Robust Nonlinear Predictive Control of a Permanent Magnet Synchronous Motor

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Abstract—A robust nonlinear predictive controller with a disturbance observer for a permanent magnet synchronous motor (PMSM) is presented. As the disturbance relative degree is less than that of the input, it is quite challenging to adopt the existing disturbance observer-based predictive control techniques. In the proposed controller, robustness of the closed-loop system with respect to mismatched parameters and unknown load torque is significantly improved. Stability of the closed-loop system is proved, and the controller is easy to implement. Validity of the proposed controller was experimentally tested on a dSPACE DS1104 board driving a 0.25 kW PMSM drive. Excellent results were obtained with respect to the speed trajectory tracking performance and robustness.

Index Terms—Permanent magnet synchronous motor (PMSM), disturbance observer, nonlinear predictive control (NPC), predictive generalized minimum variance (PGMV), stability, robustness.

I. INTRODUCTION

Because of its high efficiency and power density, the permanent magnet synchronous motor (PMSM) is widely used in many variable-speed drives applications such as robotic actuators, automotive and renewable energy conversion systems, just to name a few. However, the PMSM poses challenging control problems due to its nonlinear multivariable dynamics and generally unknown external disturbances. Also, its parameters tend to change during operation. With the advancement of Power Electronics and Digital Signal Processors (DSP) technology, it is now possible to implement advanced control techniques for PMSM in real-time.

Many robust control techniques were proposed for the PMSM [1]-[5]. Most of these works are based on the linear control theory. In [5] the voltage disturbance caused by the uncertainties is determined by an adaptive disturbance observer and utilized in the feedforward control. To take into account the nonlinearities of the PMSM, different approaches have been employed, such as nonlinear control [6] and sliding mode control [7].

Model predictive control (MPC) has been widely recognized by many researchers as one of the most robust control techniques. It is based on the optimization of a cost function pertaining to the difference between the output and the trajectory to be tracked [8]. Naturally this may give an optimal control to achieve offset-free tracking in absence of disturbance and plant model mismatch. In [9] it is shown that MPC can also be well suited for constant disturbance rejection by using Internal Model Control (IMC). However, the majority of existing methods essentially consider discrete time linear (DTLM) models in the synthesis of controllers. MPC for nonlinear control process improves insensitivity to parameters variations and external disturbances. It is also able to satisfy constraints on both state and control. The robustness of MPC has been investigated in [10] using the discrete time nonlinear model (DTNM). MPC based on DTNM, however, imposes heavy online computation burden.

Many studies have been published since 1994 on the application of MPC to nonlinear systems with fast dynamics. A methodology for constructing closed form nonlinear MPC (NMPC) is proposed in [11]-[13] where the so-called one-step ahead is adopted to predict the future system output by using Taylor series expansion up to the input relative degree. In [14] both predictive generalized minimum Variance (PGMV) and generalized predictive control (GPC) are utilized to establish a more general framework for NMPC based on continuous time model. As pointed in [12], predictive control based on Taylor series expansion cannot remove completely steady state error under mismatched parameters and external disturbance. To this end, Chen et al. [15] have developed a disturbance observer to be combined with NMPC in order to ensure the offset-free performance. However, the proposed disturbance observer is applicable only for single-input single-output (SISO) nonlinear systems having the same input and disturbance relative degree. This work is extended to multi-input multi-output (MIMO) nonlinear systems in [16]. Another shortcoming of these methods is that the closed loop system is unstable for plants whose the input relative degree is higher than four [12]. To overcome this drawback, NGPC scheme for MIMO nonlinear systems, with a same relative degree, have been proposed in [17] by approximating it with Taylor series expansion to any specified order. It is shown that the stability property depends on the so-called control order which should be chosen to be larger than zero when the input relative degree is higher than four. In [18], the above-mentioned method is extended to MIMO nonlinear systems with different relative degree.
Nowadays, MPC has been successfully applied for many applications such as power converters, drives and electrical machines [19]-[24]. An exhaustive study of application of MPC methods in power electronics and drives has been presented in [19]. In [20] and [21], the principle of the MPC is combined with the well-known direct torque control (DTC) and applied to the induction motor (IM) for a fast torque response. However, in the both publications, the load torque is considered as a known disturbance. In [22], the MPC is applied to an IM where the load torque is assumed to be a disturbance whose magnitude is computed using a Kalman filter.

The MPC of a PMSM based on DTLM has been described in [23] with the load torque again considered as a known disturbance. This approach may result in a steady state speed tracking error. In [24], the General predictive control (GPC) has been combined with the DTC technique to implement a robust speed controller. Here, the knowledge of both the electromagnetic torque and flux linkage are required. In [25], the principle of MPC is applied to tow-level inverter to select the stator voltage vector which minimizes the current error, at each sample time, over a finite prediction horizon which is taken equal to one. This strategy is also applied for multi-level inverter in [26]. Constrained MPC based on DTLM for PMSM is proposed in [27]-[30]. Although satisfactory results are obtained in [29] a significant computation effort is still required to compute the optimal control due to the need of solving on-line optimization problems. The paper [30] has proposed two cascaded explicit model predictive controllers, where a Kalman filter is used to estimate the actual speed. The use of MPC for reducing speed ripple has been treated in [31] with DTLM again considered for predicting the trend of the system’s output.

In the aforementioned works, DTLM is widely used in the synthesis of the controller whereas the PMSM model is inherently nonlinear and continuous time in nature and the use of the DTLM may lead to poor performance. On the other hand, as mentioned before, the optimal NGPC based on continuous time and on the Taylor series expansion, cannot guarantee zero steady state error under conditions of mismatched parameters and external perturbation. For improvement, the NGPC is combined with the disturbance observer to render the overall closed loop system robust [16]. Application of the disturbance observer is limited to systems having the same relative degree of input and disturbance. However, when the disturbance relative degree is less than the input one, which is the case for a PMSM, the design of the disturbance observer is, as stated in [15] nontrivial. Indeed, analysis of the closed loop system under NGPC with a nonlinear disturbance observer becomes very complicated. To tackle this problem, a newly defined design function has been proposed in [32] to construct only the load torque observer for a NGPC of PMSM. In [33], two cascaded explicit nonlinear predictive controllers have been used in order to handle the voltage and currents constraints. From the results, it can be observed that this strategy can be adopted to effectively achieve offset-free reference tracking under unknown constant disturbance. It shall be mentioned that similar idea of estimating disturbances to achieve zero offset tracking in MPC has been presented in [34], very recently. However, it only deals with linear MPC with a linear disturbance observer. In [35]-[36], an integral component has been used to increase the robustness of the closed-loop system. Thus, the knowledge of the external perturbation and uncertainties is not required. Zero steady state error is guaranteed by the integral action, which arises naturally in the controller by modifying the predicted output tracking error. The effectiveness of this strategy to the speed control of a PMSM has been proved in [37]. However, the $d$-axis current regulation is not guaranteed when the electrical parameters change. In a similar way, the derived nonlinear predictive law is specified in [38] by optimizing a newly defined design cost function. A key feature of the proposed control is that the disturbance observer is not needed to guarantee the offset free performance.

This paper proposes a robust nonlinear predictive controller (NPC) with a nonlinear disturbance observer for a PMSM. For controller synthesis, the optimal PGMV control [14] is combined with the continuous time nonlinear model (CTNM) of the motor. Furthermore, a disturbance observer is proposed for estimating the perturbations affecting the system output regulation. The closed loop system under the disturbance observer contains the error integration. As a result, zero steady state error is guaranteed as long as the closed loop system is stable.

II. MATHEMATICAL MODEL OF THE PMSM

As in [15], the model of the permanent magnet synchronous motor in the rotor reference frame is expressed in a nonlinear affine form as:

$$\begin{align*}
\dot{x}(t) &= f(x) + g_1 u(t) + g_2 b(t) \\
y(t) &= h(x)
\end{align*}$$

(1)

where

$$x = \begin{bmatrix} i_d & i_q \end{bmatrix}^T; \quad u = \begin{bmatrix} u_d & u_q \end{bmatrix}^T; \quad b = \begin{bmatrix} f_d & f_q & f_w \end{bmatrix}^T$$

(2)

The state vector $x$ is composed of the $d$-axis and $q$-axis components of the armature current, and the rotor speed. The input vector $u$ is made of the $d$-axis and $q$-axis components of the armature voltage. The disturbance vector $b$ represents perturbation resulting from parameter variations and unknown external disturbances such as load torque. The output vector $y$ consists of the $d$-axis component of the armature current and the rotor speed.

The vector function $g_1$ and $g_2$ are defined as
\[ g_1 = \begin{bmatrix} g_d \\ g_q \end{bmatrix} = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \end{bmatrix} \quad ; \quad g_2 = \begin{bmatrix} -\frac{1}{L_d} & 0 & 0 \\ 0 & -\frac{1}{L_q} & 0 \\ 0 & 0 & -\frac{1}{J} \end{bmatrix} \tag{3} \]

The vector function \( f \) is given by
\[ f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} = \begin{bmatrix} \frac{R}{L_d} i_d + \frac{L_q}{L_d} p \phi \omega_q \\ -\frac{R}{L_q} i_q - \frac{L_d}{L_q} p \phi \omega_d - \frac{q}{L_q} \phi \omega_d \\ \frac{B}{J} (\phi \omega_d + (L_d - L_q) i_d) - \frac{B}{J} \phi \omega_d \end{bmatrix} \tag{4} \]

\( i_d \) and \( i_q \) are respectively \( d \)-axis and \( q \)-axis components of the armature current; \( u_d \) and \( u_q \) are \( d \)-axis and \( q \)-axis components of the supply voltage; \( \omega_d \) is the rotor speed and \( T_L \) is the load torque, considered an unknown disturbance and ignored in the synthesis of the controller. \( R, L_d \) and \( L_q \), respectively the per-phase armature resistance and the \( d \)-axis and \( q \)-axis inductances. \( \phi \) is the permanent magnet flux; \( p \) is the number of pole pairs; \( J \) is the moment of inertia and \( B \) is the coefficient of friction.

The variables to be controlled are the rotor speed \( \omega_r \) and the \( d \)-axis component of the armature current \( i_d \).

\[
\begin{align*}
y_1(t) &= h_1(x) = i_d \\
y_2(t) &= h_2(x) = \omega_r
\end{align*}
\tag{5}
\]

### III. Predictive Generalized Minimum Variance (PGMV) of a PMSM

As in [14], the objective of the PGMV is to design a controller such that the future plant output \( y(t+T_1) \) optimally tracks a future reference trajectory \( y_{ref}(t+T_1) \) in the presence of perturbation. This can be formally stated as the problem of minimizing a finite horizon cost function defined as
\[
J = \frac{1}{2} \left\{ (y_{r'}(t+T_1) - y_1(t+T_1))^2 + (y_{r''}(t+T_2) - y_2(t+T_2))^2 \right\}
\tag{6}
\]

where
\[
\begin{bmatrix} y_1(t+T_1) \\
y_2(t+T_2) \end{bmatrix} = \begin{bmatrix} i_d(t+T_1) \\
\omega_r(t+T_2) \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} y_{r'}(t+T_1) \\
y_{r''}(t+T_2) \end{bmatrix} = \begin{bmatrix} i_{d'}(t+T_1) \\
\omega_{r'}(t+T_2) \end{bmatrix}
\tag{7}
\]

\( T_1 \) and \( T_2 \) represent the predictive times, respectively for fast (electrical) and slow (mechanical) modes, so that \( T_1 < T_2 \). To solve the nonlinear optimization problem (6), the predicted output \( y_f(t+T_1) \) and the predicted reference \( y_{ref}(t+T_1) \) are approximated by Taylor series expansion using the Lie derivative of \( h(x) \) with respect to the vector field \( f(x) \). Both \( f(x) \) and \( h(x) \) are assumed to be continuously differentiable.

#### A. Design of the controller

Consider the nonlinear model of the PMSM described in this paper. In the nonlinear predictive control design method presented in [12], [14], [15] and [17], it is necessary to calculate the relative degree of each output in order to determine the expression of the optimal control which minimizes the cost function over the prediction horizon. The relative degree of an output is the number of times that it is necessary to derive the output to reveal the input \( u \).

In the case of the output \( y_1 \) we obtain
\[
\dot{y}_1(t) = L_f h_1(x) + L_{g_1} h_1(x) u(t) + L_{g_2} h_1(x) b(t)
\tag{8}
\]

In the case of the output \( y_2 \) we get
\[
\dot{y}_2(t) = L_f h_2(x) + L_{g_1} h_2(x) u(t) + L_{g_2} h_2(x) b(t)
\tag{9}
\]

The above results are based on the assumption that the disturbance \( b(t) \) varies so slowly. This means that its time derivative is negligible. From (8) and (9), the relative degrees of \( y_1 \) and \( y_2 \) are respectively \( \rho_1 = 1 \) and \( \rho_2 = 2 \). Therefore, the relative degree of the system is \( \rho = 3 \). We note that the relative degree of the system is equal to the system’s order. Consequently, there is no zero dynamics. In addition, it is assumed that the armature current and the rotor speed are available for measurements. Therefore, the nonlinear system (1) is input-output feedback linearizable [39].

The most important part of this work is the design of an appropriate nonlinear observer to estimate the perturbation affecting the PMSM. When the PMSM model is accurate, the disturbance only includes the load torque. In this case, it is important to remark that the load torque relative degree is less than the input relative degree. On the other hand, as the dimension of the disturbance is higher than that of the system output, it is not possible to use the disturbance observer-based nonlinear predictive control [15], [16] to estimate all the perturbations. To this end, the disturbance \( f_1(t) \) is neglected in the control design and hence only the offsets \( f_2, f_3 \) affecting the system output are considered in the synthesis of the controller to guarantee offset-free performance.

Hence, the vector function \( g_2 \) is reduced to
\[
g_2(x) = \begin{bmatrix} -\frac{1}{L_d} & 0 & 0 \\ 0 & -\frac{1}{L_q} & 0 \\ 0 & 0 & -\frac{1}{J} \end{bmatrix} \tag{10}
\]

In the same fashion as in [32] after the disturbance is estimated, the resultant optimal control law is given by
\[
\begin{bmatrix} 1 \end{bmatrix} \left\{ \sum_{i=0}^{K_1} \left( L_f h_i(x) - y^{(i)}_{r'}(t) \right) + L(x) \hat{b}(t) \right\} \tag{11}
\]

5041
where \( \hat{b}(t) \) represents the estimate of the disturbance. The coefficients of the controller are given by

\[
K_l = \frac{1}{T_1}; K_l^1 = 1 \\
K_l^2 = \frac{2}{T_2}; K_l^3 = \frac{2}{T_2}; K_l^2 = 1
\]

(12)

The matrix \( G_l(x) \) in (11) is defined by

\[
G_l(x) = \begin{bmatrix}
L_{g_l}h_l(x) & L_{g_l}h_l(x) \\
L_{g_l}L_l h_l(x) & L_{g_l}L_l h_l(x)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{1}{L_d} & 0 \\
-\frac{1}{L_d} & 0
\end{bmatrix}
\]

(13)

\[
G_l(x) = \frac{p_1(L_d - L_q)i_q}{J_{L_d}} + \frac{p_2(L_d - L_q)i_d}{J_{L_d}}
\]

\[
G_l(x) \text{ is invertible if } \left( \phi_0 + (L_d - L_q)i_d \right) \neq 0
\]

Because the load torque relative degree is less than the input relative degree, the matrix \( L(x) \) is defined as

\[
L(x) = \begin{bmatrix}
L_1(x) & L_2(x) & L_{g_l}h_l(x) & L_{g_l}L_l h_l(x)
\end{bmatrix}
\]

(14)

\[
= \begin{bmatrix}
\frac{1}{L_d} & 0 & \frac{1}{J_d} & 0 \\
-\frac{1}{L_d} & 0 & \frac{1}{J_d} & 0
\end{bmatrix}
\]

As each input relative degree is less than four, the closed loop system under the predictive controller (11) is globally exponentially stable when the information about the disturbance is available [15], [17]. On the other hand, since there is no zero dynamics and the fact that the reference is bounded, the driven dynamics is always defined and bounded. It can also be shown that NPC law based on Taylor series expansion and geometric approaches [39] give similar feedback controllers. However, the controller gains for NPC are computed by minimizing a cost function over a finite horizon. This is similar to the linear case where pole placement and LQR may lead to similar state feedback law, but they are derived from different design methodologies.

B. Design of the disturbance observer

The disturbance in (11) can be estimated in the similar way as in [15], [32] and [38]. Consequently, the initial observer form is given by

\[
\hat{b}(t) = -l(x)g_{2r}(x)\hat{b}(t) + l(x)(\hat{x} - f(x) - g_l(x)u(t))
\]

(15)

\( l(x) \) is a gain to be designed to ensure the stability of the observer. In [15], the gain \( l(x) \) is chosen as

\[
l(x) = \frac{\partial p(x)}{\partial x}
\]

(16)

From the fact that \( b(t) \) varies slowly, it follows that the error dynamics of the observer is given by

\[
\dot{e}_b(t) + l(x)g_{2r}(x)e_b(t) = 0
\]

(17)

\[
e_b(t) = b(t) - \hat{b}(t)
\]

(18)

Based on (17), it can be shown that the disturbance observer (15) can be made globally asymptotically stable by choosing \( p(x) \) such that \([-h(x)g_{2r}(x)]\) is stable for all \( x \in \mathbb{R}^r \). As mentioned before, when relative degree of the disturbance is less than that of the input, which is the case for a PMSM, the design of the disturbance observer is nontrivial [15]. To address this problem, we propose the following function \( p(x) \) as

\[
p(x) = \mu \begin{bmatrix} p_1(x) \\ p_2(x) \end{bmatrix} = \mu \begin{bmatrix} h_l(x) \\ K_l^1 h_l(x) + L_l h_l(x) \end{bmatrix}
\]

(19)

where \( \mu \) is a matrix of constants.

As in [15], [32] and [38]; replace the function \( p(x) \) in (16) by its expression (19), and using the optimal control law (11) and after integration, results in a simplified disturbance observer form as follows

\[
\hat{b}(t) = \begin{bmatrix}
\hat{i}_d \\ \hat{i}_q
\end{bmatrix} = \mu \begin{bmatrix}
K_l^1 \int_{0}^{t} e_1(r)dr + K_l^2 e_1(t) \\ K_l^3 \int_{0}^{t} e_2(r)dr + K_l^4 e_2(t) + K_l^5 \dot{e}_2(t)
\end{bmatrix}
\]

(20)

where

\[
\begin{bmatrix}
e_1(t) = i_d(t) - i_d(t) \\ e_2(t) = \omega_r(t) - \omega_r(t)
\end{bmatrix}
\]

(21)

It can be shown that the combination of the optimal NPC (11) and the disturbance observer (20) contains the error integration. Thus, if the closed loop system is globally stable, the steady state error is cancelle. It remains to determine the condition on the matrix \( \mu \) such that the system (20) has uniformly globally asymptotically stable equilibrium at the origin. This would imply that the disturbance observer is uniformly globally asymptotically stable.

Now we consider again the system (17) and use the function \( p(x) \) given by (19), the corresponding error dynamics of the observer becomes

\[
\dot{e}_b(t) = -\mu L(x)e_b(t)
\]

(22)

The matrix \( L(x) \) in (14) is a lower triangular, and the diagonal elements are constants, the matrix \( \mu \) can be chosen as a diagonal matrix

\[
\mu = \begin{bmatrix}
\mu_d & 0 \\ 0 & \mu_o
\end{bmatrix}
\]

(23)

5042
From (14), (19) and (23), it follows that the resulting error dynamics of the observer is given by

\[
\dot{e}_b(t) = \begin{bmatrix}
\frac{\mu_d}{L_d} & 0 \\
p(L_d - L_q) & -\mu_a \left( \frac{B}{J} - K_i^2 \right)
\end{bmatrix} e_b(t)
\]  

(24)

**Theorem.** Consider the system (24) and suppose that the \( q \)-axis current \( i_q \) is bounded. Then the system (24) is uniformly globally asymptotically stable by choosing the parameters of the matrix \( \mu \) such that

\[-\frac{\mu_d}{L_d} > 0 \quad \text{and} \quad \frac{\mu_a}{J}\left( \frac{B}{J} - K_i^2 \right) > 0\]  

(25)

**Proof.** Let \( P = \text{diag}(P_1, P_2) \) to be a positive definite matrix. Thus the Lyapunov function for \( e_b \) system can be chosen as

\[V(e_b) = \frac{1}{2} e_b^T P e_b\]  

(26)

The derivative of the Lyapunov function \( V \) with respect to time gives

\[
\frac{dV}{dt} = -\dot{e}_b^T P e_b + e_b^T P \dot{e}_b
\]

(27)

where

\[\dot{e}_b = [f_d - \hat{f}_d, f_m - \hat{f}_m]^T\]  

(28)

Hence, the condition (25) ensures

\[
\frac{dV}{dt} \leq -\dot{e}_b^T P e_b + e_b^T P \dot{e}_b
\]

(29)

As a result, the matrix \( M \) appearing in (29) is symmetric positive definite if and only if

\[-\frac{\mu_d P_1}{L_d} > 0 \quad \text{and} \quad \frac{\mu_a}{J}\left( \frac{B}{J} - K_i^2 \right) P_2 > 0\]  

(30)

The above condition can be verified for any bounded \( i_q \), by choosing arbitrarily large \( P_1 \) and arbitrarily small \( P_2 \). Therefore, the origin \( (e_b = 0) \) is uniformly globally asymptotically stable equilibrium point for any bounded \( i_q \) and for any matrix \( \mu \) which verifies the inequalities (25). Furthermore, the desired convergence rate of the disturbance observer can be achieved by acting on the values of the matrix \( \mu \).

**C. Stability analysis for the composite controller**

As in [32] in order to check the asymptotic stability of the closed loop system, the stability analysis for the composite controller can be investigated using the principle of the connected subsystems based on the Corollaries (10.3.2) and (10.3.3) given in [40].

**IV. LABORATORY TEST SETUP**

A laboratory prototype is developed to experimentally test the validity of the proposed NGPC scheme for a PMSM drive. This scheme is shown in Fig. 1. Fig. 2 depicts the experimental setup, which consists of a 10-pole, 0.25 kW, 7A and 42V PMSM, a speed sensor, an IGBT inverter and a dSPACE DS1104 board. The proposed NGPC algorithm has been implemented on the main processor (MPC8240). The control sampling frequency is set equal to 10 kHz. The slave unit (TMS320F240 DSP) has been dedicated to the PWM signals generation unit, whose modulation frequency is set to 50 kHz, and to the management of the digital I/O signals.

![Fig. 1. Block diagram of the proposed RNGPC scheme for a PMSM.](image)

**V. EXPERIMENTAL RESULTS**

To guarantee the stability of the closed loop system, the components \( \mu_d \) and \( \mu_a \) of the matrix \( \mu \) (23) are respectively set equals to -0.1 and \( -1 \times 10^{-5} \). The load torque is considered an unknown disturbance and speed dependent. Its value is 0.4 N.m when the speed is 100 rad/s. The predictive time can be chosen based on the specifications of the desired performances according to the table in Chen et al. (2003a). In this work, the predictive times \( T_1 \) and \( T_2 \) are respectively set equal to \( 5 \times 10^{-4} \) and \( 5 \times 10^{-3} \). In the controller (11), the speed reference should be twice differentiable. This can be accomplished by passing the reference speed signal through a second order linear filter.
A. Tracking performance under unknown load torque

The first test is concerned with bidirectional speed control under variable load torque. The dynamics of the speed reference is slow such that the armature phase current doesn't exceed its maximal value during transients. According to the field-oriented control the $d$-axis current $i_d$ is forced by the control action to zero value. As shown in Fig. 3, the speed tracking performance is satisfactorily achieved. Fig. 4 shows that the $d$-axis current is maintained equal to zero. It can also be observed that the $q$-axis current is a little bit different from zero when the speed is null because of the detent torque resulting from the interaction between the rotor magnets and stator windings.

B. Uncertainty in the electrical parameters

The second test is concerned with unidirectional speed control. In addition, to verify the robustness of the drive using the proposed approach, the flux linkage and the electrical parameters are varied in the control law at time $t = 1s$: $R$ and $L_q$ are both decreased by 50%, while $L_d$ and $\phi_v$ are respectively increased by 50% and 20%. In addition, the motor started under unknown load torque and the $d$-axis current $i_d$ kept by the control action at the value of -1 $A$ in order to test the $d$-axis current regulation when the electrical resistance varies. In the same fashion as in the previous test, the dynamics of the speed reference is chosen smaller than the maximum acceleration of the drive.

As shown in Fig. 5, the motor speed quickly converges to the reference speed even if the electrical parameters are varied. Fig. 6 shows the $i_d$ and $i_q$ waveforms. As shown, $i_d$ is maintained equal to the imposed reference value. This is because the parameters variation and the external perturbation are cancelled thanks to the disturbance estimate.
current $i_d$ is forced by the control action to zero value. Fig. 7 shows that precise speed tracking is achieved and the steady state error is eliminated. From Fig. 8, it can be seen that the $d$-axis current regulation is not affected by mismatched mechanical parameters.

![Speed tracking performance under variable mechanical parameters](image1)

**Fig. 7.** Speed tracking performance under variable mechanical parameters.

![dq-axis components of the armature current response under variable mechanical parameters](image2)

**Fig. 8** $dq$-axis components of the armature current response under variable mechanical parameters

**D. Performance evaluation under quick load torque change**

This experience consists to apply a rapid change in the load torque during normal operation. The perturbation is applied at time $t = 1s$, by suddenly increasing the load torque ($q$-axis current); see Fig. 9. Fig. 10 shows that satisfactory speed tracking is achieved and the steady state error is quickly removed. Note that the rate of the disturbance rejection depends especially on the gains of the disturbance observer. Indeed, a larger observer gain will result in fast disturbance rejection and will lead to high gain controller which may excessively amplify the undesired effect of the measurement noise.

![dq-axis components of the armature current response under sudden change of the load](image3)

**Fig. 9.** $dq$-axis components of the armature current response under sudden change of the load

![Speed tracking performance under sudden change of the load](image4)

**Fig. 10.** Speed tracking performance under sudden change of the load.

**VI. CONCLUSION**

A robust nonlinear predictive controller (NPC) for a permanent magnet synchronous motor (PMSM) drive, whose load torque is unknown, has been presented. To ensure robustness against external disturbances and electrical and mechanical parameters variations, a disturbance observer is designed and integrated into the control law. The objective of this “composite” controller is high performance tracking of the rotor speed trajectory while maintaining the $d$-axis component of the armature current at the imposed reference. The relative degree of disturbance is less than that of the input, which is recognized in the control community as a difficult case. To solve this challenging problem, this work uses the existing methods and provides a disturbance observer for estimating all perturbations affecting the system output regulation for PMSM.

Moreover, the stability of the closed-loop system is analyzed. A laboratory prototype was developed to experimentally test the validity of the proposed NPC scheme. Experimental results have shown its effectiveness regarding speed trajectory tracking and robustness.
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