The Journal of Fuzzy Mathematics Vol. 27, No. 3, 2019
Los Angeles

Intuitionistic Fuzzy Translation and Multiplication of G-algebra

Mohsin Khalid

The University of Lahore, 1Km Raiwind Road, Lahore, 54000, Pakistan.
E-mail: mk4605107@gmail.com

Rakib Iqbal

University of Management and Technology (UMT), C-II, Johar Town, Lahore, 54000, Pakistan.

Sohail Zafar

University of Management and Technology (UMT), C-II, Johar Town, Lahore, 54000, Pakistan.

Hasan Khalid

National College of Business Administration and Economics, 40-E/1, Gulberg III, Lahore, 54000, Pakistan.

Abstract:
In this paper, The idea of intuitionistic fuzzy translation to intuitionistic fuzzy subalgebra and ideals over G-lgebra are introduced and some related properties are studied. Examples are also given to illustrate results. The concept of intuitionistic fuzzy extension, intuitionistic fuzzy multiplication of intuitionistic fuzzy subalgebra and ideals of G-lgebra are introduced. Also several links are investigated between intuitionistic fuzzy translation, intuitionistic fuzzy extension and intuitionistic fuzzy multiplication of intuitionistic fuzzy subalgebra and ideals in G-lgebra.

Keywords:
G-lgebra, Intuitionistic fuzzy subalgebra, Intuitionistic fuzzy ideal, Intuitionistic fuzzy translation, Intuitionistic fuzzy extension, Intuitionistic fuzzy multiplication.

1. Introduction

The study of BCI/BCK- algebras was initiated by Imai and Iseki [1], [2] as the generalization of notion of set theoretic difference and propositional calculus. Neggers and Kim [3] are introduced the concept of B-algebra, which is related to several classes

Received January, 2018

1066-8950/19 $8.50

of algebra of interest such as BCK/BCI-algebra. Jun et al. [4] fuzzyfied (normal) B-algebras and gave a characterization of a fuzzy B-algebra. Ahn and Bang[5] studied some result on fuzzy subalgebra in B-algebras. Borumand Saeid [6] give the sign of fuzzy topological B-algebras. Senapati et al. [7, 8, 9] presented the idea and basic characteristic of cubic subalgebra and cubic closed ideals of B-algebras and characteristic of fuzzy B-subalgebra according to triangular norm and dot product. R. Iqbal et al. [10] introduced the notion of neutrosophic cubic subalgebra and neutrosophic cubic closed ideals of B-algebra and investigate several properties. C. B. Kim and H. S. Kim [11] initiated the idea of BG-algebra, which is a generalization of B-algebra. Bandru and Rafi [12] give birth to a new idea, called G-algebra. The several authors have done a lot of works on BG-algebra [13, 14, 15, 16, 17, 18, 19] and G-algebras [20, 21, 22]. For more improvement of G-algebra, the ideal theory and subalgebra have great importance.

Recently in [23], the author has studied fuzzy translation, fuzzy extensions and fuzzy multiplications of fuzzy subalgebra in BG-algebra. The author introduced (intuitionistic) fuzzy translation to (intuitionistic) fuzzy H-ideal in BCK/BCI-algebra [24, 25]. In this paper, intuitionistic fuzzy translation, intuitionistic fuzzy extension and intuitionistic fuzzy multiplication of intuitionistic fuzzy subalgebra (IFSUs) and intuitionistic fuzzy ideal (IFIDs) in G-algebra are introduced. Relation among intuitionistic fuzzy translation, intuitionistic fuzzy extension and intuitionistic fuzzy multiplication of IFSUs and ideal in G-algebra are also investigated.

2. Preliminaries

We first recall some basic aspects which are necessary for this paper. A G-algebra is initiated by Bandaru and Rafi [12] and was extensively investigated by several researcher. This algebra is defined as follows.

A nonempty set Y with a constant 0 and a binary operation is said to be G-algebra [12] if it satisfies the following axioms.

$G1$: $x*x=0$

$G2$: $x*(x*y)=y$, for any $x,y\in Y$

A G-algebra is denoted by $(Y,*,0)$.

A nonempty subset S of G-algebra Y is called a G-subalgebra [12] of Y if $x*y\in S\ \forall x,y\in S$.

A non-empty subset I of a G-algebra Y is called an ideal [15] if for any $x,y\in Y$.

(i) $0\in I$,

(ii) $x*y\in I$ and $y\in I\Rightarrow x\in I$.

Let Y be a G-algebra. Then a fuzzy set [26] A in Y is defined as $A=\{\langle x,\mu_A(x)\rangle | x\in Y\}$, where $\mu_A(x)$ is called the membership value of x in A and $0\le\mu_A(x)\le 1$.

A fuzzy set A in a G -algebra Y is called a fuzzy G -subalgebra [5] of Y if $\mu(x*y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in Y$.

Let A be a fuzzy subset of Y let $\alpha \in \left[0, 1-\sup\{\mu_A(x) | x \in\}\right]$. A mapping $(\mu)_\alpha^T | Y \to [0,1]$ is called a fuzzy α-translation [24] of A if it satifies $(\mu_A)_\alpha^T(x) = \mu_A(x) + \alpha$ for all $x \in Y$.

An intuitionistic fuzzy set (IFS) [27] A over Y is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in Y\}$, where $\mu_A(x) | Y \to [0,1]$ and $\nu_A(x) | Y \to [0,1]$, with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in Y$. The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the degree of membership and the degree of non-membership of the element x in the set A.

Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in Y\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in Y\}$ be two $IFSs$ on Y. Then the intersection and union [28] of A and B are denoted by $A \cap B$ and $A \cup B$ respectively and is given by

$$A \cap B = \left\{\left\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\right\rangle | x \in Y\right\},$$

$$A \cup B = \left\{\left\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\right\rangle | x \in Y\right\}.$$

An $IFS\, A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in Y\}$ in X is called an $IFSU$ [29] of Y when it satisfies the these two conditions

(1) $\mu_A(x*y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and

(2) $\nu_A(x*y) \leq \max\{\nu_A(x), \nu_A(y)\}$ for all $x, y \in Y$

An $IFS\, A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ in Y is said to be an $IFID$ of Y when it satisfies these conditions

(1) $\mu_A(0) \geq \mu_A(x)$, $\nu_A(0) \leq \nu_A(x)$

(2) $\mu_A(x) \geq \min\{\mu_A(x*y), \mu_A(y)\}$ and

(3) $\nu_A(x) \leq \max\{\nu_A(x*y), \nu_A(y)\}$ for all $x, y \in Y$

3. Translations of intuitionistic fuzzy subalgebras

Definition 3.1. Let $A = (\mu_A, \nu_A)$ be an IFS of Y and $\alpha \in [0, Y]$. An object of the form $A_\alpha^T = \left((\mu_A)_\alpha^T, (\nu_A)_\alpha^T\right)$ is called an intuitionistic fuzzy α-translation $(IFAT)$ of A when $(\mu_A)_\alpha^T(x) = \mu_A(x) + \alpha$ and $(\nu_A)_\alpha^T(x) = \nu_A(x) - \alpha$ for all $x \in Y$.

Theorem 3.1. *Let* A *be an* $IFSUs$ *of* A *and* $\alpha \in [0, ¥\ ]$. *Then the* $IFAT\ A_{\alpha}^{T}$ *of* A *is an* $IFSUs$ *of* Y.

Proof. Let $x, y \in Y$. Then

$$(\mu_A)_{\alpha}^{T}(x * y) = \mu_A(x * y) + \alpha \geq \min\{\mu_A(x), \mu_A(y)\} + \alpha$$

$$= \min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\} = \min\left\{(\mu_A)_{\alpha}^{T}(x), (\mu_A)_{\alpha}^{T}(y)\right\}$$

and

$$(\nu_A)_{\alpha}^{T}(x * y) = \nu_A(x * y) - \alpha \leq \max\{\nu_A(x), \nu_A(y)\} - \alpha$$

$$= \max\{\nu_A(x) - \alpha, \nu_A(y) - \alpha\} = \max\left\{(\nu_A)_{\alpha}^{T}(x), (\nu_A)_{\alpha}^{T}(y)\right\}.$$

Hence $IFAT\ A_{\alpha}^{T}$ of A is an $IFSUs$ of Y.

Theorem 3.2. *Let* A *be an* IFS *of* Y *such that* $IFAT\ A_{\alpha}^{T}$ *of* A *is an* $IFSUs$ *of* Y *for some* $\alpha \in [0, ¥\ ]$. *Then* A *is an* IFSU *of* Y.

Proof. Let $A_{\alpha}^{T} = \left((\mu_A)_{\alpha}^{T}, (\nu_A)_{\alpha}^{T}\right)$ is an IFSUs of Y for some $\alpha \in [0, ¥\ ]$. Let $x, y \in Y$. We have

$$\mu_A(x * y) + \alpha = (\mu_A)_{\alpha}^{T}(x * y) \geq \min\left\{(\mu_A)_{\alpha}^{T}(x), (\mu_A)_{\alpha}^{T}(y)\right\}$$

$$= \min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\} = \min\{\mu_A(x), \mu_A(y)\} + \alpha$$

and

$$\nu_A(x * y) - \alpha = (\nu_A)_{\alpha}^{T}(x * y) \leq \min\left\{(\nu_A)_{\alpha}^{T}(x), (\nu_A)_{\alpha}^{T}(y)\right\}$$

$$= \max\{\nu_A(x) - \alpha, \nu_A(y) - \alpha\}, = \max\{\mu_A(x), \mu_A(y)\} - \alpha,$$

which implies that $\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x * y) \leq \max$ $\{\nu_A(x), \nu_A(y)\}$ for all $x, y \in Y$. Hence, A is an IFSUs of Y.

Definition 3.2. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two $IFSs$ of Y. If $A \leq B$, i.e, $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in Y$. Then we say that B is an intuitionistic fuzzy extension of A.

Definition 3.3. Let $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ be two $IFSs$ of Y. Then B is called an intutiionistic fuzzy S -extension $(IFSE)$ of A if the following assertions are valid.

(i) B is an intuitionisitc fuzzy extension of A.

(ii) If A is an $IFSU$ of Y, then B is an $IFSU$ of Y.

From the definition of $IFAT$, we get $(\mu_A)_\alpha^T(x)=\mu_A(x)+\alpha$ and $(\nu_A)_\alpha^T(x)=\nu_A(x)$ $-\alpha$ for all $x\in Y$. Therefore, we have the following theorem.

Theorem 3.3. *Let A be an IFS of Y and $\alpha\in[0,¥\ ]$. Then the $IFAT$ A_α^T of A is an $IFSE$ of A.*

The converse of the this theorem is not true in general as seen in the following example.

Example 3.1. Let $X=\{0,a,b,c,d,e\}$ be a G -algebra with the following Cayley table:

$*$	0	a	b	c	d	e
0	0	e	d	c	b	a
a	a	0	e	d	c	b
b	b	a	0	e	d	c
c	c	b	a	0	e	d
d	d	c	b	a	0	e
e	e	d	c	b	a	0

Let $A=(\mu_A,\nu_A)$ be an IFS of Y defined by

Y	0	a	b	c	d	e
μ_A	0.7	0.4	0.7	0.4	0.7	0.4
ν_A	0.5	0.6	0.5	0.6	0.5	0.6

Then A is an $IFSU$ of Y. Let $B=(\mu_B,\nu_B)$ be an IFS of Y defined by

Y	0	a	b	c	d	e
μ_B	0.68	0.36	0.68	0.36	0.68	0.36
ν_B	0.33	0.47	0.33	0.47	0.33	0.47

Then B is an $IFSE$ of A. But it is not $IFAT$ A_α^T of A for all $\alpha\in[0,¥\ ]$.

Note. Clearly, the intersection of $IFSEs$ of an $IFSU$ A of Y is an $IFSE$ of A. But the union of $IFSESs$ of an $IFSU$ A of Y is not an $IFSE$ of A as seen in the following example.

Example 3.2. Let $Y=\{0,a,b,c,d,e\}$ be a G -algebra with the following Cayley table:

$*$	0	a	b	c	d	e
0	0	b	a	c	d	e
a	a	0	b	d	e	c
b	b	a	0	e	c	d
c	c	d	e	0	b	a
d	d	e	c	a	0	b
e	e	c	d	b	a	0

Let $A=(\mu_A,\nu_A)$ be an IFS of Y defined by

Y	0	a	b	c	d	e
μ_A	0.6	0.4	0.4	0.4	0.4	0.4
ν_A	0.2	0.3	0.3	0.3	0.3	0.3

Then A is an $IFSU$ of Y. Let B and C be two $IFSs$ of Y defined by

Y	0	a	b	c	d	e
μ_A	0.7	0.5	0.5	0.7	0.5	0.5
ν_A	0.1	0.2	0.2	0.1	0.2	0.2

and

Y	0	a	b	c	d	e
μ_A	0.8	0.6	0.6	0.6	0.8	0.6
ν_A	0.0	0.1	0.1	0.1	0.0	0.1

respectively. Then B and C are $IFSEs$ of A. Obviously, the union $B\cup C$ is an intutiionistic fuzzy extension of A, but it is not an $IFSE$ of A since $\mu_{B\cap C}(d*c)=\mu_{B\cup C}(a)=0.6\not\geq 0.7=\min\{0.8,0.7\}=\min\{\mu_{B\cup C}(d),\mu_{B\cup C}(c)\}$ and $\nu_{B\cup C}(d*c)=\nu_{B\cup C}(a)=0.0\not\leq 0.1=\max\{0.0,0.1\}=\max\{\nu_{B\cup C}(d),\nu_{B\cup C}(c)\}$.

For an $IFS\,A=(\mu_A,\nu_A)$ of Y, $\alpha\in[0,¥\ ]$ and $t,s\in[0,1]$ with $t\geq\alpha$, let $U_\alpha(\mu_A;t)=\{x\,|\,\mu_A(x)\geq t-\alpha\}$ and $L_\alpha(\nu_A;s)=\{x\,|\,\nu_A(x)\leq s+\alpha\}$. If A is an $IFSU$ of Y, then it is clear that $U_\alpha(\mu_A;t)$ and $L_\alpha(\nu_A;s)$ are subalgebras of Y for all $t\in\mathrm{Im}(\mu_A)$ and $s\in\mathrm{Im}(\in_A)$ with $t\geq\alpha$. But, if we do not give a condition that A is an $IFSU$ of Y, then $U_\alpha(\mu_A;t)$ and $L_\alpha(\nu_A;s)$ are not subalgebra of Y as seen in the following example.

Example 3.3. Let $Y = \{0, a, b, c, d, e\}$ be a G -algebra in above example and $A = (\mu_A, \nu_A)$ be an IFS of Y defined by

Y	0	a	b	c	d	e
μ_A	0.67	0.44	0.44	0.55	0.55	0.55
ν_A	0.27	0.53	0.53	0.30	0.30	0.30

Since $\mu_A(d*c) = 0.44 \ngeq 0.55 = \min\{\mu_A(d), \mu_A(c)\}$ and $\nu_A(e*d) = 0.53 \nleq 0.30 = \max\{\nu_A(e), \nu_A(d)\}$, therefore, $A = (\mu_A, \nu_A)$ is not an $IFSU$ of Y.

For $\alpha = 0.15$ and $t = 0.69$, we obtain $U_\alpha(\mu_A; t) = \{0, c, d, e\}$ which is not a subalgebra of Y, since $c*d = b \notin U_\alpha(\mu_A; t)$

For $\alpha = 0.15$ and $s = 0.24$, we obtain $L_\alpha(\nu_A; s) = \{0, c, d, e\}$ which is not a subalgebra of Y, since $e*d = \notin L_\alpha(\nu_A; s)$.

Theorem 3.4. *For $\alpha \in [0, ¥\ ]$, suppose $A_\alpha^T = \left((\mu_A)_\alpha^T, (\nu_A)_\alpha^T\right)$ be the $IFAT$ of A. Then the following statements are equivalent:*

(i) $A_\alpha^T = \left((\mu_A)_\alpha^T, (\nu_A)_\alpha^T\right)$ *is an $IFSU$ of Y.*

(ii) $U_\alpha(\mu_A; t)$ *and $L_\alpha(\nu_A; s)$ are subalgebra of Y for $t \in \operatorname{Im}(\mu_A)$, $s \in \operatorname{Im}(\nu_A)$ with $t \geq \alpha$.*

Proof. Suppose that A_α^T is an $IFSU$ of Y. Then $(\mu_A)_\alpha^T$ and $(\nu_A)_\alpha^T$ are fuzzy subalgebra of Y. Let $x, y \in Y$ such that $x, y \in U_\alpha(\mu_A; t)$ and $t \in \operatorname{Im}(\mu_A)$ with $t \geq \alpha$. Then $\mu_A(x) \geq t - \alpha$ and $\mu_A(y) \geq t - \alpha$, i.e. $(\mu_A)_\alpha^T(x) = \mu_A(x) + \alpha \geq t$ and $(\mu_A)_\alpha^T(y) = \mu_A(y) + \alpha \geq t$. Since $(\mu_A)_\alpha^T$ is a fuzzy subalgebra of Y, therefore we have

$$\mu_A(x*y) + \alpha = (\mu_A)_\alpha^T(x*y) \geq \min\left\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\right\} \geq t,$$

that is, $\mu_A(x*y) \geq t - \alpha$ so that $x*y \in U_\alpha(\mu_A; t)$. Again, let $x, y \in Y$ be such that $x, y \in L_\alpha(\nu_A; s)$ and $s \in \operatorname{Im}(\nu_A)$. Then $\nu_A(x) \leq s + \alpha$ and $\nu_A(y) \leq s + \alpha$, i.e., $(\nu_A)_\alpha^T(x) = \nu_A(x) - \alpha \leq s$ and $(\nu_A)_\alpha^T(y) = \nu_A(y) - \alpha \geq s$. Since $(\nu_A)_\alpha^T$ is a fuzzy subalgebra of Y, it follows that

$$\nu_A(x*y)\alpha = (\nu_A)_\alpha^T(x*y) \leq \max\left\{(\nu_A)_\alpha^T(x), (\nu_A)_\alpha^T(y)\right\} \leq s,$$

that is, $\nu_A(x*y) \leq s + \alpha$ so that $x*y \in L_\alpha(\nu_A; s)$. Therefore, $U_\alpha(\mu; t)$ and $L_\alpha(\nu_A; s)$ are subalgebras of Y.

Conversely, assume that $U_\alpha(\mu_A;t)$ and $L_\alpha(\nu_A;s)$ are subalgebras of Y for $t\in \mathrm{Im}(\mu_A)$, $s\in \mathrm{Im}(\nu_A)$ with $t\ge\alpha$. If there exists $a,b\in Y$ such that $(\mu_A)_\alpha^T(a*b)<\beta\le \min\left\{(\mu_A)_\alpha^T(a),(\mu_A)_\alpha^T(b)\right\}$, then $\mu_A(a)\ge\beta-\alpha$ and $\mu_A(b)\ge\beta-\alpha$, but $\mu_A(a*b)<\beta-\alpha$.

This shows that $a\in U_\alpha(\mu_A;t)$ and $b\in U_\alpha(\mu_A;t)$, but $(a*b)\notin U_\alpha(\mu_A;t)$. This is a contradiction, therefore, $(\mu_A)_\alpha^T(x*y)\ge \min\left\{(\mu_A)_\alpha^T(x),(\mu_A)_\alpha^T(y)\right\}$ for all $x,y\in Y$.

Again, assume that there exist $c,d\in X$ such that $(\nu_A)_\alpha^T(c*d)>\delta\ge \max\left\{(\nu_A)_\alpha^T(c),(\nu_A)_\alpha^T(d)\right\}$. Then $\nu_A(c)\le\delta+\alpha$ and $\nu_A(d)\le\delta+\alpha$, but $\nu_A(c*d)>\delta+\alpha$. Hence, $c\in L_\alpha(\nu_A;s)$ and $d\in L_\alpha(\nu_A;s)$, but $(c*d)\notin L_\alpha(\nu_A;s)$. This is impossible and therefore, $(\nu_A)_\alpha^T(x*y)\le \max\left\{(\nu_A)_\alpha^T(x),(\nu_A)_\alpha^T(y)\right\}$ for all $x,y\in Y$. Consequently, $A_\alpha^T=\left((\mu_A)_\alpha^T,(\nu_A)_\alpha^T\right)$ is an $IFSU$ of Y.

Theorem 3.5. *Let $A=(\mu_A,\nu_A)$ be an $IFSU$ of Y and let $\alpha,\beta\in[0,¥]$. If $\alpha\ge\beta$, then the $IFAT$ $A_\alpha^T=\left((\mu_A)_\alpha^T,(\nu_A)_\alpha^T\right)$ of A is an $IFSE$ of the intuitionistic fuzzy β-translation $A_\beta^T=\left((\mu_A)_\beta^T,(\nu_A)_\beta^T\right)$ of A.*

Proof. Straightforward.

For every $IFSU$ A of Y and $\beta\in[0,¥]$, the intuitionistic fuzzy β-translation A_β^T of A is an $IFSU$ of Y. If B is an $IFSE$ of A_β^T, then there exists $\alpha\in[0,¥]$ such that $\alpha\ge\beta$ and $B\ge A_\alpha^T$, that is $\mu_\beta(x)\ge(\mu_A)_\alpha^T$ and $\nu_B(x)\le(\nu_A)_\alpha^T$ for all $x\in Y$. Hence, we have the following theorem.

Theorem 3.6. *Let A be an $IFSU$ of Y and $\beta\in[0,¥]$. For every $IFSE$ $B=(\mu_B,\nu_B)$ of the intuitionistic fuzzy β-translation A_β^T of A, there exists $\alpha\in[0,¥]$ such that $\alpha\ge\beta$ and B is an $IFSE$ of the $IFAT$ A_α^T of A.*

Let us illustrate the Theorem 3.6 using the following example.

Example 3.4. Let $Y=\{0,a,b,c,d,e\}$ be a G-algebra and $A=(\mu_A,\nu_A)$ be an IFS of Y defined in Example 3.1. Then $¥=0.5$. If we take $\beta=0.24$, then the intuitionistic fuzzy β-translation A_β^T of A is given by

Y	0	a	b	c	d	e
$(\mu_A)_\beta^T$	0.84	0.54	0.84	0.54	0.84	0.54
$(\nu_A)_\beta^T$	0.18	0.28	0.18	0.28	0.18	0.28

Let $B=(\mu_B,\nu_B)$ be an IFS of Y defined by

Y	0	a	b	c	d	e
μ_A	0.88	0.60	0.88	0.60	0.88	0.60
ν_A	0.13	0.20	0.13	0.20	0.13	0.20

Then B is clearly an $IFSU$ of Y which is an $IFSE$ of the intuitionistic fuzzy β-transloation A_β^T of A. But B is not an $IFAT$ of A for all $\alpha\in[0,\text{¥}\ ]$. If we take $\alpha=0.27$, then $\alpha=0.27>0.25=\beta$ and the $IFAT\ A_\alpha^T=\left((\mu_A)_\alpha^T,(\nu_A)_\alpha^T\right)$ of A is given as follows:

Y	0	a	b	c	d	e
$(\mu_A)_\alpha^T$	0.87	0.57	0.87	0.57	0.87	0.57
$(\nu_A)_\alpha^T$	0.15	0.25	0.15	0.25	0.15	0.25

Note that $B(x)\geq A_\alpha^T(x)$, that is $\mu_B(x)\geq(\mu_A)_\alpha^T(x)$ and $\nu_B(x)\leq(\nu_A)_\alpha^T(x)$ for all $x\in Y$. Hence, B is an $IFSE$ of the $IFAT\ A_\alpha^T$ of A.

Definition 3.4. Let A be an IFS of Y and $\alpha\in[0,1]$. An object having the form $A_\alpha^M=\left((\mu_A)_\alpha^M,(\nu_A)_\alpha^M\right)$ is called an intuitionistic fuzzy α-multiplication of A if $(\mu_A)_\alpha^M(x)=\alpha\cdot\mu_A(x)$ and $(\nu_A)_\alpha^M(x)=\alpha\cdot\nu_A(x)$ for all $x\in y$.

For any $IFS\ A=(\mu_A,\nu_A)$ of Y , an intuitionsitic fuzzy 0-multiplication $A_0^M=\left((\mu_A)_0^M,(\nu_A)_0^M\right)$ of A is an $IFSU$ of Y.

Theorem 3.7. *Let $A=(\mu_A,\nu_A)$ be an $IFSU$ of Y, then the intuitionistic fuzzy α-multiplication A_α^M of A is an $IFSU$ of Y for all $\alpha\in[0,1]$.*

Proof. For sufcient part $A=(\mu_A,\nu_A)$ is an $IFSU$ of Y. Then for all $x,y\in Y$, we have

$$(\mu_A)_\alpha^M(x*y)=\alpha\cdot\mu(x*y)\geq\alpha\cdot\min\{(\mu_A)(x),(\mu_A)(y)\}$$

$$= \min\{\alpha\cdot\mu_A(x), \alpha\cdot\mu_A(y)\} = \min\{(\mu_A)_\alpha^M(x), (\mu_A)_\alpha^M(y)\}$$

$$(\mu_A)_\alpha^M(x*y) \geq \min\{(\mu_A)_\alpha^M(x), (\mu_A)_\alpha^M(y)\}$$

and

$$(\nu_A)_\alpha^M(x*y) = \alpha\cdot\nu(x*y) \leq \alpha\cdot\max\{(\nu_A)(x), (\nu_A)(y)\}$$

$$= \max\{\alpha\cdot\nu_A(x), \alpha\cdot\nu_A(y)\} = \max\{(\nu_A)_\alpha^M(x), (\nu_A)_\alpha^M(y)\}$$

$$(\nu_A)_\alpha^M(x*y) \leq \max\{(\nu_A)_\alpha^M(x), (\nu_A)_\alpha^M(y)\}$$

Therefore, $\mu_A(x*y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x*y) \leq \max\{\nu_A(x), \nu_A(y)\}$ for all $x, y \in Y$ since $\alpha \neq 0$. Hence, A_α^T is an $IFSU$ of Y.

Theorem 3.8. *For any $IFS\, A = (\mu_A, \nu_A)$ of Y and $\alpha \in (0,1]$ if the $IFAM$ A_α^M of B is a $IFSU$ of Y then the A is also the $IFSU$ of Y.*

Proof. Assume that A_α^M of A is an $IFSU$ of Y for some $\alpha \in (0.1]$. Now for all $x, y \in Y$, we have

$$\alpha\cdot\mu_A(x*y) = (\mu_A)_\alpha^M(x*y) \geq \min\{(\mu_A)_\alpha^M(y), (\mu_A)_\alpha^M(x)\}$$

$$= \min\{\alpha\cdot\mu_A(y), \alpha\cdot\mu_A(x)\} = \alpha\cdot\min\{\mu_A(y), \mu_A(x)\}$$

and

$$\alpha\cdot\nu_A(x*y) = (\nu_A)_\alpha^M(x*y) \leq \min\{(\nu_A)_\alpha^M(y), (\nu_A)_\alpha^M(x)\}$$

$$= \min\{\alpha\cdot\nu_A(y), \alpha\cdot\nu_A(x)\} = \alpha\cdot\min\{\nu_A(y), \nu_A(x)\}$$

Therefore, $\mu_A(x*y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x*y) \leq \max\{\nu_A(y), \nu_A(x)\}$ for all $x, y \in Y$ since $\alpha \neq 0$. Hence, A is an $IFSU$ of Y.

4. Translation of intuitionistic fuzzy ideals

In this section, translation of intuitionistic fuzzy ideals are defined with some results studied.

Theorem 4.1. *If* $A=(\mu_A,\nu_A)$ *is an* $IFIDs$ *of* Y, *then the* $IFAT\ A_\alpha^T=\left((\mu_A)_\alpha^T,(\nu_A)_\alpha^T\right)$ *of* A *is an* $IFIDs$ *of* Y *for all* $\alpha\in[0,¥\ ]$.

Proof. Let $A=(\mu_A,\nu_A)$ be an $IFIDs$ of Y and $\alpha\in[0,¥\ ]$. Then $(\mu_A)_\alpha^T(0)=\mu_A(0)+\alpha\geq\mu_A(x)+\alpha=(\mu_A)_\alpha^T(x)$ and $(\nu_A)_\alpha^T(0)=\nu_A(0)-\alpha\leq\nu_A(x)-\alpha=(\nu_A)_\alpha^T(x)$ for all $x\in Y$. Now,

$$(\mu_A)_\alpha^T(x)=\mu_A(x)+\alpha\geq\min\{\mu_A(x*y),\mu_A(y)\}+\alpha$$

$$=\min\{\mu_A(x*y)+\alpha,\mu_A(y)\}+\alpha=\min\left\{(\mu_A)_\alpha^T(x*y),(\mu_A)_\alpha^T(y)\right\}$$

and

$$(\nu_A)_\alpha^T(x)=\nu_A(x)-\alpha\geq\max\{\nu_A(x*y),\nu_A(y)\}-\alpha$$

$$=\max\{\nu_A(x*y)-\alpha,\nu_A(y)-\alpha\}=\max\left\{(\nu_A)_\alpha^T(x*y),(\nu_A)_\alpha^T(y)\right\}$$

for all $x,y\in Y$. Hence, $IFAT\ A_\alpha^T$ of A is an IFIDs of Y.

Theorem 4.2. *Let* A *be an* IFS *of* Y *such that the* $IFAT\ A_\alpha^T$ *of* A *is an* IFIDs *of* Y *for some* $\alpha\in[0,¥\ ]$. *Then* A *is an* $IFID$ *of* Y.

Proof. Let A_α^T is an IFIDs of Y for some $\alpha\in[0,¥\ ]$. Let $x,y\in Y$, we have

$$\mu_A(0)+\alpha=(\mu_A)_\alpha^T(0)\geq(\mu_A)_\alpha^T(x)=\mu_A(x)+\alpha,$$

$$\nu_A(0)-\alpha=(\nu_A)_\alpha^T(0)\leq(\nu_A)_\alpha^T(x)=\nu_A(x)-\alpha,$$

which implies $\mu_A(0)\geq\mu_A(x)$ and $\nu_A(0)\leq\nu_A(x)$. Now, we have

$$\mu_A(x)+\alpha=(\mu_A)_\alpha^T(x)\geq\min\left\{(\mu_A)(x*y),(\mu_A)_\alpha^T(y)\right\}$$

$$=\min\{\mu_A(x*y)+\alpha,\mu_A(y)+\alpha\}=\min\{\mu_A(x*y),\mu_A(y)\}+\alpha,$$

and

$$\nu_A(x)\alpha=(\nu_A)_\alpha^T(x)\leq\max\left\{(\nu_A)_\alpha^T(x*y),(\nu_A)_\alpha^T(y)\right\}$$

$$=\max\{\nu_A(x*y)-\alpha,\nu_A(y)-\alpha\}=\max\{\nu_A(x*y),\nu_A(y)\}-\alpha,$$

which implies that $\mu_A(x) \geq \min\{\mu_A(x*y), \mu_A(y)\}$ and $\nu_A(x) \leq \max\{\nu_A(x*y), \nu_A(y)\}$ for all $x, y \in Y$. Hence, A is an $IFIDs$ of Y.

Definition 4.1. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two $IFSs$ of Y. Then B is called an intuitionistic fuzzy ideal extension $(IFIE)$ of A if the following assertions are valid:

(i) B is an intuitionistic fuzzy extension of A.

(ii) If A is an $IFID$ of Y, then B is an $IFID$ of Y.

Theorem 4.3. *Let A be an IFS of Y and $\alpha \in [0, ¥\ ]$. Then the $IFAT$ A_α^T of A is an $IFID$ extension of A.*

An $IFIE$ of an $IFID\,A$ may not be represented as an $IFAT$ of A, that is, the converse of this theorem is not true in general as seen in the following example.

Example 4.1. Let $Y = \{0, a, b, c, d, e\}$ be a G -algebra in 3.1 Example and $A = (\mu_A, \nu_A)$ be an IFS of Y defined by

Y	0	a	b	c	d	e
μ_A	0.62	0.53	0.44	0.44	0.44	0.44
ν_A	0.26	0.39	0.50	0.50	0.50	0.50

Then A is an $IFID$ of Y. Let $B = (\mu_B, \nu_B)$ be an IFS of Y defined by

Y	0	a	b	c	d	e
μ_B	0.66	0.58	0.47	0.47	0.47	0.47
ν_B	0.24	0.35	0.49	0.49	0.49	0.49

Then B is an $IFIE$ of A. But it is not $IFAT$ A_α^T of A for all $\alpha \in [0, ¥\ ]$.

Clearly, the intersection of $IFIEs$ of an $IFID\,A$ of Y is an $IFIE$ of A. But the union of $IFIEs$ of an $IFID\,A$ of Y is not an $IFIE$ of A as seen in the following example.

Example 4.2. Let $Y = \{0, a, b, c\}$ be a G -algebra with the following Cayley table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let $A=(\mu_A,\nu_A)$ be an IFS of Y defined by

Y	0	a	b	c
μ_A	0.66	0.44	0.44	0.44
ν_A	0.38	0.59	0.59	0.59

Then A is an $IFID$ of Y. Let B and C be two $IFSs$ of Y defined by

Y	0	a	b	c
μ_B	0.73	0.56	0.73	0.56
ν_B	0.30	0.35	0.30	0.35

and

X	0	a	b	c
μ_C	0.67	0.67	0.50	0.50
ν_C	0.32	0.32	0.47	0.47

respectively. Then B and C are $IFIEs$ of A. Obviously, the union $B\cup C$ is an intuitionistic fuzzy extension of A, but it is not an $IFIE$ of A since $\mu_{B\cup C}(c)=0.56\ngeq 0.67=\min\{\mu_{B\cup C}(c*a),\mu_{B\cup C}(a)\}$ and $\nu_{B\cup C}(c)=0.35\nleq 0.32=\max\{\nu_{B\cup C}(c*b),\nu_{B\cup C}(b)\}$.

If A is an $IFID$ of Y, then it is clear that $U_\alpha(\mu;t)$ and $L_\alpha(\nu_A;s)$ are ideals of Y for all $t\in\operatorname{Im}(\mu_A)$ and $s\in\operatorname{Im}(\nu_A)$ with $t\geq\alpha$. But, if we do not give a condition that A is an $IFID$ of Y, then $U_\alpha(\mu_A;t)$ and $L_\alpha(\nu_A;s)$ are not ideals of Y as seen in the following example.

Example 4.3. Let $Y=\{0,a,b,c\}$ be a G -algebra in Example 4.2 and $A=(\mu_A,\nu_A)$ be an IFS of Y defined by

Y	0	a	b	c
μ_A	0.76	0.51	0.60	0.60
ν_A	0.24	0.52	0.37	0.37

Since $\mu_A(a)=0.51\ngeq 0.60=\min\{\mu_A(a*b),\mu_A(b)\}$ and $\nu_A(a)=0.52\nleq 0.37=\max\{\nu_A(a*c),\nu_A(c)\}$, therefore, $A=(\mu_A,\nu_A)$ is not an $IFID$ of Y.

For $\alpha=0.17$, $t=0.74$ and $s=0.29$, we obtain $U_\alpha(\mu_A;t)=L_\alpha(\nu_A;s)=\{0,b,c\}$ which are not ideals of Y since $b*c=a\notin\{0,b,c\}$.

Theorem 4.4. *Let A be an $IFID$ of Y and let $\beta \in [0, ¥]$. For every $IFIE$ $B = (\mu_B, \nu_B)$ of the intuitionistic fuzzy β-translation A_β^T of A, there exists $\alpha \in [0, ¥]$ such that $\alpha \geq \beta$ and B is an $IFIE$ of the $IFAT$ A_α^T of A.*

Let us illustrate this Theorem using the following example.

Example 4.4. Let $Y = \{0, a, b, c, d, e\}$ be a G -algebra in 3.1 Example and $A = (\mu_A, \nu_A)$ be an IFS of Y defined by

Y	0	a	b	c	d	e
μ_A	0.75	0.60	0.75	0.60	0.75	0.60
ν_A	0.32	0.43	0.32	0.43	0.32	0.43

Then A is an $IFID$ of Y and $¥ = 0.32$. If we take $\beta = 0.15$, then the intuitionistic fuzzy β-translation $A_\beta^T = \left((\mu_A)_\beta^T, (\nu_A)_\beta^T\right)$ of A is given by

Y	0	a	b	c	d	e
$(\mu_A)_\beta^T$	0.86	0.71	0.86	0.71	0.86	0.71
$(\nu_A)_\beta^T$	0.21	0.32	0.21	0.32	0.21	0.32

Let $B = (\mu_B, \nu_B)$ be an IFS of Y defined by

Y	0	a	b	c	d	e
μ_B	0.91	0.79	0.91	0.79	0.91	0.79
ν_B	0.15	0.23	0.15	0.23	0.15	0.23

Then B is clearly an $IFID$ of Y, an $IFIE$ of the intuitionistic fuzzy β-translation A_β^T of A. But B is not an $IFAT$ of A for all $\alpha \in [0, ¥]$. If we take $\alpha = 0.19$, then $\alpha = 0.19 > 0.15 = \beta$ and the $IFAT$ $A_\alpha^T = \left((\mu_A)_\alpha^T, (\nu_A)_\alpha^T\right)$ of A is given as follows:

Y	0	a	b	c	d	e
$(\mu_A)_\alpha^T$	0.90	0.75	0.90	0.75	0.90	0.75
$(\nu_A)_\alpha^T$	0.17	0.28	0.17	0.28	0.17	0.28

Note that $B(x) \geq A_\alpha^T(x)$ that is $\mu_B(x) \geq (\mu_A)_\alpha^T$ and $\nu_B(x) \leq (\nu_A)_\alpha^T$ for all $x \in Y$. Hence, B is an $IFIE$ of the $IFAT$ A_α^T of A.

Theorem 4.5. *Let A be an intuitionistic fuzyz subset of Y such that the intuitionistic fuzzy α -multiplication A_α^M of A is an intuitionistic fuzzy ideal of Y for some $\alpha \in (0,1]$, then A is an intuitionistic fuzzy ideal of Y.*

Proof. Suppose that A_α^M is an intuitionistic fuzzy ideal of Y for some $\alpha \in (0,1]$. Let $x, y \in Y$.

$$\alpha \cdot \mu_A(x) = (\mu_A)_\alpha^M(x) \geq \min\left\{(\mu_A)_\alpha^M(y*x), (\mu_A)_\alpha^M(y)\right\}$$
$$= \min\{\alpha \cdot \mu_A(y*x), \alpha \cdot \mu_A(y)\} = \alpha \cdot \min\{\mu_A(y*x), \mu_A(y)\}$$

so $\mu_A(x) \geq \min\{\mu_A(y*x), \mu_A(y)\}$ and

$$\alpha \cdot \nu_A(x) = (\nu_A)_\alpha^M(x) \leq \max\left\{(\nu_A)_\alpha^M(y*x), (\nu_A)_\alpha^A(y)\right\}$$
$$= \max\{\alpha \cdot \nu_A(y*x), \alpha \cdot \nu_A(y)\} = \alpha \cdot \max\{\nu_A(y*x), \nu_A(y)\}$$

so $\nu_A(x) \leq \max\{\nu_A(y*x), \nu_A(y)\}$. Hence, A is an intuitionistic fuzzy ideal of Y.

Theorem 4.6. *If A is an intuitionistic fuzzy ideal of Y, then the intuitionistic fuzzy α -multiplication A_α^M of A is an intuitionistic fuzzy ideal of Y, for all $\alpha \in [0,1]$.*

Proof. Let A be an intuitionistic fuzzy ideal of Y and let $\alpha \in [0,1]$. Then

$$(\mu_A)_\alpha^M(x) = \alpha \cdot \mu_A(x) \geq \alpha \cdot \min\{\mu_A(y*x), \mu_A(y)\}$$
$$= \min\{\alpha \cdot \mu_A(y*x), \alpha \cdot \mu_A(y)\} = \min\left\{(\mu_A)_\alpha^M(y*x), (\mu_A)_\alpha^M(y)\right\}$$

and

$$(\nu_A)_\alpha^M(x) = \alpha \cdot \nu_A(x) \leq \alpha \cdot \max\{\nu_A(y*x), \nu_A(y)\}$$
$$= \max\{\alpha \cdot \nu_A(y*x), \alpha \cdot \nu_A(y)\} = \max\left\{(\nu_A)_\alpha^M(y*x), (\nu_A)_\alpha^M(y)\right\}$$

Hence, A_α^M of A is an intuitionistic fuzzy ideal of Y, for all $\alpha \in (0,1]$.

5. Conclusion

In this paper, we illustrated IFSU, IFID by using the intuitionistic fuzzy translation, intuitionistic fuzzy multiplication and intuitionistic fuzzy extension. The relationships between intuitionistic fuzzy translations and intuitionistic fuzzy extension of IFSU and

IFID have constructed. It is our hope that this work would become another foundation for further study of the theory of G -algebra.

References

[1] Y. Imai and K. Iseki, On axiom system of propositional calculi, XIV, *Proceedings of the Japan Academy*, (1966), 19-22.

[2] K. Iseki, An algebra related with a propositional calculus, *Proceedings of the Japan Academy*, 42 (1966), 26-29.

[3] J. Neggers and H. S. Kim, On B -algebras, *Matematichki Vesnik,* 54 (2002), 21-29.

[4] Y. B. Jun, E. H. Roh and H. S. Kim, On fuzzy B -algebras, *Czechoslovak Mathematical Journal,* 52 (2002), 375-384.

[5] S. S. Ahn and K. Bang, On fuzzy subalgebras in B -algebras, *Communications of the Korean Mathematical Society,* 18 (2003), 429-437.

[6] A. B. Saeid, Fuzzy topological B -algebras, *International Journal of Fuzzy Systems*, 8 (2006), 160-164.

[7] T. Senapati, C. S. Kim, M. Bhowmik and M. Pal, Cubic subalgebras and cubic closed ideals of B -algebras, *Fuzzy Information and Engineering*, 7 (2015), 129-149.

[8] T. Senapati, M. Bhowmik and M. Pal, Fuzzy dot subalgebras and fuzzy dot ideals of B-algebras, *Journalof Uncertain Systems*, 8 (2014), 22-30.

[9] T. Senapati, M. Bhowmik and M. Pal, Fuzzy B -subalgebras of B -algebras with respect to t -norm, Journal of Fuzzy Set Valued Analysis, 2012, (2012), doi: 10.5899/2012/jfsva-00111.

[10] R. Iqbal, S. Zafar and M. S. Sardar, Neutrosophic Cubic Subalgebras and Neutrosophic Cubic Closed Ideals of B -algebras, *Neutrosophic Sets and Systems*, 14 (2016), 47-60.

[11] C. B. Kim and H. S. Kim, On BG -algebras, *Demonstratio Mathematica*, 41 (2008), 497-505.

[12] R. K. Bandaru and N. Rafi, On G -algebras, *Scientia Magna*, 8 (2012), 1-7.

[13] M. Bhowmik, T. Senapati and M. Pal, Intuitionistic L -fuzzy ideals of BG -algebras, *Afrika Matematika*, 25 (2014), 577-590.

[14] T. Senapati, M. Bhowmik and M. Pal, Triangular norm based fuzzy BG -algebras, *Afrika Matematika*, (2015), doi: 10.1007/s13370-015-0330-y.

[15] T. Senapati, M. Bhowmik and M. Pal, Fuzzy dot structure of BG -algebras, *Fuzzy Information and Engi-neering*, 6 (2014), 315-329.

[16] T. Senapati, Bipolar fuzzy structure of BG -subalgebras, *The Journal of Fuzzy Mathematics*, 23 (2015)209-220.

[17] T. Senapati, M. Bhowmik and M. Pal, Interval-valued intuitionistic fuzzy BG -subalgebras, *The Journalof Fuzzy Mathematics*, 20 (2012), 707-720.

[18] T. Senapati, M. Bhowmik and M. Pal, Interval-valued intuitionistic fuzzy closed ideals BG -algebras andtheir products, *International Journal of Fuzzy Logic Systems*, 2 (2012), 27-44.

[19] T. Senapati, M. Bhowmik and M. Pal, Intuitionistic fuzzications of ideals in BG -algebras, *MathematicaAeterna*, 2 (2012) 761-778.

[20] C. Jana, T. Senapati, M. Bhowmik and M. Pal, On intuitionistic fuzzy G -subalgebras of G -algebras, *Fuzzy Information and Engineering*, 7 (2015), 195-209.

[21] C. Jana, M. Pal, T. Senapati and M. Bhowmik, Atanassovs intutionistic L -fuzzy G -subalgebras of G -algebras, *The Journal of Fuzzy Mathematics*, 23 (2015), 325-340.

[22] T. Senapati, C. Jana, M. Bhowmik and M. Pal, L -fuzzy G -subalgebras of G -algebras, *Journal of the Egyptian Mathematical Society*, 23 (2015), 219-223.

[23] M. Bhowmika and T. Senapati, Fuzzy Translations of Fuzzy Subalgebras in BG -algebras, *Journal of Mathematics and Informatics*, Vol. 4, (2015), 1-8.

[24] T. Senapati, M. Bhowmik, M. Pal and B. Davvaz, Fuzzy translations of fuzzy H -ideals in BCK/BCI -algebras, *Journal of the Indonesian Mathematical Society*, 21 (2015), 45-58.

[25] T. Senapati, M. Bhowmik and M. Pal, Atanassovs intuitionistic fuzzy translations of intuitionistic fuzzy H -ideals in BCK/BCI -algebras, *Notes on Intuitionistic Fuzzy Sets*, 19 (2013), 32-47.

[26] L. A. Zadeh, Fuzzy sets, *Information and Control*, 8 (1965), 338-353.

[27] K.T. Atanassov, Intuitionistic Fuzzy Sets: Theory and Applications, Studies in Fuzziness and Soft Computing, Vol. 35 Physica-Verlag, Heidel-berg, New York, (1999).

[28] K. T. Atanassov, Intuitionistic Fuzzy Sets: Theory and Applications, Studies in Fuzziness and Soft Computing, Vol. 35, Physica-Verlag, Heidel-berg, New York, (1999).

[29] Y. H. Kim and T. E. Jeong, Intuitionistic fuzzy structure of B -algebras, *Journal of Applied Mathematicsand Computing*, 22 (2006), 491-500.