Development of joint maintenance and production strategies in a subcontracting environment

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This article, inspired by an industrial problem, develops efficient maintenance and just-in-time production policies in a subcontracting environment according to two orientations. The first invokes subcontracting with the objective of satisfying a constant customer demand knowing that our production system, composed of a machine $M_1$, cannot satisfy the totality of demand. Subcontracting is represented by a machine $M_2$ which has a constant failure rate, while three maintenance policies for $M_1$ are tested and evaluated. The second orientation takes the perspective of our production system as a supplier which is obliged to allocate part of its production capacity to subcontracting so as to satisfy a constant demand. We consider a production system made up of two machines, both of which produce a single type of product, are subject to breakdowns and can carry out subcontracting tasks. The objective of this part of the article is to prove the efficiency of the so-called integrated maintenance policy, which combines production and maintenance decisions in a subcontracting environment.

Keywords: preventive maintenance, subcontracting, production management, simulation, experimental design

1. Introduction

The performance of manufacturing systems is strongly influenced by machine breakdowns together with the maintenance and production policies in force. The traditional approach, such as just-in-time, which dissociates the maintenance and production decisions, is limited to dealing with situations in which the system is permanently available.

This approach started in studies which considered the use of a buffer stock in a production system without preventive maintenance. For instance, Buzacott (1967), using Markov chains, noted that a buffer stock increases the productivity of the system. In the same spirit, Conway et al. (1988) analysed by simulation the number of units produced relative to the size of the buffer stock.

Buzacott and Shanthikumar (1993) proved the importance of the choice of the maintenance policy for the minimisation of the total cost. Van der Dyun Schouten and Vanneste (1995) proposed a preventive maintenance policy based on the age of the machine and the storage capacity for a production line made up of two machines separated by a buffer stock. Meller and Kim (1996) studied the impact of preventive maintenance

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On another front, the practice of subcontracting has become widespread in production management, which Bertrand and Sridharan (2001) justify by the lack of adequate internal manufacturing technology or by the inability to deliver the requested quantities within the specified deadlines. Liang and May (2006) describe subcontracting as an increasingly popular method to organise production so as to increase competitiveness and reduce production costs.

The above-mentioned advantages of subcontracting were summarised by Bradley (2005) as follows: reduced operating costs, increased responsiveness to peak demand, deferral or avoidance of capital expenditures and reduced inventory.

In this study we will develop joint maintenance and production strategies in a subcontracting environment. Indeed, the work presented in this paper is undertaken according to two orientations. The first relates to the use of subcontracting for the purpose of satisfying customer demand knowing that our production system cannot satisfy the totality of the demand. The second orientation takes the point of view that our production system is a supplier which subcontracts part of its capacity to an external customer.

The remainder of the paper is organised as follows. Section 2 formally defines the problems to be tackled. In Section 3 we study the first problem, that is, that which makes use of the subcontractor. We define a simple maintenance policy (SMP), which is considered as a reference, following which we define two additional maintenance policies, namely, the improved maintenance policy (IMP) and the production maintenance policy (PMP). We analyse and compare the performance of these policies.

Section 4 deals with the second problem, that which considers our production system to be the supplier of subcontracting services. The proposed policies, the simple maintenance policy (SiMP) and the integrated maintenance policy (InMP), are analysed by simulation via a numerical example. We provide numerical evidence as to the higher performance of InMP compared to SiMP. The conclusion is presented in Section 5.

2. Problem definition
The manufacturing system under consideration in the first part of this study is composed of a machine $M_1$ which produces a single product. To satisfy a constant demand $d$ exceeding the maximal production capacity of $M_1$, $U_{\text{max}}$, the system uses a subcontractor
composed of a machine $M_2$ which has a maximal production rate $U_{\text{max}}^2$. The use of two machines is justified by the fact that $U_{\text{max}}^1 < d$, $U_{\text{max}}^2 < d$ and $U_{\text{max}}^1 + U_{\text{max}}^2 > d$.

Both machines are subject to random failures. Being outside our own manufacturing system, and hence little reliability information being known about it, we consider that $M_2$ has a constant failure rate and that its failures cannot be prevented by preventive maintenance. By contrast, machine $M_1$ has a failure rate that is increasing with its age and its failures can be prevented by preventive maintenance actions. An age-limit policy is used for preventive maintenance planning and machine $M_1$ stops for preventive maintenance when it reaches a given age $m$. In the maintenance periods of the two machines, the production capacity decreases and the two machines become unable to satisfy the demand. In order to minimise the resulting product shortage we suggest two different strategies.

The first strategy is called the improved maintenance policy (IMP) and entails producing, in a just-in-time fashion, schedules preventive maintenance actions according to both the history of $M_1$ and the state of $M_2$. The second strategy, called production maintenance policy (PMP), consists of building up a safety stock level $h$ in order to satisfy the demand occurring during the maintenance period.

In the second part of this article, our system is composed of two machines producing one type of product to satisfy a constant demand $d$. The use of two machines is justified by the fact that $U_{\text{max}}^1 < d$, $U_{\text{max}}^2 < d$ and $U_{\text{max}}^1 + U_{\text{max}}^2 > d$, such that machine $M_1$ functions at a single production rate $U_{\text{max}}^1$ while machine $M_2$ can function at a variable rate $0 \leq U_2 \leq U_{\text{max}}^2$. Moreover, the production cost of $M_2$ is higher than that of $M_1$. To minimise the use of machine $M_2$ and to reduce the production costs, it would be interesting to control the rate $U_2$ of $M_2$ according to rate $U_1$ of $M_1$.

Machines $M_1$ and $M_2$ can be allocated to periodically carry out subcontracting tasks for a fixed duration. To decrease the amount of lost demand due to machine breakdowns and subcontracting, a safety stock with a fixed capacity will be used. The objective of this part of our work is to study the effectiveness of an integrated maintenance policy (InMP), which combines maintenance and production management decisions, compared to a simple maintenance policy (SiMP), which considers them individually.

3. The joint use of internal manufacturing and subcontracting to fulfil product demand

3.1 Problem statement

The manufacturing system under consideration consists of a machine $M_1$ which produces a single product whose demand $d$ is constant. Since the maximal production rate of machine $M_1$ is less than the demand rate, the system calls upon another machine $M_2$, the so-called subcontractor machine, in order to fulfil the remainder of the demand (Figure 1).

From an availability point of view, machine $M_1$ has three states: running, in repair and in preventive maintenance. Machine $M_2$ has only two states: running and in repair.

The degradation law of machine $M_1$ is described by the probability density function of time to failure $f_1(t)$ whose failure rate $\lambda_1(t)$ increases with age. Failures of machine $M_1$ can be prevented by preventive maintenance actions which are scheduled according to its history. We suppose that the degradation law of the subcontractor machine $M_2$ is described by the probability density function of time to failure $f_2(t)$ whose failure rate is constant.

Next, we define the two production and maintenance policies, SMP and IMP.

Let $U(t) = (U_1(t), U_2(t))$ be the production rate of the system at time $t$. The production control policy is defined as follows, under the constraints $U_{\text{max}}^1 < d$, $U_{\text{max}}^2 < d$ and

$$U_{\text{max}}^1 + U_{\text{max}}^2 > d.$$
$U_{\text{max}}^1 + U_{\text{max}}^2 > d$

$$U(t) = (u_1(t), u_2(t)) = \begin{cases} (U_{\text{max}}^1, d - U_{\text{max}}^1) & \text{if } M_1 \text{ is up and } M_2 \text{ is up}, \\ (U_{\text{max}}^1, 0) & \text{if } M_1 \text{ is up and } M_2 \text{ is down}, \\ (0, U_{\text{max}}^2) & \text{if } M_1 \text{ is down and } M_2 \text{ is up}. \end{cases}$$

### 3.2 Notation and data for numerical example

The following notation will be used throughout the paper:

- $C_{mp}$: Preventive maintenance cost of machine $M_1$.
- $C_{mc1}$: Corrective maintenance cost of machine $M_1$.
- $\xi^k_1$: Time of the $k$th failure occurrence of machine $M_1$.
- $\xi^k_2$: Time of the $k'$th failure occurrence of machine $M_2$.
- $m$: Age-limit for preventive maintenance of machine $M_1$.
- $z^k_p$: Time needed for preventive maintenance on machine $M_1$ in cycle $T_k$.
- $z^k_{c1}$: Time needed for corrective maintenance on machine $M_1$ in cycle $T_k$.
- $z^k_{c2}$: Time needed for corrective maintenance on machine $M_2$ upon its $k'$th failure.
- $\mu_p$: Mean time for preventive maintenance on machine $M_1$.
- $\mu_{ci}$: Mean time for corrective maintenance on machine $M_i$, ($i = 1, 2$).
- $C_p$: Unit loss cost of demand.
- $U_{\text{max}}^i$: The maximal production rate of machine $M_i$, ($i = 1, 2$).
- $d$: Demand.
- $tu$, $mu$: Time unit, monetary unit.

Table 1 gives the machine data for the numerical example at hand. The demand is defined by $d = (Q_d, T_d) = (30, 1)$ with $Q_d$ being the demand quantity and $T_d$ the demand frequency. The unit loss cost is equal to $C_p = 250\text{ um}$. The preventive maintenance age $m$ is one of the decision variables.

### 3.3 Simple maintenance policy (SMP)

This policy schedules the preventive maintenance of machine $M_1$ without taking into account the state of the subcontractor machine $M_2$ and only according to the history of $M_1$, denoted $H^1$ (Figures 2 and 3). Formally, we can define the simple maintenance policy.
policy, SMP, as follows:

\[ \delta_m = \begin{cases} 
(\text{To perform the preventive maintenance of machine } M_1 \text{ at the age } m) \\
\text{or} \\
(\text{To perform a corrective maintenance if the machine } M_1 \text{ fails before the age } m(\xi_1^e \leq m))
\end{cases} \]

We denote \( C_t(SMP) \) the total average cost of maintenance and demand loss of this simple maintenance policy. The latter will be compared with the two policies IMP and

<table>
<thead>
<tr>
<th>Machine</th>
<th>Law of failures</th>
<th>MTBF</th>
<th>Maximal production/tu</th>
<th>( \mu_{ci} ) (tu)</th>
<th>( \mu_p ) (tu)</th>
<th>( C_{mi} ) (mu)</th>
<th>( C_{mp} ) (mu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>( W(2,100) )</td>
<td>88.6</td>
<td>20</td>
<td>( EXP(30) )</td>
<td>( EXP(20) )</td>
<td>2000</td>
<td>500</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>( EXP(50) )</td>
<td>50.0</td>
<td>20</td>
<td>( EXP(10) )</td>
<td>–</td>
<td>2000</td>
<td>–</td>
</tr>
</tbody>
</table>

Figure 2. Scenario 1 in SMP.

Figure 3. Scenario 2 in SMP.
PMP, defined hereafter. Looking at the coinciding maintenance age $m$ of the $M_1$ machine and the downtime of machine $M_2$ we have two possible scenarios.

### 3.4 Improved maintenance policy (IMP)

This policy schedules preventive maintenance actions according to both the history of $M_1$ and the state of the subcontractor machine $M_2$. First, a preventive maintenance is tentatively scheduled at age $m$ of machine $M_1$. When machine $M_1$ reaches age $m$, the scheduled preventive maintenance action takes place if the subcontractor machine $M_2$ is up and it is postponed to age $m + \Delta m$ if the subcontractor machine $M_2$ is down. Formally, we can define an improved maintenance policy, IMP, as follows:

$$
\delta_m = \begin{cases} 
(\text{To perform the preventive maintenance of machine } M_1 \text{ at age } m') & \\
(\text{To perform a corrective maintenance if the machine } M_1 \text{ fails before age } m' (\xi_{1i} \leq m')) & 
\end{cases}
$$

where

$$
 m' = \begin{cases} 
m & \text{if } m \notin \left\lfloor \xi_2^k, \xi_2^{k'} + z_{c2} \right\rfloor \\
 m + \Delta m & \text{if } m \in \left\lfloor \xi_2^k, \xi_2^{k'} + z_{c2} \right\rfloor.
\end{cases}
$$

We have two possible scenarios in this policy, as illustrated in Figures 4 and 5 below.

![Figure 4. Scenario 1 in IMP.](image1)

![Figure 5. Scenario 2 in IMP.](image2)
We denote $C_t$ (IMP) the total average cost of maintenance and demand loss of this improved maintenance policy. The aim of the IMP is to minimise the average total cost of maintenance and demand loss by delaying the preventive maintenance action on $M_1$ if the subcontractor machine $M_2$ is in the repair state, thereby making it possible to continue the production and satisfy more demand since the unfulfilled demand will be reduced to $d - U_{\max}^j$ instead of $d$.

We recall Theorems 1 and 2, which were proved in Dellagi et al. (2007).

**Theorem 1:** $C_t$(IMP) $- C_t$(SMP) $\leq 0$ if the following conditions are all satisfied:

- (C1) : $E(\xi^f_2) + \mu_2 - \mu_p \leq m \leq E(\xi^f_2) + \mu_2$
- (C2) : $C_{mc1} - C_{mp} + C_p d(\mu_{c1} - \mu_p) \leq C_p U_{\max}^2 (E(\xi^f_2) + \mu_2 + \mu_{c1} - m - \mu_p)$
- (C3) : $C_{mc1} - C_{mp} + C_p d(\mu_{c1} - \mu_p) \leq C_p U_{\max}^2 (E(\xi^f_2) + \mu_{c1} - m - \mu_p)$

**Theorem 2:** $\forall m$ satisfying conditions (C1)–(C3) in Theorem 1, there exists a $\Delta m^*$ for which IMP admits a maximal performance measure $|(C_t$(IMP) $- C_t$(SMP))($\Delta m^*$)|.

From this analytical study we conclude that if the four conditions cited in Theorem 1 are satisfied, the IMP is more economical than the SMP for all delay periods $\Delta m$. Furthermore, we note in Theorem 2 that there exists an optimal delay period $\Delta m^*$ for which the performance $H$ of the IMP compared to SMP is maximal. For the numerical data given in Subsection 3.2, Theorem 1 yields:

For $m \in [40, 54.7]$ $\Rightarrow$ $H \leq 0$ $\Rightarrow$ $PC$(IMP) $- PC$(SMP) $\leq 0$ $\forall \Delta m$.

Assuming that the delay period $\Delta m$ is inferior or equal to the age $m$, and taking into account the above result from Theorem 1, problem $P(1)$ can be stated as follows:

For $m \in [40, 54.7], m^*$?

For $\Delta m \in [0, m], \Delta m^*$?

$\Rightarrow$ $CT$(IMP) is minimal $P(1)$

As will be demonstrated later on, the complexity of the analytical model led us to opt for a simulation methodology to solve $P(1)$.

### 3.5 Production maintenance policy (PMP)

#### 3.5.1 Description of PMP

In order to decrease the demand loss caused by the maintenance actions on both machine $M_1$ and the subcontractor machine $M_2$, we present the PMP, illustrated in Figure 6. This policy consists of building a safety stock $S(t)$, whose maximal level is $h$.

![Figure 6. Machine $M_1$ and the subcontractor machine $M_2$ (Policy PMP).](image-url)
The two machines produce at their maximal pace until a given safety stock level \( h \) is reached. When the latter occurs, the two machines produce just enough to satisfy the exact demand rate. Let \( U(t) = (U_1(t), U_2(t)) \) be the controlled production rate of the system at time \( t \). The production control policy is thus as specified below, under the constraints \( U_{\text{max}}^1 < d, U_{\text{max}}^2 < d \) and \( U_{\text{max}}^1 + U_{\text{max}}^2 > d \):

\[
U(t) = (u_1(t), u_2(t)) = \begin{cases} 
(U_{\text{max}}^1, U_{\text{max}}^2) & \text{if } M_1 \text{ is up, } M_2 \text{ is up and } S(t) < h \\
(U_{\text{max}}^1, d - U_{\text{max}}^1) & \text{if } M_1 \text{ is up, } M_2 \text{ is up and } S(t) = h \\
(U_{\text{max}}^1, 0) & \text{if } M_1 \text{ is up and } M_2 \text{ is down,} \\
(0, U_{\text{max}}^2) & \text{if } M_1 \text{ is down and } M_2 \text{ is up,} 
\end{cases}
\]

### 3.5.2 An analytical model of the PMP

In the maintenance periods the machines are unable to satisfy the demand; therefore, the demand will be filled from the safety stock, provided it is sufficiently large. To describe the various possible production cycles we introduce the following definitions:

- \( W_{K}^{\text{1(c)}} \) Uptime of machine \( M_1 \) before failing in cycle \( K \).
- \( W_{K}^{\text{1(p)}} \) Uptime of machine \( M_1 \) before the preventive maintenance actions in cycle \( K \).
- \( M \) Preventive maintenance age of machine \( M_1 \).
- \( W_{K'}^{2} \) Uptime of machine \( M_2 \) before failing in cycle \( K' \).

At the beginning of the cycle the two machines produce at their maximal rate

\[
(U_{\text{max}}^1 + U_{\text{max}}^2 - d)
\]

until the maximal safety stock level \( h \) is attained. The end of the cycle is characterised by the end of the maintenance actions applied on machines \( M_1 \) and \( M_2 \), at which point the two machines are assumed to be as good as new.

It is further assumed that the two machines do not fail before the safety level stock \( h \) is attained, that is,

\[
h \leq (U_{\text{max}}^1 + U_{\text{max}}^2 - d) \times \left( \min \left( W_{K}^{\text{1(c)}}, W_{K}^{\text{1(p)}}, W_{K'}^{2} \right) \right).
\]

Observe that the maintenance periods of the two machines can either coincide or be separated. From these possibilities we can classify the scenarios into four types presented in Tables 2(a) and 2(b).

We conclude that we have 64 different possible scenarios.

Recall that the goal of the PMP is to reduce the demand loss during the maintenance periods of the two machines. In order to estimate the gain obtained with this policy we must minimise the average total cost of maintenance, inventory and demand loss. The average total cost of PMP is denoted \( Ct(PMP) \). The decision variables are the maximal safety stock level \( h \) and the preventive maintenance age \( m \). The other problem data were given in Subsection 3.2. The problem is to determine the optimal preventive maintenance age \( m^* \) and optimal stock level \( h^* \) minimising the average total cost \( Ct(PMP) \), as formulated below.

\[
Ct(m, h) = \frac{C_{\text{mc1}} \times F_1(m) + C_{\text{mp}} \times R_1(m) + \delta(h, m)}{E(\text{cycle length})}
\]
Table 2(a). Possible scenarios with no demand loss.

<table>
<thead>
<tr>
<th>Separate maintenance periods of $M_1$ and $M_2$</th>
<th>Coinciding maintenance periods of $M_1$ and $M_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No demand loss</strong></td>
<td><strong>Type A:</strong> In this type the maintenance periods of the two machines do not coincide and we do not have a demand loss during these maintenance periods. In this case we noted that once we reach the first maintenance period the stock level $h$ decreases. Between the end of this first maintenance period and the beginning of the second maintenance period the stock level $h$ can be rebuilt. Therefore, we have two different cases in the evolution of the stock level $S(t)$. Due to the maintenance actions possible for the machine $M_1$ and the possibility of the first machine stopped for maintenance action, we have eight possible scenarios.</td>
</tr>
<tr>
<td><strong>Type B:</strong> In this type the two maintenance periods of machines $M_1$ and $M_2$ coincide. During the maintenance period of the first machine which is stopped, the maintenance period of the other machine begins. In addition there is no demand loss during these maintenance periods and we noted the existence of two cases. In the first case, the maintenance action of the first stopped machine was finished before the end of the maintenance action of the second stopped machine. In the second case, the maintenance action of the first machine stopped was finished after the end of the maintenance action of the second machine stopped. Due to the maintenance actions possible for machine $M_1$, the possibility of the first machine stopped for maintenance action, and the first machine restored after the maintenance actions, in this type we have eight possible scenarios.</td>
<td></td>
</tr>
</tbody>
</table>

Table 2(b). Possible scenarios with demand loss.

<table>
<thead>
<tr>
<th>Separate maintenance periods of $M_1$ and $M_2$</th>
<th>Coinciding maintenance periods of $M_1$ and $M_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>With demand loss</strong></td>
<td><strong>Type C:</strong> In this type the maintenance periods of the two machines do not coincide and we have a demand loss during these maintenance periods. Between the end of the maintenance period of the first machine stopped and the beginning of the second machine stopped, both machines produced at their maximal rate in order to rebuild the safety stock level $h$. We noted at this point that level $h$ cannot be reached if the period separating the two machine maintenance actions is small. In addition the demand loss in this type of cycle can be caused by the maintenance action on machine $M_1$ or machine $M_2$ or both. From these various possibilities in this type we have 24 possible scenarios.</td>
</tr>
<tr>
<td><strong>Type D:</strong> In this type the two maintenance periods of the two machines coincide. During the maintenance period of the first machine stopped, the maintenance period of the other machine begins. In addition there is a demand loss during these periods. The demand loss can be started in the maintenance period of the first machine stopped for maintenance or during the simultaneous maintenance periods of the two machines or after one of the two machines is fixed. From the various possibilities, in this type we have 24 scenarios.</td>
<td></td>
</tr>
</tbody>
</table>
where

\[
\delta(m,h) = C_s \sum_{i=1}^{N_{sp}} P_{spi} E(S_{spi}) + \sum_{j=1}^{N_{ap}} P_{api} ((C_s \times E(S_{api})) + (C_p \times E(dp_{api})))
\]  \tag{2}

\[
E(\text{cycle length}) = \begin{cases}
(E(W_{k,c}^{k(c)})) + \mu_{c1} \times P(\xi_{1}^{k(c)} < m) \\
(E(W_{1}^{k(p)})) + \mu_{p} \times P(m < \xi_{1}^{k}) \\
(E(W_{2}^{k}) + \mu_{c2} \times P(\xi^{k(c)} < m) \times P(W_{2}^{k} + D_{2}^{k} > W_{1}^{k(c)} + D_{1}^{k(c)})
\end{cases}
\]  \tag{3}

\(C_s, C_p\) Unit storage cost, unit loss cost, \(r = C_p/C_s\)

\(N_{sp}\) Number of scenarios without demand loss = 16.

\(N_{ap}\) Number of scenarios with demand loss = 48.

\(P_{spi}\) Probability for every scenario without demand loss.

\(P_{api}\) Probability of every scenario with demand loss.

\(S_{spi}\) The storage area for every scenario without demand loss.

\(S_{api}\) The storage area for every scenario with demand loss.

\(dp_{api}\) The demand loss for every scenarios with demand loss.

In light of the 64 possible scenarios, together with the complexity of the analytical expression of \(C_t(PMP)\), we will use simulation in order to find \(m^*\) and \(h^*\).

3.6 Performance analysis of SMP, IMP and PMP

We developed a simulation model in order to calculate the average cost incorporating maintenance and demand loss for the three policies SMP, IMP and PMP. Formally, the simulation approach is described as shown in Figure 7.

The logic of the simulation model is based on the discrete event concept and is described in detail in Appendix I.

Remark: \(C_t(SMP)\) is obtained from IMP with \(\Delta m = 0\).

3.6.1 The performance of IMP compared to SMP

Using problem \(P(1)\) formulated in Subsection 3.4, we plot the curves of the average cost as a function of \(\Delta m\), with a fixed preventive maintenance age \(m \in [40, 54.7]\). From these
curves we note that the optimal average cost is obtained for $\Delta m \in [0, 16]$. We can thus infer that $(m, \Delta m) \in ([40, 54], [0, 16])$.

The correspondence between the levels used in the experimental design and the real values is given in Table 3.

We want to express $C_t(IMP)$ as a function of the decision variables $m$ and $\Delta m$:

$$C_t(IMP) = a_0 + a_1 X_m + a_2 X_{\Delta m} + a_3 (X_m)^2 + a_4 (X_{\Delta m})^2 + a_5 X_m X_{\Delta m}$$

By multiple linear regression we obtain:

$$C_t(IMP) = 1359.4 - 65.12 X_m - 22.24 X_{\Delta m} + 12.02 (X_m)^2 + 32.02 (X_{\Delta m})^2 + 14.48 X_m X_{\Delta m}$$

The ANOVA analysis is given in Table 4.

By eliminating the insignificant coefficients we obtain:

$$C_t(IMP) = 1359.4 - 65.12 X_m - 22.24 X_{\Delta m} + 32.02 (X_{\Delta m})^2$$

$$C_t(IMP) = 1833.03 - 8.84 m + 10.78 \Delta m + 0.5 \Delta m^2$$ (4)

From Equation (4) we plotted $C_t(IMP)$ as a function of $m$ and $\Delta m$, as illustrated in Figure 8.

From Equation (4) it is clear that the minimum value of $C_t(IMP)$ is obtained for $m = 54.7$. The minimum cost is obtained by deriving $C_t(IMP)$ with respect to $\Delta m$:

$$\left. \frac{d(C_t(IMP))}{d\Delta m} \right|_{\Delta m^*} = 0 \Rightarrow \Delta m^* = 10.78$$

We conclude that $m^* = 54.7$, $\Delta m^* = 10.75$, and $C_t^*(IMP) = 1291.37 \text{um/ut.}$
Applying simulation \((5 \times 1,000,000 \text{ hours})\) for \(m^* = 54.7\) and \(\Delta m^* = 10.75\), we get \(C_t^*(IMP)_{\text{sim}} = 1298.16 \text{ um/ut}\).

Moreover, the average cost of the SMP at \(m = 54.7\) is \(C_t(SMP)_{\text{sim}} = 1343.09 \text{ um/ut}\). Since that the optimal gain \(G_{\text{IMP}}\) of the IMP compared to the SMP is hence:

\[
G_{\text{IMP}} = \frac{C_t(SMP)_{\text{sim}} - C_t^*(IMP)_{\text{sim}}}{C_t(SMP)_{\text{sim}}} = 3.34\%
\]

### 3.6.2 The performance of PMP compared to SMP and IMP

The problem is to determine the optimal preventive maintenance age \(m^*\) and stock level \(h^*\) which minimise \(C_t(PMP)\). Let \(r = 1500\). In order to compare PMP with IMP and SMP, for the preventive maintenance \(m\) age we will consider the analytic results established in Section 3.4. Specifically, for a given \(m \in [40, 54.7]\) we plotted \(C_t(PMP)\) as a function of \(h\). From the convex curves obtained we noted that \(h^* \in [5, 19]\). Formally, the problem to be solved is:

\[
\begin{align*}
\text{For} & \quad m \in [40, 54.7], m^*? \\
\text{For} & \quad h \in [5, 19], h^*? \\
\Rightarrow & \quad C_t(PMP) \text{ is minimal} \quad P(2)
\end{align*}
\]

We use the following experimental design based on \(P(2)\) (Table 5).

Consequently, with simulation and multiple linear regression we get:

\[
C_t(PMP) = 1165.04 - 53.47X_m - 19.94X_h + 12.75X_m^2 + 42.25X_h^2 + 0.74X_mX_h
\]

The ANOVA analysis is given in Table 6.

By eliminating the insignificant coefficients we obtain:

\[
C_t(PMP)_{\text{sim}} = 1165.04 - 53.47X_m - 19.94X_h + 42.25X_h^2
\]

\[
\Rightarrow C_t(PMP)_{\text{sim}} = 1673.62 - 7.27m + 0.9h^2 - 24.51h
\]
From Equation (5) we plotted $C_t(PMP)$ as a function of $m$ and $h$, as illustrated in Figure 9.

We want to minimise the profit $C_t(PMP)$ with respect to $h$ and $m$. From Equation (5) it is clear that the minimum value of $C_t(PMP)$ obtained for $m = 54.7$. Furthermore, the minimum cost as a function of $h$ is obtained as follows:

$$\frac{d(C_t(IMP))}{dh} = 0 \Rightarrow h^* = 13.61$$

We conclude that $m^* = 54.7$, $h^* = 13.61$, and $C_t^*(PMP) = 1109.07$ um/ut.

Applying simulation ($5 \times 1,000,000$ hours) for $m = 54.7$, $h^* = 13.61$ and $r = 1500$ we obtained $C_t^*(PMP)_{sim} = 1115.34$ um/ut. Meantime, the average cost of the SMP at $m = 54.7$ is $C_t(SMP)_{sim} = 1343.09$ um/ut. The optimal gain $G_{PMP/SMP}$ of the PMP compared to the SMP is thus:

$$G_{PMP/SMP} = \frac{C_t(SMP)_{sim} - C_t^*(PMP)_{sim}}{C_t(SMP)_{sim}} = 16.95\%$$

Using the result of Subsection 3.6.2, we determined the optimal gain $G_{PMP/IMP}$ of PMP compared to IMP:

$$G_{PMP/IMP} = \frac{C_t(IMP)_{sim} - C_t^*(PMP)_{sim}}{C_t(IMP)_{sim}} = 14.08\%$$

3.6.3 Analysis of results

From the analysis presented in Subsections 3.6.1 and 3.6.2, we conclude that the PMP is more economical than IMP and SMP. But it’s noted that the results of PMP are obtained with a ratio $r = C_p/C_s = 1500$. From that it is clear that if $r$ decreases, the unit storage cost
Cs increases, the optimal cost \( C^*(PMP) \) increases and the gains \( GPMP/SMP \) and \( GPMP/IMP \) decrease to zero. The gains \( GPMP/SMP \) and \( GPMP/IMP \) become equal to zero for threshold values of \( r \) which are respectively \( r_{th}/SMP = 316.45 \) and \( r_{th}/IMP = 342.46 \). Consequently, if \( r < r_{th}/SMP \), SMP is more economical than PMP. Likewise, if \( r < r_{th}/IMP \), IMP is more economical than PMP. In summary, the evolution of \( GPMP/SMP \) and \( GPMP/IMP \) with respect to \( r \) is illustrated in Figures 10 and 11.

\[ \begin{array}{c|c|c}
\hline
r & r_{th}/SMP = 316.45 & r = 1500 \\
\hline
GPMP/SMP & SMP is more economical than PMP & PMP is more economical than SMP \\
\hline
\end{array} \]

\[ \text{Figure 10. Evolution of } GPMP/SMP \text{ with respect to } r. \]

4. Production control policies under a constraint to perform subcontracting tasks

4.1 Problem statement

In this part of our study, in order to fulfil all the demand, it is necessary to use two machines, \( M_1 \) and \( M_2 \), whose law of degradation respectively follow Weibull and
exponential distributions. Moreover, the unit production cost on $M_2$ is greater than that on $M_1$. Thus, we aim to optimise the production rate of $M_2$, given by

$$U_2(t) = \alpha U_1^{\text{max}},$$

such that

$$0 \leq \alpha \leq \alpha_{\text{max}} = \frac{U_2^{\text{max}}}{U_1^{\text{max}}} \leq 1,$$

where $\alpha$ is called the rating coefficient of $M_2$.

Machines $M_1$ and $M_2$ can be assigned to periodically carry out subcontracting tasks (ST) for a fixed duration. The two maintenance policies which will be compared, i.e. integrated maintenance policy (InMP) and simple maintenance policy (SiMP), are based mainly on the same production control policies, maintenance and assignment of machines to subcontracting tasks.

(i) The production control policy

$$U(t) = (U_1(t), U_2(t))$$

$$= \begin{cases} 
(U_1^{\text{max}}, \alpha U_1^{\text{max}}) & \text{If } M_1 \text{ and } M_2 \text{ are under operating.} \\
(U_1^{\text{max}}, 0) & \text{If } M_1 \text{ is operating, but } M_2 \text{ is down or in ST.} \\
(0, U_2^{\text{max}}) & \text{If } M_2 \text{ is operating, but } M_1 \text{ is down or in ST.}
\end{cases}$$

If, at a moment $t$, the stock level reaches its maximum capacity $h$, it is necessary to switch to a just-in-time production mode. In this case, the total production rate of the system will be given by the following relation, where

$$\alpha = \frac{d - U_1^{\text{max}}}{U_1^{\text{max}}};$$

$$U(t) = (U_1^{\text{max}}, d - U_1^{\text{max}}) = (U_1^{\text{max}}, \alpha U_1^{\text{max}}).$$

In this case, the production control policy incorporating the stock level is defined by:

$$U(t) = (U_1(t), U_2(t))$$

$$= \begin{cases} 
(U_1^{\text{max}}, \alpha U_1^{\text{max}}) & \text{If } M_1 \text{ and } M_2 \text{ are operating and } S(t) < h. \\
(U_1^{\text{max}}, d - U_1^{\text{max}}) & \text{If } M_1 \text{ and } M_2 \text{ are operating and } S(t) = h. \\
(U_1^{\text{max}}, 0) & \text{If } M_1 \text{ is operating, but } M_2 \text{ is down or in ST.} \\
(0, U_2^{\text{max}}) & \text{If } M_2 \text{ is operating, but } M_1 \text{ is down or in ST.}
\end{cases}$$
Let $C_S$ and $C_p$ respectively denote the unit holding cost of inventory and the unit cost of lost demand.

(ii) The maintenance policy

Machine $M_1$ is subjected to preventive maintenance actions every $m$ units of use. These actions are prohibited during periods of subcontracting and each incurs cost $C_{mp}$. The corrective maintenance actions on $M_1$ and $M_2$ each incur a cost $C_{mc}$, such that $C_{mp} << C_{mc}$. In addition, each repair that takes place during a subcontracting period costs $C_{mcs}$, such that $C_{mcs} > C_{mc}$. The durations of the maintenance actions are random.

We define $\eta_i(t)$, indicating the state of the machine $M_i$ at time $t$, as follows:

$$\eta_i(t) = \begin{cases} 
0 & \text{Machine } M_i \text{ is down.} \\
1 & \text{Machine } M_i \text{ is operating with the maximum production rate.} \\
2 & \text{Machine } M_i \text{ is operating in just-in-time.}
\end{cases}$$

(iii) Assignment to subcontracting policy

The policy of assignment of the machines to carry out subcontracting tasks is based on the machines’ age, since a repair during the time a machine is subcontracted more expensive. Functions $\Delta(t)$ and $\Phi(t)$ respectively indicate, at time $t$, the status of the machines as well as which one is subcontracted.

$$\Delta(t) = \begin{cases} 
0 & \text{If two machines } M_1 \text{ and } M_2 \text{ are broken down.} \\
1 & \text{If the machine } M_1 \text{ is the least old.} \\
2 & \text{If the machine } M_2 \text{ is the least old.} \\
0 & \text{If no machine is allocated to ST}
\end{cases}$$

$$\Phi(t) = \begin{cases} 
1 & \text{If machine } M_1 \text{ is allocated to ST.} \\
0 & \text{If machine } M_2 \text{ is allocated to ST.}
\end{cases}$$

The assignment of machines to subcontracting is made periodically each $A_1$ units of time, for a fixed period $A_2$; see Figure 12. If the planned subcontracting period begins at a time when both machines are down, then this subcontracting opportunity is lost at a cost $C_C$. If, on the other hand, exactly one machine is functioning, it will be directly assigned to subcontracting.

Figure 12. Assignment to subcontracting policy.
With regards to machine \( M_1 \), recall that preventive maintenance actions are forbidden while the machine is subcontracted. Nevertheless, we can carry out corrective maintenance. Because the subcontracted machine is broken down, the repair period is excluded from the remaining subcontracting period. However, if preventive maintenance is scheduled on machine \( M_1 \) while it is subcontracted, we defer the preventive maintenance to the completion date of subcontracting, i.e. until \( t + A_1 + A_2 \). According to this strategy, two situations can arise:

**Situation 1:** If \( M_1 \) survives until \( t + A_1 + A_2 \), then the preventive maintenance is carried out at instant \( t = t + A_1 + A_2 \) (Figure 13(a)).

**Situation 2:** If \( M_1 \) breaks down before \( t + A_1 + A_2 \), preventive maintenance will be carried out at the scheduled time following corrective maintenance (i.e. after \( m \) time unit of use) (Figure 13(b)).

Characteristics of the machines are detailed in Table 7.

<table>
<thead>
<tr>
<th>Machines</th>
<th>( f(t) )</th>
<th>( U_{\text{max}} )</th>
<th>( gc(t) )</th>
<th>( gp(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>Weibull (2,100)</td>
<td>20/tu parts</td>
<td>LogNormal (15.2)</td>
<td>LogNormal (4,0.5)</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>Exponential (80)</td>
<td>20/tu parts</td>
<td>LogNormal (9.2)</td>
<td>–</td>
</tr>
</tbody>
</table>

With regards to machine \( M_1 \), recall that preventive maintenance actions are forbidden while the machine is subcontracted. Nevertheless, we can carry out corrective maintenance. Because the subcontracted machine is broken down, the repair period is excluded from the remaining subcontracting period. However, if preventive maintenance is scheduled on machine \( M_1 \) while it is subcontracted, we defer the preventive maintenance to the completion date of subcontracting, i.e. until \( t + A_1 + A_2 \). According to this strategy, two situations can arise:

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**Situation 2:** If \( M_1 \) breaks down before \( t + A_1 + A_2 \), preventive maintenance will be carried out at the scheduled time following corrective maintenance (i.e. after \( m \) time unit of use) (Figure 13(b)).

Characteristics of the machines are detailed in Table 7.

The other parameters are: \( d = 30 \) pieces/tu, \( A_1 = 10 \) tu, \( A_2 = 5 \) tu, \( C_{mc} = 3500 \) mu, \( C_{mp} = 350 \) mu, \( C_{mcs} = 5000 \) mu, \( C_p = 4 \) mu, \( C_s = 2.66 \) mu and \( C_c = 1200 \) mu.

In the following subsections, we define two policies, SiMP and InMP, which we optimise based on a cost minimisation criterion.

### 4.2 Simple maintenance policy (SiMP)

The SiMP policy is founded on the separation of the management of maintenance and production. Consequently, the optimisation of the SiMP rests on the independent
optimisation of the costs pertaining to inventory control and the loss of subcontracting revenue, on the one hand, and the cost of maintenance on machine $M_1$, on the other hand. Let us define:

- $L_1(m_1)$ average total cost of the maintenance actions as a function of the age $m$.
- $L_2(h_1, \alpha_1)$ average total cost of subcontracting losses and inventory control as a function of $h$ and $\alpha$.

Let $L_{SIMP}(h_1^*, \alpha_1^*, m_1^*) = L_1(m_1^*) + L_2(h_1^*, \alpha_1^*)$, the optimal average total cost of SiMP given that the terms $L_1(m_1)$ and $L_2(h_1, \alpha_1)$ are optimised independently.

### 4.3 Integrated maintenance policy (InMP)

The InMP is the management of maintenance and production. Thus, the optimisation here rests on the global optimisation of $L_{InMP}(m_2, h_2, \alpha_2)$, incorporating all costs pertaining to maintenance, inventory control and losses of subcontracting revenue.

### 4.4 Performance analysis of InMP and SiMP

The performance of InMP compared to SiMP is measured by:

$$H = \frac{L_{SIMP}(h_1^*, \alpha_1^*, m_1^*)}{L_{SIMP}(h_1^*, \alpha_1^*, m_1^*)} - \frac{L_{InMP}(h_2^*, \alpha_2^*, m_2^*)}{L_{InMP}(h_2^*, \alpha_2^*, m_2^*)}$$

To calculate $H$, we must optimise $L_{SIMP}(h, \alpha, m)$ and $L_{InMP}(h, \alpha, m)$, which, considering the difficulty of dealing with an analytical model, we accomplish by means of simulation combined with experimental design.

#### 4.4.1 Simulation approach

To compare the alternative policies, the simulation program was run on a horizon of 1,000,000 time units for three replications. Various performance measures were obtained, namely: the average total cost per unit time ($C_t$), the availability of machine $M_1$ ($\Lambda_1$) and the production rate of $M_2$ ($\alpha_2$). Then, by using experimental design, we obtain an analytical the expression of $C_t$ which can then be optimised. Finally, we use the simulation program to determine the optimal average total cost. This is illustrated by Figure 14.

As in Section 3, the logic of the simulation model is based on the discrete event concept, while algorithmic details are provided in Appendix II.

#### 4.4.2 Optimisation of SiMP

(i) Optimisation of $L_1 (m)$

To determine $m^*$, we use the well-known result of Barlow and Proschan (1965) concerning the optimal age-based maintenance strategy:

$$\left. \frac{dL_1(m)}{dm} \right|_{m^*} = 0$$
where

\[ L_1(m) = \frac{R(m)C_{np} + F(m)C_{mc}}{\int_0^m R(t)\,dt + F(m)\mu_c + R(m)\mu_p}, \]

\( R(m) \) is the reliability function of the system, \( F(m) = 1 - R(m) \), and \( \mu_p \) and \( \mu_c \) are respectively the average durations of preventive and corrective maintenance actions.

The optimisation of \( L_1(m) \) yields \( m^* = 27 \).

(ii) Optimisation of \( L_2(\alpha, h) \)

For the purpose of optimising \( L_2(\alpha, h) \) using simulation and experimental design. Table 8 gives the correspondence between the real values of \( \alpha \) and \( h \), and their levels in the experimental design.

Consequently:

\[ X_\alpha = \frac{10}{3} \alpha - \frac{7}{3} \quad (6) \]

\[ X_h = \frac{1}{10} h - 3 \quad (7) \]

(iii) Analytical model of \( L_2(\alpha, h) \) derived from simulation and experimental design

Collecting outputs from the simulation model which was run for various values of \( \alpha \) and \( h \) with \( m^* = 27 \) (resulting from the optimisation of \( L_1(m) \)), the following general expression of \( L_2(X_\alpha, X_h) \) was obtained:

\[ L_2(X_\alpha, X_h) = a_0 + a_1X_\alpha + a_2X_h + a_3X_\alpha^2 + a_4X_h^2 + a_5X_\alpha X_h \]
Table 9 presents an analysis of variance to determine the influential factors on $L_2(X_\alpha, X_h)$.

After eliminating the non-significant coefficients, and by using Equations (6) and (7), the analytical model of $L_2(X_\alpha, h)$ is as follows:

$$L_2(X_\alpha, h) = 154.94 - 13.26\alpha + 0.054h^2 - 3.858h - 0.65\alpha h.$$  

(iv) Optimisation phase

$$\begin{align*}
\left\{ \frac{dL_2}{dh} \right|_{h^*} &= 0 \Rightarrow 0.108h - 3.858 - 0.65\alpha = 0 \\
\left\{ \frac{dL_2}{d\alpha} \right|_{\alpha^*} &= 0 \Rightarrow -13.26 - 0.65h = 0 \\
\end{align*}$$

Hence,

$$h^* = \frac{3.858}{0.108} + \frac{0.65}{0.108}\alpha^*.$$  

According to Table 10, observe that $L_2(\alpha^*, h^*)$ is optimal for $\alpha^* = 1$ and $h^* = 41.74$. Thus, from the results of the independent optimisation of $L_1(m)$ and $L_2(\alpha, h)$, we conclude that for SiMP, the optimal values of the decision variables are:

$$m^* = 27, \quad \alpha^* = 1 \quad \text{and} \quad h^* = 41.74.$$  

4.4.3 Optimisation of InMP

We seek to find the values of $h^*, \alpha^*, \text{ and } m^*$ yielding $L_{\text{InMP}}(h^*, \alpha^*, m^*)$. Table 11 shows the correspondence between the real values of $m$, $\alpha$ and $h$ and their levels used in the experimental design. The latter consists of 33 tests.
Therefore,

\[ X_m = \frac{1}{15} m - \frac{4}{3} \]  \hfill (8)

\[ X_\alpha = 4\alpha - 3 \]  \hfill (9)

\[ X_h = \frac{1}{10} h - 3 \]  \hfill (10)

(i) **Analytical model of** \( L_{\text{InMP}}(h, \alpha, m) \)** **derived from simulation and experimental design**

Following the same logic as in Subsection 4.4.2, let:

\[ L_{\text{InMP}}(X_m, X_\alpha, X_h) = a_0 + a_1 X_m + a_2 X_\alpha + a_3 X_h + a_4 X_m^2 + a_5 X_\alpha^2 + a_6 X_h^2 + a_7 X_m X_\alpha + a_8 X_m X_h + a_9 X_h X_\alpha \]

This quadratic model represents, maintenance, inventory, shortage and lost subcontracting costs. In order to determine the degree of influence of each parameter, we will present in Table 12 the analysis of variance for \( L_{\text{InMP}}(X_h, X_\alpha, X_m) \).

After the elimination of the non-significant coefficients we obtain:

\[ L_{\text{InMP}}(X_m, X_\alpha, X_h) = a_0 + a_3 X_h + a_4 X_m^2 + a_6 X_h^2 + a_9 X_h X_\alpha \]

where \( a_0 = 90.84, a_3 = -1.34, a_4 = 8, a_6 = 3.05 \) and \( a_9 = 2.88 \).
By replacing each variable by its real value, using Equations (7), (8) and (9), we obtain:

\[ L_{InMP}(m, \alpha, h) = 159.72 + \frac{8}{225} m^2 - \frac{64}{45} m + 0.0305h^2 - 2.828h - 34.56\alpha + 1.152h\alpha \]

(ii) **Optimisation phase**

\[
\begin{align*}
\frac{dL_{InMP}}{dm} &= 0 \Rightarrow \frac{16}{225} m - \frac{64}{45} = 0 \\
\frac{dL_{InMP}}{d\alpha} &= 0 \Rightarrow -34.56 - 1.152h = 0 \\
\frac{dL_{InMP}}{dh} &= 0 \Rightarrow 0.061h - 2.828 + 1.152\alpha = 0
\end{align*}
\]

Thus, \( m^* = 20, \alpha^* = 0.86 \) and \( h^* = 30 \).

4.4.4 **Performance of InMP compared to SiMP**

Table 13 presents a comparison between InMP and SiMP in terms of performance. Note that InMP allows a better availability for \( M_1 \) than does SiMP. According to this table, it is clear that InMP makes less use of machine \( M_2 \), which reduces production costs (which were not explicitly considered in this study).

The performance in terms of cost of InMP compared to SiMP is:

\[ H = \frac{L_{SiMP}(h^*, \alpha^*, m^*_1) - L_{InMP}(h^*_2, \alpha^*_2, m^*_2)}{L_{SiMP}(h^*_1, \alpha^*_1, m^*_1)} = 15.16\% \]

Moreover, the other performance measures of InMP are summarised in Table 14, where

\[ P_p = \alpha^*_{SiMP}C^{M2} - \alpha^*_{InMP}C^{M2}, \quad P_\Delta = \frac{\Delta^*_1}{\Delta^{SiMP}_1}, \]

while \( \Delta^{InMP}_2 \) and \( \Delta^{SiMP}_2 \) are respectively the availability of \( M_1 \) under InMP and SiMP.
5. Conclusion

In this paper, we developed simulation and analytical models which enabled us to compare the performance of alternative operating policies in a manufacturing system in the context of subcontracting. Subcontracting was considered with two perspectives: either the internal manufacturing capacity was supplemented by employing an outside subcontractor, or else the internal manufacturing capacity had to be shared between fulfilling our own customer demand and providing subcontracting services to a third party.

In the first part of this paper, the manufacturing system was composed of a machine $M_1$ which produces a single product. In order to satisfy a constant demand $d$ exceeding the capacity of $M_1$, the system uses a subcontractor composed of a machine $M_2$ which produces at a certain rate the same type of product. Both machines are subject to random failures. Corrective maintenance actions are carried out on each machine upon their failure. Machine $M_1$ having an increasing failure rate, failures can be reduced by preventive maintenance actions. An age-limit policy of preventive maintenance is used for $M_1$, implying that $M_1$ stops for preventive maintenance when it reaches a given age.

In order to minimise the demand loss, we proposed two maintenance policies, denoted IMP and PMP. The former schedules preventive maintenance actions on $M_1$ considering the state of $M_2$, while the latter builds a safety stock for the purpose of supplying demand during the downtimes of $M_1$. The conditions under which IMP is efficient were specified analytically and assuming that these were satisfied we determined, by means of simulation, the optimal profit and cost obtained by IMP. Next, using simulation, we established the conditions for which PMP is profitable and moreover determined its optimum. Finally, we compared the two policies at their respective optima and concluded that the efficacy conditions of the IMP are more flexible than those of the PMP.

In the second part of this paper, we studied a system made up of two machines satisfying a constant demand $d$ under a constraint that subcontracting services must also be provided. Costs optimised jointly in the integrated maintenance policy (InMP) pertain to maintenance, inventory control and losses of subcontracting revenue. Compared with the simple maintenance policy (SiMP), which considers the aforementioned factors separately, the InMP yielded improvements on the availability of the manufacturing system as well as total costs.

References


Appendix I: Simulation model of IMP

Inputs:

- Lifetime distribution of each machine: $f_i(\cdot)$.
- Preventive maintenance distribution of $M_{1gp}$ and Repair time distribution of each machine $g_{ci}(\cdot)$.
- Parameters of the demand $d$: Frequency of $d$: $td$ and quantity of $d$: $Q_d$.
- Costs of maintenance actions: $C_{mp}$, $C_{mc}$.
- Inventory cost: $C_i$.
- Shortage cost: $C_{sp}$.
- Time of simulation: $tsim$.

Outputs: average total cost per time unit.

Beginning:

For each value of $(m, \Delta m)$ do:

- Initialise the state of the system in the operating state.
- Simulate the operation of the system during a simulation time $tsim$, do:
  - Initialise the system in operating state and outside subcontracting.
  
  - Read the state of the system $s$.
  - Determine the whole of events $E = \{ev_1, ev_2, \ldots, ev_n\}$ corresponding to the state $s$.
  - An event can be: failure, end of corrective maintenance, end of preventive maintenance, production of a piece, demand, shortage.
  - Determine the next event of $E$: $ne$.
  - Carry out the appropriate updates to the event $ne$. 

Switch towards the next state of the system: $ns$.

Record the resulting cost.

End.

Let us note that the simulation algorithms of SMP and PMP are based on the same logic as that of the IMP.

**Appendix II: Simulation model of SiMP**

**Inputs:**
- Lifetime distribution of each machine: $f_i(\cdot)$.
- Preventive maintenance distribution of $M_1gp(\cdot)$ and Repair time distribution of each machine $gei(\cdot)$.
- The age of the machine $M_1$: $m_0^*$ (obtained analytically).
- Frequency of subcontracting: $A_1$ and its duration: $A_2$.
- Parameters of the demand $d$: Frequency of $d$: $td$ and quantity of $d$: $Qd$.
- Costs of maintenances: $C_{mp}$, $C_{mc}$, $C_{smc}$, and cost of loss of subcontracting: $C_c$, inventory cost: $Cs$ and shortage cost: $C_p$.
- Time of simulation: $tsim$.

**Outputs:** average total cost per time unit.

**Beginning:**
For each value of $(h, a)$ do:

- Initialise the state of the system in the operating state.
- Simulate the operation of the system during a simulation time $tsim$, do:

  - Initialise the system in operating state and outside subcontracting.
  - Read the state of the system $s$.
  - Determine the whole of events $E = \{ev_1, ev_2, \ldots, ev_n\}$ corresponding to the state $s$.
    An event can be: failure, end of maintenance, end of preventive maintenance, production of a piece, demand, shortage, subcontracting.
  - Determine the next event of $E$: $ne$.
  - Carry out the appropriate updates to the event $ne$.
  - Switch towards the next state of the system: $ns$.

  Record the resulting cost.

End.

Note that the simulation algorithm of InMP is based on the same logic as that of the SiMP.