and Pan (2004), Ouyang et al. (2007), El Saadany and Jaber (2008), Chen and Kang (2010a) and Khan and Jaber (2011).

The aim of this paper is decreasing the discrepancy between theory and practice of inventory management by releasing some unrealistic assumptions. Therefore, the approaches of imperfect quality items and pricing are incorporated into the integrated single-vendor and single-buyer inventory model. The main contributions of the paper can be summarized as follow: 1) The demand rate is sensitive to the selling price with constant elasticity. 2) There are imperfect quality items, which are created during production as well as transportation. 3) In order to detect the imperfect items, two inspection scenarios are introduced. In the first inspection scenario, the buyer inspects the received shipments and returns the imperfect items to the vendor in a single batch at the end of the 100% screening process. However, in the second scenario, not only the buyer but also the vendor inspects the shipments. 4) The unit purchasing price which is paid by the buyer to the vendor is a linear decreasing function of the received shipments’ defective rate. For each inspection scenario, the corresponding problem is formulated as a mixed integer nonlinear programming problem with one integer and two non-integer variables, and an optimization algorithm is presented to obtain optimal solution. A numerical example, including sensitivity analysis is carried out to evaluate the proposed models. The computational results show that the supply chain’s profit can be increased by coordination and also by selecting the best inspection scenario. The paper is organized as follows: In Section 2, assumptions and notations are provided. In section 3, the models are mathematically formulated for the first and second inspection scenarios. In section 4, the solution algorithm is described. Section 5 presents numerical example and sensitivity analysis. Conclusions and future research are summarized in Section 6.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used to develop integrated inventory models.

1. Single vendor-single buyer supply chain with single product is considered.

2. Shortage is not allowed.

3. The demand rate is a decreasing function of the selling price, \( D(p) = \alpha p^{-\beta} \), where \( \alpha > 0 \) is a scaling factor and \( \beta > 1 \) is the index of price elasticity. This type of demand is called the Iso-elastic demand function. For notational simplicity, \( D(p) \) and \( D \) will be used interchangeably in this paper.

4. The defective items are created during production and transportation with the known rates \( \delta y_p \) and \( y_T \), respectively.

5. In order to recognize these defective items, full inspection is done under two scenarios. In the first scenario, the buyer inspects the received shipments to separate and return back the imperfect items in a single batch at the end of the 100% screening process. In the second scenario, at first, the vendor inspects the shipments before dispatching them to the buyer to distinguish and separate the defective items which are created during production. Then, the received shipments by the buyer are checked again to detect and give back the defective ones which are made during delivery. Therefore, in the first and second inspection scenarios, the buyer receives the shipments which contain defective items with rates \( y = y_p + y_T (1 - y_p) \), and \( y = y_T \), respectively.

6. Each imperfect item which is detected by the buyer costs \( C_w \) dollars for the vendor. This cost involves not only return cost but also penalty cost due to goodwill loss from the buyer’s quality dissatisfaction.

7. The unit purchasing price which is paid by the buyer to the vendor is a linear decreasing function of the received shipments’ defective rate, \( w(y) = m - ky \) (\( m > k > 0 \)).

8. The capacity utilization \( \rho \) is defined as the ratio of the market demand rate, \( D \), to the production rate, \( R \), i.e., \( \rho = D/R \), where \( \rho \leq 1 \) and...
is fixed and given. In addition, in order to satisfy the demand in full to have the feasible problem, it is assumed that the \( \frac{D}{(1 - y)} \). In the other words, \( \frac{D}{(1 - y)} \leq 1 \).

9. It is considered the given and fixed values \( \rho_b = D/x_b \) and \( \rho_v = D/x_v \), where \( x_b \) and \( x_v \), which are greater than the demand rate, are the screening rates of the buyer and vendor, respectively. In order to avoid shortages, as Salameh and Jaber (2000) assumed, the number of perfect items is at least equal to the demand during the screening time. So, \( \frac{\rho_b}{(1 - y)} \leq 1 \) and \( \frac{\rho_v}{(1 - y)} \leq 1 \).

10. The time horizon is infinite.

11. The inventory is continuously reviewed.

The other parameters are:

\( S \) vendor’s set up; \( A \) buyer’s ordering cost; \( C \) unit production cost \( (C < w(y)) \); \( r \) annual inventory holding charge, expressed as fraction of dollar value; \( h_v \) vendor’s inventory holding cost per item per unit time \( (h_v = rC) \); \( h_b \) buyer’s inventory holding cost per item per unit time \( (h_b = r w(y)) \); \( C_{v, e} \) vendor’s unit screening cost; \( C_{b, u} \) buyer’s unit screening cost; \( F \) fixed transportation cost per shipment; \( v \) unit variable cost for handling or receiving each item; \( C_w \) unit warranty cost for defective items, including the penalty cost for the vendor; \( p_t \) the buyer unit selling price for \( i \)’th scenario \( (i = b, v) \); \( Q_i \) buyer’s order quantity for \( i \)’th scenario; \( n_i \) number of shipment for \( i \)’th scenario; \( Q_P_i \) vendor’s production quantity for \( i \)’th scenario \((Q_P_i = n_i Q_i)\); \( TPB_i \) buyer’s profit function per unit time under independent optimization for \( i \)’th scenario; \( TPV_i \) vendor’s profit function per unit time under independent optimization for \( i \)’th scenario; \( TPI_i \) total system profit per unit time under independent optimization for \( i \)’th scenario; \( JTP_i \) joint total profit per unit time for \( i \)’th scenario

### 3. MODEL DEVELOPMENT

In this section, for each inspection scenario, the corresponding integrated inventory model is formulated. We first obtain the buyer’s and the vendor’s profit functions under independently optimization and then the integrated model are developed.

#### 3.1. Mathematical models under the first inspection scenario

In the first inspection scenario, the buyer inspects the received shipments and then returns the imperfect items to the vendor in a single batch at the end of the fully inspection. Hence, the buyer’s and the vendor’s inventory behavior is as a Fig.1.

#### 3.1.1. Buyer’s total profit per unit time under the first inspection scenario

Under this inspection scenario, each received shipment by the buyer contains \( y = y_p + y_r(1 - y_p) \) percent defective items. According to Fig.1a, the buyer’s profit per unit time can be expressed as

\[
TPB_b(p_b, Q_b) = \left( p_b - w(y) - \frac{C_{b, u} + v}{(1 - y)} \right) \alpha p_b^{-\beta} - \frac{h_b Q_b}{2} \left( 1 - y + \frac{2 p_b y}{(1 - y)} \right)
\]

#### 3.1.2. Vendor’s total profit per unit time under the first inspection scenario

The vendor’s profit function per unit time is as follow where \( \rho = D/R \) (see Fig.1b).

\[
TPV_b(n_b) = (w(y) - \frac{C + y C_w}{(1 - y)}) \alpha p_b^{-\beta} - \frac{S \alpha p_b^{-\beta}}{n_b (1 - y) Q_b} - \frac{h_v Q_b}{2} \left( \frac{2 - n_b \rho}{(1 - y)} + n_b - 1 \right)
\]
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3.1.3 The joint total profit per unit time under the first inspection scenario

Now, suppose that the vendor and the buyer cooperate and share information with each other to determine the best scenario for the whole supply chain. Therefore, the joint total profit function per unit time is given by

\[
\text{Maximize } JTP_b(p_b, Q_b, n_b) \\
= \left( p_b - \frac{Cl_b + C + v + yC_w}{(1 - y)} \right) \alpha p_b^{-\beta} - \frac{\left( \frac{s}{n_b} + A + F \right) \alpha p_b^{-\beta}}{Q_b(1 - y)} \\
- \frac{h_v Q_b}{2} \left( \frac{2 - n_b}{(1 - y)} + n_b - 1 \right) - \frac{h_b Q_b}{2} \left( (1 - y) + \frac{2 \rho_y y}{(1 - y)} \right) \\
\text{S.t.} \\
\begin{align*}
Q_b &> 0 \\
n_b &\in N \\
p_b &> \frac{Cl_b + C + v + yC_w}{(1 - y)} \\
\end{align*}
\]

(3)

First, the effect of \( n_b \) on the joint total profit will be examined. Taking the second-order partial derivative of \( JTP_b(p_b, Q_b, n_b) \) with respect to \( n_b \) implies that \( JTP_b(p_b, Q_b, n_b) \) is a concave function in \( n_b \) for fixed \( p_b \) and \( Q_b \). Therefore, the search for the optimal shipment number, \( n_b^* \), is reduced to find a local optimal solution. In addition, \( JTP_b(p_b, Q_b, n_b) \) is also a concave function in \( Q_b \) for fixed \( n_b \) and \( p_b \). Thus, there exists a unique value of \( Q_b \) (denoted by \( Q_b^* \)) which maximizes \( JTP_b(p_b, Q_b, n_b) \), and the value of \( Q_b^* \) can be obtained by solving the equation \( \frac{\partial JTP_b(p_b, Q_b, n_b)}{\partial Q_b} = 0 \).
\[
Q_b^* = \frac{2\alpha p_b - \beta (S/n_b + A + F)}{(1 - y)\left[h_v \left(\frac{(2-n_b)p}{(1-y)} + n_b - 1\right) + h_b \left(1 - y + 2\rho b y\right)\right]}.
\]

Substituting (4) into (3), we can get the following joint total profit which is the function of the two variables \(p_b\) and \(n_b\).

\[
JTP_b(p_b, n_b) = \left( \frac{C + v + y c_w}{Cl_b + C + v + y c_w} \right) \alpha p_b^{-\beta} - \frac{2\alpha p_b - \beta (S/n_b + A + F)}{(1 - y)\left[h_v \left(\frac{(2-n_b)p}{(1-y)} + n_b - 1\right) + h_b \left(1 - y + 2\rho b y\right)\right]}.
\]

The optimal value for the selling price \(p_b\) for a fixed value of \(n_b\) can be obtained by taking the first-order partial derivative of \(JTP_b(p_b, n_b)\) in Eq. (5) with respect to \(p_b\) and setting it equal to zero (this is the necessary condition for optimality), and solving for \(p_b\) by a numerical method. For example, the \texttt{fsolve} procedure of MATLAB could be used to solve this equation, as was done in this paper.

\[
\frac{\partial JTP_b(p_b, n_b)}{\partial p_b} = \alpha p_b^{-\beta} - \alpha \beta p_b^{-\beta - 1} \left(\frac{Cl_b + C + v + yc_w}{(1 - y)}\right) + \frac{2\alpha p_b - \beta (S/n_b + A + F)}{(1 - y)\left[h_v \left(\frac{(2-n_b)p}{(1-y)} + n_b - 1\right) + h_b \left(1 - y + 2\rho b y\right)\right]} = 0.
\]

Subsequently, we need to check the second-order condition for concavity.

\[
\frac{\partial^2 JTP_b(p_b)}{\partial p_b^2} = -\alpha \beta p_b^{-\beta - 1} + \alpha \beta^2 p_b^{-\beta - 1} - \alpha \beta (\beta + 1) p_b^{-\beta - 2} \left(\frac{Cl_b + C + v + yc_w}{(1 - y)}\right) + \frac{2\alpha p_b - \beta (S/n_b + A + F)}{(1 - y)\left[h_v \left(\frac{(2-n_b)p}{(1-y)} + n_b - 1\right) + h_b \left(1 - y + 2\rho b y\right)\right]} \beta (\beta + 2) \left(\frac{2\alpha p_b - \beta (S/n_b + A + F)}{(1 - y)\left[h_v \left(\frac{(2-n_b)p}{(1-y)} + n_b - 1\right) + h_b \left(1 - y + 2\rho b y\right)\right]}\right)^{3/2}.
\]

It can be shown that the second derivative of \(JTP_b(p_b, n_b)\) in Eq. (7) is smaller than zero for \(p_b > \frac{Cl_b + C + v + yc_w}{(1 - y)}\). Hence, \(JTP_b(p_b, n_b)\) is a concave function in \(p_b\), and the obtained value for the selling price from Eq. (6) is the optimal value.

### 3.2. Mathematical models under the second inspection scenario

In the second inspection scenario, each shipment is inspected twice, first, by the vendor before dispatching in order to separate the defective items which are created during production, and then, by the buyer after receiving to detect defective items which are made during transportation.

#### 3.2.1. Buyer’s total profit per unit time under the second inspection scenario

Here, the buyer’s inventory behavior is like before (see Fig. 1a). However, each received shipment by the buyer contains \(sy\) percent defective items. So, the buyer’s total profit per unit time under the second inspection scenario is...
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\[
TPB_v(p_v, Q_v) = \left( p_v - w(y_T) - \frac{CL_b + v}{(1 - y_T)} \right) \alpha p_v^{-\beta} - \frac{(A + F)\alpha p_v^{-\beta}}{(1 - y_T)Q_v} - \frac{h_b Q_v}{2} \left\{ \left( 1 - y_T \right) + \frac{2\rho_b y_T}{(1 - y_T)} \right\} 
\]

(8)

3.2.2. **Vendor’s total profit per unit time under the second inspection scenario**

When the buyer orders \( Q_v \) units, the vendor produces \( \hat{Q} = Q_v / (1 - y_p) \) units, and after inspection, he delivers \( Q_v \) units to the buyer. Therefore, the vendor’s inventory behavior is as Fig. 2, and its profit function per unit time can be presented by the expression

\[
TPV_v(v) = (w(y_T) - \frac{CL_v + C}{(1 - y_p)(1 - y_T)} - \frac{y_T C_W}{(1 - y_T)} \alpha p_v^{-\beta} - \frac{S \alpha p_v^{-\beta}}{n_v Q_v (1 - y_T)} - \frac{h_v Q_v}{2(1 - y_p)} \left( \frac{2 - n_v}{(1 - y_T)(1 - y_T)} + \frac{n_v - 1}{(1 - y_T)(1 - y_T) + n_v - 1} \right) 
\]

(9)

**Fig. 2:** the vendor’s inventory system under the second inspection scenario
3.2.3. The joint total profit per unit time under the second inspection scenario

The joint total profit function per unit time under the second inspection scenario is:

\[
\begin{align*}
\text{Maximize } & JTP_v(p_v, Q_v, n_v) \\
= & \left( p_v - \frac{CI_v + C}{(1-y_P)(1-y_T)} - \frac{CI_b + v + y_T C_W}{(1-y_T)} \right) \alpha p_v^{-\beta} \\
& - \frac{Q_v(1-y_T)}{Q_v(1-y_T) - \frac{Q_v}{2} \left( \frac{2-n_v}{(1-y_P)(1-y_T)} \right) + \frac{2\rho v}{(1-y_P)(1-y_T)} + n_v - 1} \\
& + h_b \left( 1 - y_T + \frac{2\rho y_T}{(1-y_T)} \right) \\
\text{s.t.} & \\
p_v & > \frac{CI_v + C}{(1-y_P)(1-y_T)} - \frac{CI_b + v + y_T C_W}{(1-y_T)} \\
Q_v & > 0 \\
n_v & \text{integer}
\end{align*}
\]

(10)

\[
\text{For simplification we show } \left\{ \frac{h_v}{(1-y_P)} \left( 2 - n_v \right) + \frac{2\rho v}{(1-y_P)(1-y_T)} + n_v - 1 \right\} + h_b \left( 1 - y_T + \frac{2\rho y_T}{(1-y_T)} \right) \text{ with } M.
\]

\[
Q_v^* = \sqrt{\frac{2\alpha p_v^{-\beta}(S/n_v + A + F)}{(1-y_T)M}}
\]

(11)

With the same procedure like the previous section, by substituting (11) into the joint total profit (10), and then taking the first-order derivative respect to \( p_v \) and setting the result equal to zero, the optimal selling price for fixed \( n_v \) can be found because it can be shown that the sufficient condition for optimality is met.

\[
\frac{\partial JTP_v(p_v, n_v)}{\partial p_v} = \alpha p_v^{-\beta} - \alpha \beta p_v^{-\beta} + \alpha \beta p_v^{-\beta-1} \left( \frac{CI_v + C}{(1-y_P)(1-y_T)} - \frac{CI_b + v + y_T C_W}{(1-y_T)} \right)
\]

(12)

4. SOLUTION ALGORITHM

Referring to the iterative algorithm to obtain the optimal solution described in the existing literature such as Ho et al. (2008) and Chen and Kang (2010b), we can develop the following algorithm to determine the optimal selling price, order quantity and number of shipment for the both inspection scenarios.

Step 1. Set \( n_i = 1 \) (\( i = b, v \)).

Step 2. Determine \( p_v^{(n_i)} \) by solving Eq. (6) for the first inspection scenario or Eq. (12) for the second inspection scenario.
Step 3. Compute the value of \( Q_i^{(n_i)} \) using Eq. (4) for the first inspection scenario or Eq. (11) for the second inspection scenario.

Step 4. Calculate \( JTP_i(p_i^{(n_i)}, Q_i^{(n_i)}, n_i) \) using Eq. (3) for the first inspection scenario or Eq. (10) for the second inspection scenario.

Step 5. Let \( n_i = n_i + 1 \), repeat steps 2-4 to find \( JTP_i(p_i^{(n_i)}, Q_i^{(n_i)}, n_i) \).

Step 6. If \( JTP_i(p_i^{(n_i)}, Q_i^{(n_i)}, n_i) \geq JTP_i(p_i^{(n_i-1)}, Q_i^{(n_i-1)}, n_i - 1) \), go to step 5. Otherwise, the optimal solution is \( (p_i^*, Q_i^*, n_i^*) = (p_i^{(n_i-1)}, Q_i^{(n_i-1)}, n_i - 1) \).

5. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

Similar to the presented papers in the literature, a numerical example and sensitivity analysis are performed in order to gain further insights of the supply chains, and to compare the obtained results with the literature. The numerical example, which adopts some parameters from Chen and Kang (2010b) and Huang (2004) is solved with the proposed analytic solution method. The values of parameters are: \( A = $100/\text{order} \), \( F = $100/\text{shipment} \), \( S = $1200/\text{setup} \), \( C = \frac{2.5/\text{unit} \cdot v}{1/\text{unit}} \cdot C_W = \frac{11/\text{unit} \cdot \rho}{0.8} = 0.3, \rho_v = 0.3, y_p = 1\%, y_T = 1\%, C_I = \frac{0.1/\text{unit}}{1/\text{unit}} \), \( r = 0.1, m = 9, k = 20, \alpha = 300,000 \), and \( \beta = 1.25 \). Therefore, \( D(p) = 300,000p^{-1.25} \) and \( w(y) = 9 - 20y \).

With the purpose of clarifying the benefits of joint optimization under each inspection scenario, the percentage improvement i.e., \( Pl = \frac{JTP_i - TPI_i}{TPI_i} \times 100 \), \( i = v, b \) is calculated. It’s noticeable that \( TP_i \) and \( TPI_i \) represent the total system profit under joint and independent optimization. In addition, the joint total profits’ difference, \( L = JTP_o - JTP_b \), is computed to compare the obtained profits under the two inspection scenarios. Indisputably, the positive value of \( L \) proves that under the second inspection scenario, which the buyer and also the vendor inspect the shipments, more profits for the supply chain can be achieved, whereas the negative value of \( L \) shows that the first inspection scenario is more beneficial.

The mathematical models explained in equations (1)-(7) (Scenario 1), and (8)-(12) (Scenario 2) are solved for the above values of the input parameters for the cases of coordination and no coordination between the vendor and the buyer. The results are summarized in Table 1.

<table>
<thead>
<tr>
<th>Inspection scenario</th>
<th>Independent optimization</th>
<th>Joint optimization</th>
<th>( Pl )</th>
<th>( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p )</td>
<td>( Q )</td>
<td>( n )</td>
<td>( TP_B )</td>
</tr>
<tr>
<td>The first one</td>
<td>49.6</td>
<td>1043.9</td>
<td>11</td>
<td>89,991</td>
</tr>
<tr>
<td>The second one</td>
<td>50.6</td>
<td>1012.9</td>
<td>11</td>
<td>89,563</td>
</tr>
</tbody>
</table>

In Table 1, the values of \( Pl \) prove that cooperation between the buyer and the vendor, regardless of inspection scenario, brings more profits for the supply chain. Besides carrying out an investigation into how coordination influences the supply chain’s profit; we can explore the effects of the different inspection scenarios on the optimal decisions. For the given example, Table 1 shows that under the second inspection scenario, the supply chain can acquire more profits and also more percentage improvement from coordination rather than under the first inspection scenario. In addition, in the paper of Chen and Kang (2010b), coordination results in 5% increase in the total profit. Therefore, in the both inspection scenarios, our paper shows more improvement in the total profit in comparison to the paper of Chen and Kang (2010b).

Sensitivity analysis is performed over realistic ranges of every parameter, and the effect of critical parameters on the optimal solution and total system profit are summarized in the following.

5.1. Sensitivity analysis for the index of price elasticity \( \beta \)

As can be seen in Table 2 and Fig.3, the percentage improvement, \( Pl \), increases by \( \beta \) in the both inspection scenarios. It means that for the more price sensitive demand, joint optimization shows more improvement, and it is more beneficial. Another deduction from Table 2 is that as demand sensitivity to price increases, the optimal selling price decreases. These results are similar to which Sajadieh and Jokar (2009) deduced for the inventory model with linear price sensitive
demand. Therefore, without considering the type of the demand function, more price sensitive demands always gain more benefits through coordination.

Table 2: Sensitivity analysis for the index of price elasticity $\beta$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Inspection scenario</th>
<th>Independent optimization</th>
<th>Joint optimization</th>
<th>$PI$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p$</td>
<td>$Q$</td>
<td>$n$</td>
<td>$TP_B$</td>
</tr>
<tr>
<td>1.05</td>
<td>The first</td>
<td>210.1</td>
<td>722.8</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>The second</td>
<td>214.2</td>
<td>702.7</td>
<td>11</td>
</tr>
<tr>
<td>1.1</td>
<td>The first</td>
<td>109.4</td>
<td>905.3</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>The second</td>
<td>111.5</td>
<td>879.6</td>
<td>11</td>
</tr>
<tr>
<td>1.25</td>
<td>The first</td>
<td>49.6</td>
<td>1043.9</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>The second</td>
<td>50.6</td>
<td>1012.9</td>
<td>11</td>
</tr>
<tr>
<td>1.5</td>
<td>The first</td>
<td>29.8</td>
<td>938.4</td>
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<td>572.9</td>
<td>11</td>
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<td></td>
<td>The second</td>
<td>17.5</td>
<td>327.4</td>
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</tr>
</tbody>
</table>

Fig.3: Effect of the index of price elasticity on the percentage improvement

5.2. Sensitivity analysis for defective rates

In order to investigate the effects of defective items on the joint total profits, the sensitivity of the models to $y_P$ and $y_T$ in rather practical ranges are tested, and the obtained results are depicted in Fig. 4. It can be discovered that the joint total profit tends to be decreasing with the percentage of defectives, $y_P$ and also $y_T$, in the both inspection scenarios. This result confirms the obtained result in Liu and Çetinkaya (2010). However, when the production defective rate, $y_P$, increases, the reduction in $JTP$ under the second inspection scenario is much more than its reduction value under the first scenario. Therefore, when the production defective rate, $y_P$, is high, it is more profitable for the supply chain to check the shipments by the vendor and also by the buyer, but when $y_P$ is low, it is better to inspect only by the buyer because the production defective rate isn’t as high to worth to inspect twice, i.e., by the vendor and then by the buyer (see Fig. 4 & 5). In addition, it can be inferred from Fig. 5 that although the joint total profits’ difference, $L$, decreases by $y_T$, its sensitivity to $y_T$ isn’t as high to change the decision about the best inspection scenario.
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5.3. Sensitivity analysis for the warranty cost \( C_W \)

The joint total profits’ difference, \( L \), increases by increase in the unit warranty cost, \( C_W \) (see Fig.6). Consequently, in the situation in which the unit warranty cost is high, it’s more profitable for the supply chain to inspect the shipments twice, once by the vendor before dispatching and once by the buyer after receiving because an increase in the inspection cost is outweighed by a decrease in the warranty cost. However, it is vice versa for the lower values of \( C_W \). According to the result, making decision about who inspect the shipments is dependent on the unit warranty cost.

5.4. Sensitivity analysis for \( \delta = \frac{C_{I_V}}{C_{I_B}} \)

Now, the value of \( \delta = \frac{C_{I_V}}{C_{I_B}} \) is varied from 0.25 to 4 to explore the sensitivity of the models to the unit inspection costs. The outcomes, depicted in Fig.7, clarify the joint total profits’ difference, \( L \), drops with \( \delta \). It implies when the vendor’s unit inspection cost becomes greater than its value for the buyer, the best inspection scenario tends to switch from the second scenario to the first scenario. It is rational because when the vendor’s inspection cost is high, the supply chain prefers not to inspect the shipments by him/her.

Fig. 4: Effect of the defective rates on the joint total profit

Fig. 5: Effect of the defective rates on \( L \)
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L against $C_W$

6. CONCLUSION

This paper proposed an integrated inventory model for items with imperfect quality in the single-vendor and single-buyer supply chain. Furthermore, it was assumed that the demand is sensitive to the selling price, and two inspection scenarios were considered to detect the imperfect ones. Then, for each inspection scenario, the supply chain’s profit function was extracted, and then the optimal selling price, order quantity and number of shipments were derived under independent and also joint optimization. Finally, a numerical example, including sensitivity analysis was presented to illustrate the models and the effects of the critical parameters on the optimal solution and the total system profit. The analytical and numerical results disclosed that, in the both inspection scenarios, the coordinated model performs better than the uncoordinated one, and results in the lower selling price and higher overall channel profit. Besides coordination, the supply chains’ profit can be improved by choosing the best inspection scenario. The comparison of the joint total profits of the two inspection scenarios reveals that it is sometimes more beneficial for the supply chain to inspect the shipments twice, first by the vendor and then by the buyer. Specifying the best inspection scenario depends on the defective rates, unit warranty cost and unit inspection costs, and can be determined by comparing the obtained joint total profits. The main contribution of the present paper to the existing body of knowledge lies in developing more realistic and practical models for the integrated inventory, which incorporates imperfect production-transportation into marketing, along with providing richer managerial insights into practices through sensitivity analyses.

In further research, we would like to extend the model for uncertain defective rates and demand. Other extensions of this model may be directed in several ways, for example, multi-vendors and multi-buyers supply chains.

REFERENCES

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