Integrated Optimisation of In-House Production and Outsourcing Strategy:
Genetic Algorithm Based Approach

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Abstract: In this paper, we deal with the problem of balance between two alternative modes for production: in-house production and outsourcing, under a combined approach of maintenance management and production control in multi-periods plan. The considered production system consists of a single randomly failing and repairable machine $M_{in}$ producing a single product. The production system aims to satisfy the market demands with variable production rates during non-homogeneous periodicities and decides to outsource all or part of demands. Outsourcing is justified by the lack of the production capacity. We consider that the contractor company produces for the manufacturer and offers a multiple pricing schedules that depend on the outsourced production rates requested by the manufacturer.

The main objective is to determine simultaneously for each period, the optimal in-house production and the outsourcing rates that should be adopted, in order to satisfy all demands while minimizing the generated total cost of the manufacturer over the plan horizon, including costs of maintenance, production, outsourcing, shortage and holding. We propose a genetic algorithm based optimization and conduct computational experiments to study the managerial insights for the developed strategy.

Keywords: production control, maintenance management, in-house production, outsourcing, multi-period setting, failure rate, inventory, multiple pricing schedules, genetic algorithms.

1. INTRODUCTION AND LITERATURE REVIEW

In the last few years, with the economy globalization and the development of information technology, outsourcing is increasingly developing in manufacturing industries (Boulaksil et al., 2011). Many manufacturing enterprises are outsourcing production operations or materials to their partners as one way of contract. This is in order to cope with varied market demand or keep core competition.

Outsourcing is commonly required as a tool to improve overall planning effectively and efficiently in different companies. Proper outsourcing can shorten lead times, reduce total costs, and make an organization more flexible (Lee and Choi, 2011; Lee and Sung, 2008). Thus, well-utilized outsourcing can make a company more competitive (Cachon and Harker, 2002).

There are a lot of papers in the literature dealing with outsourcing strategies between different parties of the supply chain. For example, in their study, Coman and Ronen, (2000) have formulated the outsourcing problem as a Linear Programming problem and have identified an analytical solution. They proceed with an example examining three decision models: standard cost accounting, standard theory of constraints and their own solution. Bertrand and Sridharan, (2001) have studied a situation where demand rate greater than the production rate, thus outsourcing of some of the orders. They have considered that order lead times are exogenous and highly variable. Authors have developed heuristics decision rules in order to minimize costs and maximize delivery reliability.

Kim (2003) have considered a situation in which a manufacturing company outsources its assembly operations to two different contract manufacturers, each one has owner level of improvement capability of inducing supply cost reduction. Author have proposed an optimal control model to determine how much should be outsourced to each contract manufacturer and how processed should the semi-finished units be when returned from the contract manufacturers. Liu et al., (2008) have developed a genetic algorithm heuristic method to solve a dynamic capacitated production planning problem with consideration of outsourcing. In this paper, authors consider that all demands are met by in-house manufacturing or outsourcing without postponement or backlog. They also consider that levels of production, inventory, and outsourcing are limited. The cost functions are considered time-varying. Other researchers have considered outsourcing in the context of decision multi-levels. For example, Boulaksil and Fransoo, (2009) have discuss and compare the performance of three different order release strategies to control outsourced operations between an original equipment manufacturer and a contract manufacturer.
that serves several clients. In another work, Boulaksil et al., (2011) consider a contract manufacturer that serves a number of outsourcee with different levels of demand uncertainty. The authors are formulating a hierarchical model as an integer program that consists of two decision levels. They perform a numerical study based on simulation for solving the problem iteratively.

At the operational level, Lee and Choi (2011) have analyzed the model’s computational complexity for a two-stage production scheduling problem. Two options for operations processing are considered: produce by utilizing in-house resources or outsource to a subcontractor. For in-house operations, the performance of a schedule is measured by the makespan however the outsourced operations require only outsourcing cost. Qi (2011) studied the production scheduling problem for a two-stage flow shop in which there are options of outsourcing some operations to subcontractors. The basic objective is to look for a production schedule that can optimally utilize the resources of both the in-house production and outsourcing so that the makespan is minimized. In (Chen and Li, 2008; Lee and Sung 2008) outsourcing is allowed in parallel to the associated in-house scheduling as to promote the overall scheduling quality. Ruiz-Torres et al. (2006) study the case with parallel in-house machines/parallel outsourcing machines.

Recently, Zhen (2012) investigates a problem where an enterprise that manufactures multiple products in multiple periods and compares between two alternative modes: outsource parts or in-house manufacture parts and then assembles them in order to satisfy stochastic demands. The author proposes an analytical approach to choose the optimal decision during each plan period. Following the same reasoning of (Kim, 2003), Chiao et al., (2012) answer the question: how a manufacturer determines the outsourcing quantity to be allocated to each outsourcer? Authors identify two types of outsourcers: the first one offers a lower outsourcing price but has inferior facilities which result in a higher deteriorating rate, the other has advanced facilities causing a lower deteriorating rate but requires a higher outsourcing price.

The most of proposed models have considered mainly the outsourcing in production scheduling and planning, lot-sizing and production capacity reservation view points. While the outsourcing in those contexts has received much attention in the literature, but under a combined approach of control production and maintenance management has been less considered. Other researchers have treated outsourcing in the context of combined approach of control production and maintenance management. For example, in (Dahane et al., 2010) authors develop a combined approach of production and maintenance for a unit subject to an outsourcing constraint. Haoues et al (2011) propose mathematical models for sharing the outsourcing request between the contractors. The objective is to find the quantity to be outsourced to each contractor and the best outsourcing periods. Different multi-contractor strategies are adopted. Those researches rested limited on single period. However, there are only few papers considering the issue of outsourcing under combining approach of maintenance and production planning in multi-period setting.

The objective of this paper is to study the outsourcing activities problem in two-stage supply chain under combined approach of maintenance management and production control. We focus on the problem of mixture of the in-house production and outsourcing where a demand quantity or a specific quantity of a demand can be produced by either the in-house production or outsourcing.

The rest of the paper is organized as follows: in the next section we present the considered system, notations and assumptions. The global mathematical model is developed in section 3. The solution procedure is discussed in section 4. Section 5 presents and discusses a numerical example. Finally, summary, conclusions and future scope of work are provided in section 6.

2. PROBLEM STATEMENT

This paper deals with the determination of the mixed optimal in-house production and outsourcing plan in multi-period setting from a manufacturer perspective that has a relationship with a contractor with different production levels.

Figure 1 shows the supply-chain structure that we are considering in this paper. The manufacturer company is composed of a single randomly failing and repairable machine that produces a single product. It should satisfy the demand of its clients $d_k$ with variable production rates in non-homogeneous periodicities $k (k = 1, \ldots, N)$.

![Diagram](image_url)

Fig. 1. Supply-chain structure under study.

The in-house production capacity is less than the global market demand. In this case, in order to satisfy very high demands beyond predictions, without postponement or backlog, the company has to resort to the outsourcing to fill in the gap between in-house production and demands. However, the recourse to outsourcing is not always justified by the inability of in-house production, but also due to the consideration of minimization costs issue. Based on the principle of comparative advantage, the manufacturer chooses the in-house production plan that minimizes the total costs (Zhen, 2012). The results demonstrate that manufacturer’s high in-house costs motivate firms to outsource to an independent contractor (Ni et al., 2009). Nowadays, in the global supply chain network, few firms manufacture products by in-house manufacturing all the parts of the product without outsourcing.
From reliability and maintenance point of view, we consider dependence between the production and failure rates, such as the failure rate \( \lambda(t) \) during each interval depends on the production rate \( u(t) \). So, the production at high rate accelerates the machine degradation and therefore increases the number and the total cost of repairs (Haoues et al., 2011).

For a given interval \( I = [\tau_{k-1}, \tau_k] \), \( k \geq 1 \) the failure rate is written as follows:

\[
\lambda_k(t) = \alpha_k \lambda_{\max}(t)
\]

(1)

Where \( \alpha_k = \frac{u_{\max}}{u_m} \) and \( 0 \leq t \leq (\tau_k - \tau_{k-1}) \)

\( \lambda_{\max}(t) \) is the nominal failure rate corresponding to the maximal production rate \( u_{\max} \). Where \( \lambda_{\max}(0) = 0 \).

Suppose we use the machine with production rates \( u_1, ..., u_p, ..., u_L \). Such as: \( u_1 < ... < u_i < ... < u_L \) (i)

According to the dependence relationship between failure rate and production rate we have:

\[
\lambda_1(t) < ... < \lambda_i(t) < ... < \lambda_L(t)
\]

(\( \alpha \))
i.e. if we use the machine with a production rate \( u_i \) a failure rate \( \lambda_i(t) \) is generated. On the other hand we have the following expression of the maintenance costs:

\[
MC_i = C_r M_i(t) + C_m
\]

(2)

Therefore, \( MC_1 < ... < MC_i < ... < MC_L \)

(\( \alpha \))

Thus, the in-house production rates will affect the total cost.

From outsourcing point of view, the contractor company consists of a single machine \( M_e \). It produces the requested quantities to satisfy the manufacturer under contractual obligations. The contractor offers different outsourcing rates. Each outsourcing rate corresponds on a production rate and a specific cost.

We are given a plan horizon of manufacturer \( H \) including \( N \) periods of non-homogeneous duration \( \Delta \tau_e \) (\( t = 1, ..., N \)).

Our objective is to determine for each period the optimal in-house production rates and the outsourcing rates those minimize the total costs of manufacturer. Below, we present notations which will be used throughout the paper:

The failure probability density function of machine \( M_m \) \( f(t) \). And the cumulative distributions function of the machine \( F(t) \), such as:

\[
F(t) = \int_0^t f(x) dx
\]

(3)

Let be \( \lambda(t) \) the failure rate of a machine at a given instant \( t \):

\[
\lambda(t) = \frac{f(t)}{1-F(t)}
\]

(4)

At each random failure of the manufacturer’s machine \( M_m \) we carry out a minimal repair action.

At the end of each period, i.e. at \( T = \tau_1, ..., \tau_N \) machine \( M_m \) undergoes a preventive maintenance action.

All maintenance actions are supposed to be perfectly performed and each action restores the machine to the “As Good As New” configuration. It is also assumed that the maintenance actions durations are negligible (Fig. 2)

![Fig. 2. Integrated maintenance strategy.](image)

**Model Indices**

- \( k \) The index of Period.
- \( l \) The rank in price schedule.

**Model decision variables**

- \( u_{m,k} \) The in-house production rate in period \( k \).
- \( u_{o,l,k} \) The production rate of the contractor during period \( k \) and rank \( l \) of price schedule.

**Model parameters**

- \( d_k \) Demand during period \( k, k = 1, ..., N \).
- \( \tau_k \) Delivery time of demand in period \( k \).
- \( \Delta \tau_k \) Duration of each plan period (\( k = 1, ..., N \)).

\( \Delta \tau_k = \tau_k - \tau_{k-1} \).

- \( S_k \) Inventory level at the end of period \( k, k = 1, ..., N \).
- \( C_p \) Unit cost of production.
- \( C_h \) Unit cost of holding.
- \( C_s \) Unit cost of shortage.
- \( C_m \) Unit cost of preventive maintenance action.
- \( C_r \) Unit cost of corrective maintenance action.
- \( C_{o,l} \) Unit cost of outsourcing during one time unit corresponding in rank \( l \) in pricing schedule.

**TC, IC, MC, SC and OC**

- \( u_{\min} \) Minimum production rate of \( M_e \) (the contractor).
- \( u_{\max} \) Maximum production rate of \( M_e \) (the contractor).

To describe the problem more clearly, we provide the following assumptions:

1. The unmet demand is lost and generates a shortage cost.
2. Outsourcing cost is variable and depends on the production capacity (rate) requested by the manufacturer.
3. The contractor delivers the outsourced quantity to the manufacturer at the end of each period.
4. The demand vector \( (d_k, \tau_k) \) arrives at the beginning of production plan horizon. Where \( k = 1, ..., N \).
5. The production plan can start with a not-null inventory level: \( (S_0) \). Where \( S_0 = 0 \).
6. Failures are detected instantaneously.
7. The product is imperishable with time.
We mention that manufacturer has different options of outsourcing:
- It must outsource the shortage quantity.
- In addition to the shortage quantity, it may outsource an additional quantity.

3. MATHEMATICAL MODEL FORMULATION

In proposed model, the total expected cost is including costs of production, holding, maintenance, shortage and outsourcing.

3.1 Holding cost

The total holding cost $IC$ includes the inventory cost of each period. It is given by the following expression:

$$IC = \sum_{k=1}^{N} IC_k = C_h \sum_{k=1}^{N} Z(k)$$

$Z(k)$ Is the area generated by the inventory level evolution during the period $(k = 1, ..., N)$. The generated area during a period $k$ is given as follows:

$$Z(k) = u_m k (\Delta r_k)^2 / 2 + \Delta r_k S_{k-1}. $$

The evolution of the inventory level can be expressed as:

$$S_k = S_{k-1} + \left( u_m k + u_{s,l,k} \right) \Delta r_k - d_k.$$  

Where $S_0 = 0$.

Thus:

$$IC = C_h \sum_{k=1}^{N} u_m k (\Delta r_k)^2 / 2 + \Delta r_k S_{k-1} - d_k.$$  

3.2 Production cost

The production unit produces with variable production rates for each production period. Thus, the total production cost is given by the following expression:

$$PC = \sum_{k=1}^{N} PC_k = C_p \sum_{k=1}^{N} u_m k \Delta r_k$$

3.3 Shortage cost

When the selected production and outsourcing rates do not meet the demand, a shortage cost is generated. It’s expressed by the following formula:

$$SC = C_s \sum_{k=1}^{N} \max \{0, d_k - (S_{k-1} + \left( u_m k + u_{s,l,k} \right) \Delta r_k)\}$$

3.4 Outsourcing cost

We consider dependence between the outsourcing price and outsourced production rate, such as the contractor offers multiple pricing schedules. The manufacturer is always looking to outsource with a minimal cost, especially if there will be enough time. However, in some cases, it is too expensive to outsource a great quantity within during very short period.

Let be a pricing model with $L$ schedule $\{C_{o,l, u_{s,l,k}}\}$, $l = 1, 2, ..., L$. It consists of outsourcing price $C_{o,l}$ and the corresponding common/group production rate $u_{s,l,k}$. The following formula is the general form of the price schedule offered by the contractor. This schedule is based on the outsourced capacity requested by the manufacturer.

$$C_o = \begin{cases} C_{o,1} \quad \text{if} \quad u_{s}^{\min} < u_{s,l,k} \leq u_1 \\ \vdots \\ C_{o,L} \quad \text{if} \quad u_{l-1} < u_{s,l,k} \leq u_l \\ \vdots \\ C_{o,L} \quad \text{if} \quad u_{l-1} < u_{s,l,k} \leq u_{s}^{\max} \end{cases}$$

Where $C_{o,1} < \cdots < C_{o,l} < \cdots < C_{o,L}$

And $u_s^{\min} < \cdots < u_{l-1} < u_1 < \cdots < u_L < u_s^{\max}$

The total outsourcing cost $OC$ can be written as follows:

$$OC = \sum_{k=1}^{N} OC_k = \sum_{k=1}^{N} \Delta r_k u_{s,l,k}$$

3.5 Maintenance cost

The expected maintenance cost is given by the following expression:

$$MC = C_m M(H) + N.C_m$$

Where $M(H)$ is the average number of failures during the plan horizon $H$. $N$ is the number of preventive maintenance.

$$M(H) = \sum_{k=1}^{N} \int_{0}^{\Delta r_k} \lambda_{\max}(t) dt$$

Thus,

$$MC = N.C_m + C_r \sum_{k=1}^{N} \left( \frac{u_{m,k}}{U_{m}^{\max}} \right) \int_{0}^{\Delta r_k} \lambda_{\max}(t) dt$$

The total cost can be formulated by the following expression:

$$\begin{array}{ll}
\text{Min} C & = C_s \sum_{k=1}^{N} \max \{0, d_k - (S_{k-1} + \left( u_{m,k} + u_{s,l,k} \right) \Delta r_k)\} + C_p \sum_{k=1}^{N} u_{m,k} \Delta r_k + C_o \sum_{k=1}^{N} \Delta r_k u_{s,l,k} + C_s \sum_{k=1}^{N} \max \{0, d_k - (S_{k-1} + \left( u_{m,k} + u_{s,l,k} \right) \Delta r_k)\} + C_o \sum_{k=1}^{N} \Delta r_k u_{s,l,k} \\
\end{array}$$

Subject to:

$$S_k = S_{k-1} + \left( u_{m,k} + u_{s,l,k} \right) \Delta r_k - d_k$$

$$S_{k-1} + \Delta r_k \left( u_{m,k} + u_{s,l,k} \right) \geq d_k$$

$$0 \leq u_{m,k} \leq u_{m}^{\max}$$

$$u_s^{\min} \leq u_{s,l,k} \leq u_s^{\max}$$

$$u_{m,k} u_{s,l,k} \geq 0$$

$\forall k = 1, ..., N$ and $l = 1, ..., L$
holding cost, maintenance cost, shortage cost and outsourcing cost. The constraint (15) shows that the stock level at the beginning of each period $k$ equals in the stock of previous production period $k - 1$ minus the demand for this period. The Constraints (16) ensure that the manufacturer’s capacity, plus the capacity allocated by the contractor must meet all demands. The manufacturer’s machine can operate with a production rate that depends on outsourcing options. The production rate of manufacturer is required within the lower and upper bounds by constraints (17). The constraints (18) show that the contractor can use his machine with a production rate included between minimal and maximal production rate. Finally, the constraints (19) represent non negativity constraints.

4. SOLUTION PROCEDURE

We recall that our objective is to determine for each period, the optimal mixed plan of in-house production and the outsourcing, i.e. we determine the rates that should be adopted in order to satisfy the combinations of demand: 

\[(\text{requested demand, delivery time})\]. The optimization criterion is the minimization of the total cost over the plan horizon $H$. It is difficult to solve the above mentioned model by using the exact optimization methods. The complexity also increases with the increase of the number of periods or pricing schedules. In such cases, heuristic or evolutionary algorithms have been widely used by many researchers – as these algorithms are found to be more efficient in generating optimal/near-optimal solution for complex problems with less computational time (Liu et al., 2008 and Sinha and Sarmah, 2010).

Accordingly, we have applied Genetic Algorithm “GA, evolutionary computation techniques to solve efficiently the proposed mathematical programming model.

The Fig. 3. illustrates the chromosome structure:

\[
\begin{align*}
\text{In-house production rates} & : u_{m,1} \ldots u_{m,k} \ldots u_{m,N} \\
\text{Outsourcing rates} & : u_{s,1} \ldots u_{s,k} \ldots u_{s,N}
\end{align*}
\]

Fig. 3. Chromosome structure.

Each gene of the first line of chromosome represents the in-house production rate $u_{m,k}$, which is assigned to a period $k$ of the manufacturer plan ($k = 1, \ldots, N$). In second line of the chromosome, the genes represent the outsourcing rates $u_{s,k}$ which should be allocated by the contractor company for each period.

5. NUMERICAL EXAMPLE

In this section, we provide a numerical example to illustrate our results. Let’s consider a manufacturer and a contractor described as follows:

The manufacturer company receives the demands at the beginning of production plan horizon. Each component $(d_k, r_k)$ of this combination contains the demand quantity and its delivery time.

We consider for the manufacturer company a nominal failure probability density function of machine $M_s f(t) =$ Weibull$(2,100)$ when the machine operates with the maximum production rate $U_{max} = 3$ units/ t.u.

Thus, the nominal failure rate function is:

\[
\lambda_{max}(t) = (2/100)(t/100)
\]

The rest of parameters represent the unit costs of the manufacturer where:

\[
C_p = 2 \text{ m.u/unit}; C_s = 0.18 \text{ m.u/unit/t.u}; C_s = 22 \text{ m.u/unit}; C_m = 22 \text{ m.u}; C_r = 40 \text{ m.u.}
\]

Such as t.u: time unit, m.u: monetary unit.

In the other side, the contractor offers multiple pricing schedules. Here, we consider a pricing model with four schedules $C_{o.1} = 7$ m.u. if $0 < u_{s,1,k} \leq 1$; $C_{o.2} = 16$ m.u. if $1 < u_{s,2,k} \leq 2$; $C_{o.3} = 35.5$ m.u. if $2 < u_{s,3,k} \leq 3$ and $C_{o.4} = 45$ m.u. if $3 < u_{s,4,k} \leq 4$. Where $U_{s} = 4$ units/t.u.

Using the procedure described above, we obtain the optimal results presented in Table 2:

<table>
<thead>
<tr>
<th>Period</th>
<th>In-house production rates</th>
<th>Outsourcing rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3,0000</td>
<td>2,9254</td>
</tr>
<tr>
<td>2</td>
<td>3,0000</td>
<td>4,0000</td>
</tr>
<tr>
<td>3</td>
<td>1,9502</td>
<td>2,6756</td>
</tr>
<tr>
<td>4</td>
<td>2,4224</td>
<td>1,5602</td>
</tr>
<tr>
<td>5</td>
<td>3,0000</td>
<td>3,9944</td>
</tr>
<tr>
<td>6</td>
<td>0,8199</td>
<td>0,8199</td>
</tr>
<tr>
<td>7</td>
<td>2,4598</td>
<td>2,4598</td>
</tr>
<tr>
<td>8</td>
<td>1,9825</td>
<td>3,1295</td>
</tr>
<tr>
<td>9</td>
<td>3,0000</td>
<td>2,0000</td>
</tr>
<tr>
<td>10</td>
<td>3,0000</td>
<td>2,0625</td>
</tr>
</tbody>
</table>

In table 2, the second column represents the in-house production plan; each element of this table is the optimal production rate to be adopted by the manufacturer for each period. Similarly, the second column represents the outsourcing plan to be adopted by the contractor. The set of two plans, called the mixed plan production/outsourcing generates a minimal cost to the manufacturer company equal to 9238,7842 m.u.

6. CONCLUSIONS AND FUTURE DIRECTION RESEARCHS

In this paper, we have studied a problem of mixture of the in-house production and outsourcing under a combined approach of maintenance management and production control.

Table 1. Input data vector

<table>
<thead>
<tr>
<th>k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_k$</td>
<td>70</td>
<td>54</td>
<td>30</td>
<td>62</td>
<td>58</td>
<td>40</td>
<td>105</td>
<td>70</td>
<td>70</td>
<td>81</td>
</tr>
<tr>
<td>$r_k$</td>
<td>15</td>
<td>22</td>
<td>32</td>
<td>40</td>
<td>60</td>
<td>65</td>
<td>80</td>
<td>90</td>
<td>104</td>
<td>120</td>
</tr>
</tbody>
</table>
in multi-period setting. The considered manufacturing system consists of a single randomly failing and repairable machine producing a single product. In order to meet the market demand the manufacturer decides to outsource all of a part of demand to a contractor company. The contractor produces by offering outsourcing services to the manufacturer under a multiple pricing schedules.

A mathematical model and genetic optimization algorithm have been implemented in order to determine simultaneously the optimal in-house production and outsourcing plans for the manufacturer while minimizing the total costs over the plan horizon.

However, this research may be further extended by taking into account the cooperation with the contractor under stochastic demands.

REFERENCES