Network Sum-Rate Maximization via Joint Relay Selection and Power Allocation in Energy-Harvesting NOMA Multicast Cognitive Radio Networks

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***Abstract*—This paper studies the problem of joint relay selection and power allocation (J-RS-PA) for energy-harvesting (EH) non-orthogonal multiple-access (NOMA) multicast cognitive radio networks. Particularly, primary and secondary transmitters communicate with their receivers over EH amplify-and-forward relays, with the aim of maximizing the network sum-rate, while satisfying quality-of-service (QoS) constraints. The formulated J-RS-PA problem happens to be non-convex and computationally-intensive. Alternatively, it is solved optimally via a low-complexity solution procedure, which optimally solves the power allocation problem over each relay, and then selects the relay that maximizes the network sum-rate. Simulation results are presented to evaluate the proposed solution procedure, which is shown to efficiently yield the optimal network sum-rate solution, in agreement with the J-RS-PA scheme (solved via a global optimization package), but with lower computational-complexity, while satisfying the QoS requirements at the primary and secondary receivers.**

***Index Terms*—Cognitive radio, cooperation, energy-harvesting, non-orthogonal multiple-access, power allocation, relay selection.**

I. Introduction

Non-orthogonal multiple-access (NOMA) has recently posed itself as a paradigm shift from the conventional orthogonal multiple-access (OMA) techniques, due to its ability to improve spectrum utilization of 5G cellular networks [1,2]. In NOMA, multiple users are multiplexed in the power-domain by exploiting channel gains' differences, while employing successive interference cancellation (SIC) for multiuser detection. Cognitive radio (CR) has also emerged as an effective means for intelligent spectrum sharing, where a secondary transmitter (ST) can access the spectrum occupied by the primary transmitter (PT), while maintaining quality-of-service (QoS) for the primary receivers (PRs) as well as the secondary receivers (SRs) [3]. Additionally, cooperative relay communications have been proven to effectively extend network coverage and exploit diversity gains [4]. Furthermore energy-harvesting (EH) has emerged as an effective means to energize cellular networks and reduce dependency on the electrical grid [5]. Undoubtedly, combining NOMA with cognitive radio, energy-harvesting, and cooperative relaying can further improve spectrum utilization and transmission reliability, ultimately fulfilling the requirements of future generation cellular networks.

Recently, several works have considered resource allocation in NOMA cognitive radio networks [3,6]. In [7], a spectrum sharing protocol is proposed, where a base-station (BS) serves a primary user multicast group (PU-MG), and a secondary user multicast group (SU-MG). In particular, the SU-MG is located between the BS and the PU-MG, where the BS seeks the cooperation of the SUs to relay the PUs' signal. Outage probabilities of the PU-MG and SU-MG are derived and validated via simulations. The authors in [8] propose a novel cooperative relaying scheme, where the secondary BS—following the detection of an idle channel—transmits a NOMA signal to the first nearby SU, which applies a decode-and-forward (DF) strategy to relay the signal to the intended second SU. Analytical expressions for the outage probability and ergodic rate are derived, with simulation results illustrating the effectiveness of the proposed scheme and the accuracy of the derivations. In [9], the authors study the problem of relay selection in EH-assisted CR-NOMA network with simultaneous energy and information transmission. Particularly, the studied scenario consists of a PT and a ST, two PRs, a SR, and a set of amplify-and-forward (AF) relays. The outage performance over Rayleigh fading channels has been analyzed for the cases of a single relay as well as multiple relays, where the outage performance has been shown to improve with the increase in the number of selected relays.

In this paper, the problem of joint relay selection and power allocation (J-RS-PA) for NOMA multicast cognitive radio networks is studied. Specifically, the primary and secondary transmitters communicate with their receivers over EH AF relays, with the aim of maximizing the network sum-rate, while satisfying QoS constraints for all primary and secondary receivers. However, the formulated J-RS-PA problem happens to be non-convex and computationally-intensive. Alternatively, an optimal low-complexity solution procedure is proposed to solve the power allocation problem over each relay, and then select the relay that maximizes the network sum-rate. Simulation results are presented to validate the efficacy of the proposed solution procedure, where it will be shown to yield the optimal network sum-rate, in agreement with the J-RS-PA scheme (solved via a global optimization package), but with lower computational-complexity, while satisfying the QoS requirements at all primary and secondary receivers. To the best of our knowledge, no prior work has considered the problem of joint relay selection and power allocation for network sum-rate maximization in NOMA multicast cognitive radio networks.

The rest of this paper is organized as follows. In Section II, the system model is presented, while Section III presents the joint relay selection and power allocation problem formulation. Section IV discusses the proposed solution procedure, while Section V presents the simulation results. Lastly, Section VI draws the conclusions.

II. System Model

A. Network Model

Consider a cooperative NOMA network with a primary transmitter (PT) and a set of N primary receivers

(PRs), denoted $\mathcal{PR} = \{PR_1, \ldots, PR_n, \ldots, PR_N\}$. Additionally, let there be a secondary transmitter (SR) and a set of M secondary receivers (SRs), denoted $\mathcal{SR} = \{SR_1, \ldots, SR_m, \ldots, SR_M\}$. The primary and secondary transmitters communicate with their respective receivers in a multicast fashion over a set of K EH AF relays, denoted $\mathcal{R} = \{R_1, \ldots, R_k, \ldots, R_K\}$. The channel between any two network nodes follows narrowband Rayleigh fading with zero-mean N_0-variance additive white Gaussian noise (AWGN). Particularly, let $h_{i,j} \sim \mathcal{CN}\left(0, d_{i,j}^{-\nu}\right)$ be the channel coefficient between nodes i and j (for $i \neq j$), where $d_{i,j}$ is the inter-node distance, and ν is the path-loss exponent. The maximum transmit energy per time-slot is set to $E_{\max}$. For convenience, let $\mathcal{I}_{R_k}$ be a binary decision variable defined as

$$\mathcal{I}_{R_k} = \begin{cases} 1, & \text{if relay } R_k \text{ is selected,} \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where $\sum_{k=1}^{K} \mathcal{I}_{R_k} \leq 1$ (i.e. single relay selection).

In this work, no direct link is assumed between the PT and any PR, or the ST and any of the SRs. In addition, communication between the PT and ST and their respective receivers is performed over two phases (of one time-slot each) [4], namely the broadcasting phase, and the cooperation phase (see Fig. 1). Also, let a transmission frame $\tau \geq 0$ comprise the two communication phases (i.e. of two time-slots)[1].

Remark 1. For $\tau = 0$, no transmission takes place, as the relays harvest energy from the surrounding environment. However, for $\tau \geq 1$, transmission occurs by utilizing the harvested energy in the previous transmission frames.

B. Energy-Harvesting Model

In this work, it is assumed that the relays are solely powered by a renewable energy resource, such as photo-voltaic (PV) solar panels. Thus, let $\mathcal{E}_{R_k}^{\tau}$ be the amount of harvested energy during transmission frame $\tau \geq 0$, at each relay $R_k \in \mathcal{R}$. Particularly, $\mathcal{E}_{R_k}^{\tau}$ is modeled as an independent uniformly distributed random variable as $\mathcal{E}_{R_k}^{\tau} \sim U\left(0, \mathcal{E}_{R_k}^{\max}\right)$, with $\mathcal{E}_{R_k}^{\max}$ being the maximum harvested energy in any transmission frame [10]. Furthermore, let $B_{R_k}^{\max}$ be the maximum battery capacity of each relay R_k, which is used to store any leftover energy from previous transmission frames. Now, let $E_{R_k}^{\tau}$ be relay R_k's available transmit energy, for $\tau \geq 1$. Now, if a relay is selected for cooperative transmission (i.e. $\mathcal{I}_{R_k}^{\tau} = 1$), then it utilizes its harvested energy, subject to the maximum transmit energy per time-slot $E_{\max}$ (i.e. $E_{R_k}^{\tau} = \min\left(\mathcal{E}_{R_k}^{\tau-1}, E_{\max}\right)$). To be specific, if $\mathcal{E}_{R_k}^{\tau-1} > E_{\max}$, then the leftover energy (i.e. $\mathcal{E}_{R_k}^{\tau-1} - E_{\max}$) is stored in the battery for later use. Contrarily, if that relay is not selected, then it stores it harvested energy in the battery. Hence, the leftover energy can be written as

$$\Delta\mathcal{E}_{R_k}^{\tau} = \left(1 - \mathcal{I}_{R_k}^{\tau}\right) E_{R_k}^{\tau} + \max\left(0, \mathcal{E}_{R_k}^{\tau-1} - E_{\max}\right), \tag{2}$$

where $\Delta\mathcal{E}_{R_k}^{0} = 0$. In summary, the total amount of harvested energy along with any leftover energy at each relay $R_k \in \mathcal{R}$, subject to battery capacity, can be expressed as

[1]Each time-slot is assumed to be of unit duration, and hence the terms "power" and "energy" can be used interchangeably.

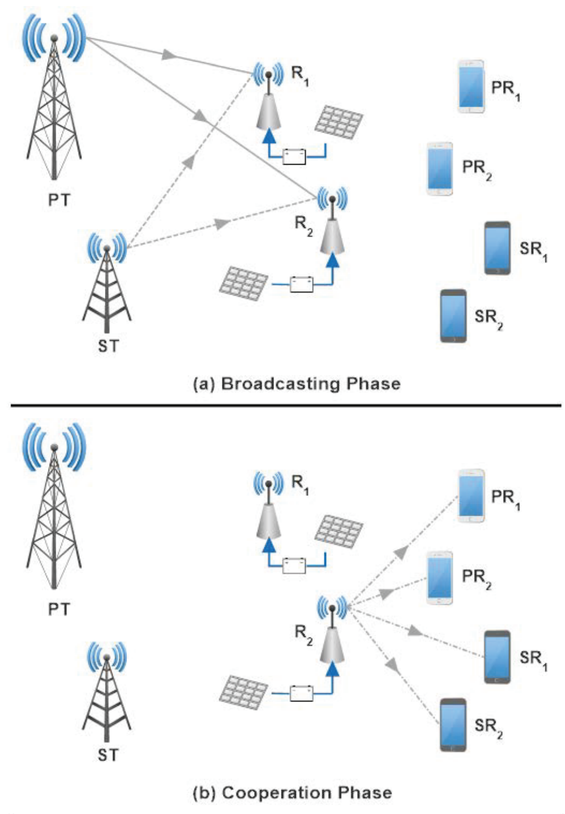

Fig. 1. A NOMA Multicast Cognitive Radio Network with $K = 2$ EH AF Relays and $N = M = 2$ Primary and Secondary Receivers

$$\mathcal{E}_{R_k}^{\tau} = \begin{cases} \min\left(\mathcal{E}_{R_k}^{0}, B_{R_k}^{\max}\right), & \text{for } \tau = 0, \\ \min\left(\mathcal{E}_{R_k}^{\tau} + \Delta\mathcal{E}_{R_k}^{\tau}, B_{R_k}^{\max}\right), & \text{for } \tau \geq 1. \end{cases} \tag{3}$$

From this point onwards, the superscript τ is dropped for notational convenience, while taking into account the battery dynamics during network operation.

C. Communication Model

In the broadcasting phase, both the PT and the ST transmit their signals via uplink NOMA. Particularly, the received signal at each relay R_k is expressed as

$$y_{R_k} = \sqrt{E_P} x_P h_{PT,R_k} + \sqrt{E_S} x_S h_{ST,R_k} + \eta_{R_k}, \tag{4}$$

where η_{R_k} is the received AWGN at relay R_k. Moreover, E_P and E_S are the transmit energy of the PT and ST, respectively, such that $E_P + E_S \leq E_{\max}$ and[2] $E_P \geq E_S$. Furthermore, x_P and x_S are the data symbols of the PT and ST, respectively.

In the cooperation phase, the relay amplifies-and-forwards the received signal as

$$x_{R_k} = \sqrt{E_{R_k} G_{R_k}} y_{R_k}, \tag{5}$$

where G_{R_k} is the AF normalization factor, written as [4]

$$G_{R_k} = \frac{1}{E_P |h_{PT,R_k}|^2 + E_S |h_{ST,R_k}|^2 + N_0}. \tag{6}$$

In turn, the received signal at the PR_n is given by

[2]Notably, the PT must always be assigned higher transmit energy than the ST, since it is licensed transmitter (i.e. has higher transmission priority [11]).

$$\begin{aligned} y_{R_k,PR_n} &= x_{R_k} h_{R_k,PR_n} + \eta_{PR_n} \\ &= \sqrt{E_{R_k} E_P G_{R_k}} x_P h_{PT,R_k} h_{R_k,PR_n} \\ &\quad + \sqrt{E_{R_k} E_S G_{R_k}} x_S h_{ST,R_k} h_{R_k,PR_n} \\ &\quad\quad + \sqrt{E_{R_k} G_{R_k}} h_{R_k,PR_n} \eta_{R_k} + \eta_{PR_n}. \end{aligned} \tag{7}$$

Similarly, the received signal at SR_m is written as

$$\begin{aligned} y_{R_k,SR_m} &= x_{R_k} h_{R_k,SR_m} + \eta_{SR_m} \\ &= \sqrt{E_{R_k} E_P G_{R_k}} x_P h_{PT,R_k} h_{R_k,SR_m} \\ &\quad + \sqrt{E_{R_k} E_S G_{R_k}} x_S h_{ST,R_k} h_{R_k,SR_m} \\ &\quad\quad + \sqrt{E_{R_k} G_{R_k}} h_{R_k,SR_m} \eta_{R_k} + \eta_{SR_m}. \end{aligned} \tag{8}$$

The received signal-to-interference-plus-noise ratio (SINR) at PR_n over relay R_k is obtained as [12]

$$\gamma_{R_k,PR_n}(\mathbf{E}) = \frac{E_{R_k} E_P G_{R_k} |h_{PT,R_k}|^2 |h_{R_k,PR_n}|^2}{E_{R_k} E_S G_{R_k} |h_{ST,R_k}|^2 |h_{R_k,PR_n}|^2 + E_{R_k} G_{R_k} |h_{R_k,PR_n}|^2 N_0 + N_0}. \tag{9}$$

By substituting (6) into (9), the received SINR is obtained as

$$\gamma_{R_k,PR_n}(\mathbf{E}) = \frac{\overline{\gamma}_{R_k,PR_n}(\mathbf{E})}{\underline{\gamma}_{R_k,PR_n}(\mathbf{E})}, \tag{10}$$

where

$$\overline{\gamma}_{R_k,PR_n}(\mathbf{E}) = E_{R_k} E_P \xi_{PT,R_k} \xi_{R_k,PR_n}, \tag{11}$$

and

$$\begin{aligned} \underline{\gamma}_{R_k,PR_n}(\mathbf{E}) &= E_{R_k} E_S \xi_{ST,R_k} \xi_{R_k,PR_n} + E_{R_k} \xi_{R_k,PR_n} \\ &\quad + E_P \xi_{PT,R_k} + E_S \xi_{ST,R_k} + 1. \end{aligned} \tag{12}$$

Additionally, $\xi_{PT,R_k} = |h_{PT,R_k}|^2/N_0$, $\xi_{R_k,PR_n} = |h_{R_k,PR_n}|^2/N_0$, and $\xi_{ST,R_k} = |h_{ST,R_k}|^2/N_0$. It should be noted that $\overline{\gamma}_{R_k,PR_n}(\mathbf{E})$ and $\underline{\gamma}_{R_k,PR_n}(\mathbf{E})$ define the numerator and denominator of the SINR function $\gamma_{R_k,PR_n}(\mathbf{E})$, respectively. Moreover, $\mathbf{E} \triangleq [E_P, E_S]$ is the transmit energy vector.

On the other hand, by the principle of NOMA, and since $E_P \geq E_S$, then by assuming perfect SIC, the received SINR at SR_m over relay $R_k \in \mathcal{R}$ can be shown to be [11,12]

$$\begin{aligned} \gamma_{R_k,SR_m}(\mathbf{E}) &= \frac{E_{R_k} E_S \xi_{ST,R_k} \xi_{R_k,SR_m}}{E_{R_k} \xi_{R_k,SR_m} + E_P \xi_{PT,R_k} + E_S \xi_{ST,R_k} + 1} \\ &\triangleq \frac{\overline{\gamma}_{R_k,SR_m}(\mathbf{E})}{\underline{\gamma}_{R_k,SR_m}(\mathbf{E})}, \end{aligned} \tag{13}$$

where $\xi_{R_k,SR_m} = |h_{R_k,SR_m}|^2/N_0$, while $\overline{\gamma}_{R_k,SR_m}(\mathbf{E})$ and $\underline{\gamma}_{R_k,SR_m}(\mathbf{E})$ are the numerator and denominator of the SINR function $\gamma_{R_k,SR_m}(\mathbf{E})$, respectively. Thus, the achievable rate at PR_n is determined as

$$\mathbb{R}_{PR_n}(\mathbf{E},\boldsymbol{\mathcal{I}}) = \frac{1}{2}\log_2\left(1 + \sum_{R_k \in \mathcal{R}} \mathcal{I}_{R_k} \gamma_{R_k,PR_n}(\mathbf{E})\right), \tag{14}$$

while that at SR_m is given by

$$\mathbb{R}_{SR_m}(\mathbf{E},\boldsymbol{\mathcal{I}}) = \frac{1}{2}\log_2\left(1 + \sum_{R_k \in \mathcal{R}} \mathcal{I}_{R_k} \gamma_{R_k,SR_m}(\mathbf{E})\right), \tag{15}$$

where $\boldsymbol{\mathcal{I}} = [\mathcal{I}_{R_1}, \ldots, \mathcal{I}_{R_k}, \ldots, \mathcal{I}_{R_K}]$. To ensure QoS, each PR_n and SR_m must satisfy a minimum rate constraint as

$$\begin{aligned} \mathbb{R}_{PR_n}(\mathbf{E},\boldsymbol{\mathcal{I}}) &\geq \mathbb{R}^P_{\min}, \quad \forall PR_n \in \mathcal{PR}, \\ \mathbb{R}_{SR_m}(\mathbf{E},\boldsymbol{\mathcal{I}}) &\geq \mathbb{R}^S_{\min}, \quad \forall SR_m \in \mathcal{SR}, \end{aligned} \tag{16}$$

where $\mathbb{R}^P_{\min}$ and $\mathbb{R}^S_{\min}$ are the minimum rates per primary and secondary receiver, respectively.

The network sum-rate (in bits/s/Hz) is determined as

$$\mathbb{R}_T(\mathbf{E},\boldsymbol{\mathcal{I}}) = \sum_{PR_n \in \mathcal{PR}} \mathbb{R}_{PR_n}(\mathbf{E},\boldsymbol{\mathcal{I}}) + \sum_{SR_m \in \mathcal{SR}} \mathbb{R}_{SR_m}(\mathbf{E},\boldsymbol{\mathcal{I}}), \tag{17}$$

In this work, the aim is to jointly perform relay selection and power allocation so as to maximize the network sum-rate, as detailed in the following section.

III. Joint Relay Selection and Power Allocation

The joint relay selection and power allocation (J-RS-PA) problem is formulated as

J-RS-PA$(\mathbf{E},\boldsymbol{\mathcal{I}})$**:** (18)

$$\max_{\mathbf{E},\boldsymbol{\mathcal{I}}} \quad \mathbb{R}_T(\mathbf{E},\boldsymbol{\mathcal{I}}) \tag{18a}$$

$$\text{s.t.} \quad \sum_{R_k \in \mathcal{R}} \mathcal{I}_{R_k} \leq 1, \tag{18b}$$

$$\mathcal{I}_{R_k} \cdot \left(\mathbb{R}_{PR_n}(\mathbf{E},\boldsymbol{\mathcal{I}}) - \mathbb{R}^P_{\min}\right) \geq 0, \forall PR_n \in \mathcal{PR}, \forall R_k \in \mathcal{R}, \tag{18c}$$

$$\mathcal{I}_{R_k} \cdot \left(\mathbb{R}_{SR_m}(\mathbf{E},\boldsymbol{\mathcal{I}}) - \mathbb{R}^S_{\min}\right) \geq 0, \forall SR_m \in \mathcal{SR}, \forall R_k \in \mathcal{R}, \tag{18d}$$

$$E_P + E_S \leq E_{\max}, \tag{18e}$$

$$E_P \geq E_S, \tag{18f}$$

$$0 \leq E_P, E_S \leq E_{\max}, \tag{18g}$$

$$\mathcal{I}_{R_k} \in \{0,1\}, \quad \forall R_k \in \mathcal{R}. \tag{18h}$$

In problem **J-RS-PA**, the objective function in (18a) is the network sum-rate. Constraint (18b) ensures that at most one relay is selected. Constraints (18c) and (18d) ensure that if a relay is selected, then all the PRs and SRs satisfy their minimum rate requirement, respectively. Constraint (18e) ensures that the sum of transmit energy of the PT and ST does not exceed $E_{\max}$, while Constraint (18f) ensures that the PT is assigned higher energy than the ST, which also enforces the SIC decoding order at the SRs. The last two constraints define the range of values the decision variables take.

Remark 2. Problem **J-RS-PA** is a mixed-integer nonlinear programming (MINLP) problem, which is NP-hard [13].

Based on **Remark 2**, solving problem **J-RS-PA** is computationally-intensive. Alternatively, it can be solved via a low-complexity algorithm that decouples the power allocation and relay selection problems, but guarantees global optimality.

IV. Solution Procedure

In this section, an optimal low-complexity solution procedure is developed to solve problem **J-RS-PA**. Now, the aim is determine the optimal power allocation per relay. Specifically, let $R_k \in \mathcal{R}$ be the selected relay (i.e. $\mathcal{I}_{R_k} = 1$, while $\mathcal{I}_{R_l} = 0$, $\forall l \neq k$). In turn, problem **J-RS-PA** reduces to the power allocation problem given by

$$\underline{\textbf{PA}(\mathbf{E},k)\textbf{:}} \tag{19}$$

$$\max_{\mathbf{E}} \quad \mathbb{R}_T(\mathbf{E},k) \tag{19a}$$

$$\text{s.t.} \quad \mathbb{R}_{PR_n}(\mathbf{E},k) \geq \mathbb{R}^P_{\min}, \quad \forall PR_n \in \mathcal{PR}, \tag{19b}$$

$$\mathbb{R}_{SR_m}(\mathbf{E},k) \geq \mathbb{R}^S_{\min}, \quad \forall SR_m \in \mathcal{SR}, \tag{19c}$$

$$E_P + E_S \leq E_{\max}, \tag{19d}$$

$$E_P \geq E_S, \tag{19e}$$

$$0 \leq E_P, E_S \leq E_{\max}, \tag{19f}$$

where

$$\mathbb{R}_T(\mathbf{E},k) = \sum_{PR_n \in \mathcal{PR}} \mathbb{R}_{PR_n}(\mathbf{E},k) + \sum_{SR_m \in \mathcal{SR}} \mathbb{R}_{SR_m}(\mathbf{E},k) \triangleq \sum_{PR_n \in \mathcal{PR}} \frac{1}{2}\log_2\left(\Gamma_{R_k,PR_n}\right) + \sum_{SR_m \in \mathcal{SR}} \frac{1}{2}\log_2\left(\Gamma_{R_k,SR_m}\right). \tag{20}$$

Moreover,

$$\Gamma_{R_k,PR_n}(\mathbf{E}) = 1 + \gamma_{R_k,PR_n}(\mathbf{E}) \triangleq \frac{\overline{\gamma}_{R_k,PR_n}(\mathbf{E})}{\underline{\gamma}_{R_k,PR_n}(\mathbf{E})}, \tag{21}$$

while

$$\Gamma_{R_k,SR_m}(\mathbf{E}) = 1 + \gamma_{R_k,SR_m}(\mathbf{E}) \triangleq \frac{\overline{\gamma}_{R_k,SR_m}(\mathbf{E})}{\underline{\gamma}_{R_k,SR_m}(\mathbf{E})}. \tag{22}$$

Additionally,

$$\overline{\underline{\gamma}}_{R_k,PR_n}(\mathbf{E}) \triangleq \overline{\gamma}_{R_k,PR_n}(\mathbf{E}) + \underline{\gamma}_{R_k,PR_n}(\mathbf{E}), \tag{23}$$

while

$$\overline{\underline{\gamma}}_{R_k,SR_m}(\mathbf{E}) \triangleq \overline{\gamma}_{R_k,SR_m}(\mathbf{E}) + \underline{\gamma}_{R_k,SR_m}(\mathbf{E}), \tag{24}$$

where it can be verified that $\overline{\underline{\gamma}}_{R_k,PR_n}(\mathbf{E})$ and $\overline{\underline{\gamma}}_{R_k,SR_m}(\mathbf{E})$ are linear in $\mathbf{E}$.

Remark 3. Γ_{R_k,PR_n} and Γ_{R_k,SR_m} are linear-fractional functions, with $\underline{\gamma}_{R_k,PR_n}(\mathbf{E}) > 0$ and $\underline{\gamma}_{R_k,SR_m}(\mathbf{E}) > 0$, $\forall PR_n \in \mathcal{PR}$, and $\forall SR_m \in \mathcal{SR}$, respectively. More importantly, Γ_{R_k,PR_n} and Γ_{R_k,SR_m} are pseudo-linear functions (i.e. both pseudo-convex and pseudo-concave) [14,15].

To facilitate the solution of problem $\textbf{PA}(\mathbf{E},k)$, note that the objective function can be re-written as

$$\mathbb{R}_T(\mathbf{E},k) = f(\mathbf{E},k) - g(\mathbf{E},k), \tag{25}$$

where

$$f(\mathbf{E},k) \triangleq \frac{1}{2}\left(\sum_{PR_n \in \mathcal{PR}} \log_2\left(\overline{\underline{\gamma}}_{R_k,PR_n}(\mathbf{E})\right) + \sum_{SR_m \in \mathcal{SR}} \log_2\left(\overline{\underline{\gamma}}_{R_k,SR_m}(\mathbf{E})\right)\right), \tag{26}$$

and

$$g(\mathbf{E},k) \triangleq \frac{1}{2}\left(\sum_{PR_n \in \mathcal{PR}} \log_2\left(\underline{\gamma}_{R_k,PR_n}(\mathbf{E})\right) + \sum_{SR_m \in \mathcal{SR}} \log_2\left(\underline{\gamma}_{R_k,SR_m}(\mathbf{E})\right)\right), \tag{27}$$

where it can be easily verified that $f(\mathbf{E},k)$ and $g(\mathbf{E},k)$ are twice continuously differentiable and increasing concave functions in $\mathbf{E}$, $\forall R_k \in \mathcal{R}$ [16].

Thus, problem $\textbf{PA}(\mathbf{E},k)$ can be *reformulated* as [17]

$$\underline{\textbf{R-PA}(\mathbf{E},k)\textbf{:}}$$

$$\max_{\mathbf{E}} \quad \mathbb{R}_T(\mathbf{E},k) = f(\mathbf{E},k) - g(\mathbf{E},k)$$
$$\text{s.t.} \quad \text{Constraints (19b)—(19f).} \tag{28}$$

Remark 4. The objective function of problem $\textbf{R-PA}(\mathbf{E},k)$ is a difference of concave functions, which is not necessarily concave [18,19]. Hence, problem $\textbf{R-PA}(\mathbf{E},k)$ in its current form cannot guarantee global optimality.

Based **Remark 4**, problem $\textbf{R-PA}(\mathbf{E},k)$ must be transformed, such that concavity of the objective function as well as the constraints can be guaranteed. To this end, note that Constraints (19b) and (19c) can be transformed into their linear forms as

$$\overline{\underline{\gamma}}_{R_k,PR_n}(\mathbf{E}) \geq \overline{\mathbb{R}}^P_{\min} \underline{\gamma}_{R_k,PR_n}(\mathbf{E}), \tag{29}$$

and

$$\overline{\underline{\gamma}}_{R_k,SR_m}(\mathbf{E}) \geq \overline{\mathbb{R}}^S_{\min} \underline{\gamma}_{R_k,SR_m}(\mathbf{E}), \tag{30}$$

where $\overline{\mathbb{R}}^P_{\min} \triangleq 2^{2\mathbb{R}^P_{\min}} - 1$, and $\overline{\mathbb{R}}^S_{\min} \triangleq 2^{2\mathbb{R}^S_{\min}} - 1$. Now, by introducing the auxiliary variable λ, problem $\textbf{R-PA}(\mathbf{E},k)$ can be *transformed* into [20]

$$\underline{\textbf{T-PA}(\mathbf{E},k)\textbf{:}}$$

$$\max_{\mathbf{E}} \quad f(\mathbf{E},k) - \lambda \tag{31a}$$

$$\text{s.t.} \quad g(\mathbf{E},k) \leq \lambda,, \tag{31b}$$

$$\overline{\underline{\gamma}}_{R_k,PR_n}(\mathbf{E}) \geq \overline{\mathbb{R}}^P_{\min} \underline{\gamma}_{R_k,PR_n}(\mathbf{E}), \forall PR_n \in \mathcal{PR}, \tag{31c}$$

$$\overline{\underline{\gamma}}_{R_k,SR_m}(\mathbf{E}) \geq \overline{\mathbb{R}}^S_{\min} \underline{\gamma}_{R_k,SR_m}(\mathbf{E}), \forall SR_m \in \mathcal{SR}, \tag{31d}$$

$$E_P + E_S \leq E_{\max}, \tag{31e}$$

$$E_P \geq E_S, \tag{31f}$$

$$0 \leq E_P, E_S \leq E_{\max}, \tag{31g}$$

$$\lambda \geq 0. \tag{31h}$$

Intuitively, the objective function aims at maximizing $f(\mathbf{E},k) - \lambda$, which entails maximizing $f(\mathbf{E},k)$ and minimizing λ. By constraint (31b), this also minimizes $g(\mathbf{E},k)$, such that the difference $f(\mathbf{E},k) - g(\mathbf{E},k)$ is maximized, in agreement with the objective function of problem $\textbf{R-PA}(\mathbf{E},k)$. Lastly, it can be verified that the objective function and the first constraint are concave in $\mathbf{E}$, while the remaining constraints are linear (i.e. a convex constraints set) [21].

Remark 5. Problem $\textbf{T-PA}(\mathbf{E},k)$ is a concave maximization problem, and thus can be solved efficiently to obtain the global optimal solution $\mathbf{E}^*$—within polynomial-time complexity [21]—via any standard convex optimization package.

A. Algorithm Description

Problem **J-RS-PA** can be solved by iteratively solving problem $\textbf{T-PA}(\mathbf{E},k)$ for each relay $R_k \in \mathcal{R}$, and then selecting the relay that yields the maximum $\overline{\mathbb{R}}^*_{T,R_k} \triangleq f(\mathbf{E}^*,k) - g(\mathbf{E}^*,k)$ network sum-rate value, as

$$R^* = \arg\max_{R_k \in \mathcal{R}} \overline{\mathbb{R}}^*_{T,R_k}. \tag{32}$$

Algorithm 1 outlines the proposed solution procedure for joint relay selection and power allocation (SP-J-RS-PA).

Algorithm 1: SP-J-RS-PA

Input: Sets $\mathcal{PR}$, $\mathcal{SR}$, and $\mathcal{R}$.

Power Allocation:

```
FOR R_k ∈ R
    Solve problem T-PA(E, k) to get E*;
    Compute R*_{T,R_k} = f(E*, k) − g(E*, k);
END FOR
```

$\text{FOR } R_k \in \mathcal{R}$

$\quad$ Solve problem $\textbf{T-PA}(\mathbf{E}, k)$ to get $\mathbf{E}^*$;

$\quad$ Compute $\overline{\mathbb{R}}^*_{T,R_k} = f(\mathbf{E}^*, k) - g(\mathbf{E}^*, k)$;

END FOR

Relay Selection:

$R^* = \arg\max_{R_k \in \mathcal{R}} \overline{\mathbb{R}}^*_{T,R_k}$;

Output: Optimal transmit energy vector $\mathbf{E}^*$ and selected relay R^*.

Remark 6. The **SP-J-RS-PA** algorithm yields the global optimal joint relay selection and power allocation solution. This is because for each relay $R_k \in \mathcal{R}$, problem **T-PA**$(\mathbf{E}, k)$ is globally optimally solved, as per **Remark 5**. In addition, by iterating over each relay, and determining the global optimal power allocation solution $\mathbf{E}^*$, the relay $R_k \in \mathcal{R}$ that yields the maximum value of the objective function $\overline{\mathbb{R}}^*_{T,R_k}$ is selected.

Remark 7. SP-J-RS-PA requires $|\mathcal{R}| = K$ iterations, in each of which a concave optimization problem is solved efficiently within polynomial-time complexity.

V. Simulation Results

In this section, the performance of the proposed **SP-J-RS-PA** is evaluated. Particularly, the simulations assume $K = 4$ relays, and $N = 3$ and $M = 3$ primary and secondary receivers, respectively, which are located as shown in Fig. 2. The maximum transmit energy per time-slot is set to $E_{\max} = 0.5$ J, while the noise variance is set to $N_0 = 10^{-8}$ J. The path-loss exponent is set to $\nu = 3$. In addition, the minimum rate requirements for the primary and secondary receivers are set as $\mathbb{R}^P_{\min} = 1$ and $\mathbb{R}^S_{\min} = 0.5$ bits/s/Hz, respectively[3]. Moreover, the battery capacity is $B^{\max}_{R_k} = 5$ J, $\forall R_k \in \mathcal{R}$. Furthermore, the simulations are averaged over 10^3 independent network instances, where each network instance comprises 10 transmission frame, with randomly generated harvested energy in each frame. The channel coefficients randomly change from one network instance to another, but remain constant during each network instance.

Remark 8: In the simulations, the maximum harvested energy of each relay per transmission frame is set to $\mathcal{E}^{\max}_{R_k} = 0.5$ J for $k = 2$ and $k = 4$, and $\mathcal{E}^{\max}_{R_k} = 0.25$ J, for $k = 1$ and $k = 3$. That is, relays R_2 and R_4 harvest—on average—relatively higher amounts of energy than relays R_1 and R_3.

The proposed **SP-J-RS-PA** scheme is compared with the **J-RS-PA**[4]. In addition, the following schemes are considered:

Maximum Harvested Energy RS and PA (MHE-RS-PA):
In this scheme, the relay with the maximum amount of harvested energy in each transmission frame is selected, followed by optimal power allocation.

Random RS and PA (R-RS-PA):
This scheme randomly selects a relay in each transmission frame, and then optimizes power allocation.

[3]In alignment with $E_P \geq E_S$, and the higher priority of the primary network, $\mathbb{R}^P_{\min} > \mathbb{R}^S_{\min}$.

[4]All optimization problems are solved via MIDACO [22], with tolerance set to 0.001.

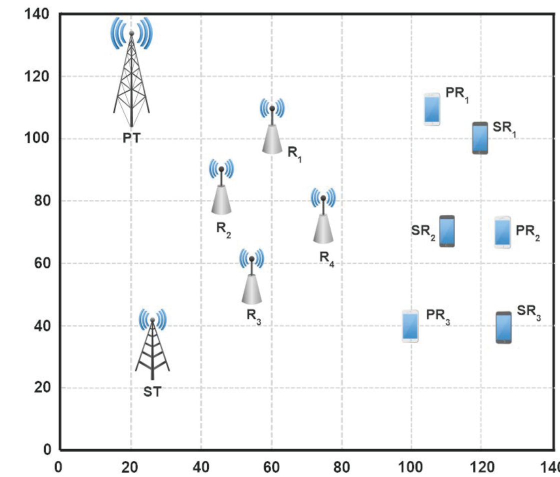

Fig. 2. Simulated Network Topology

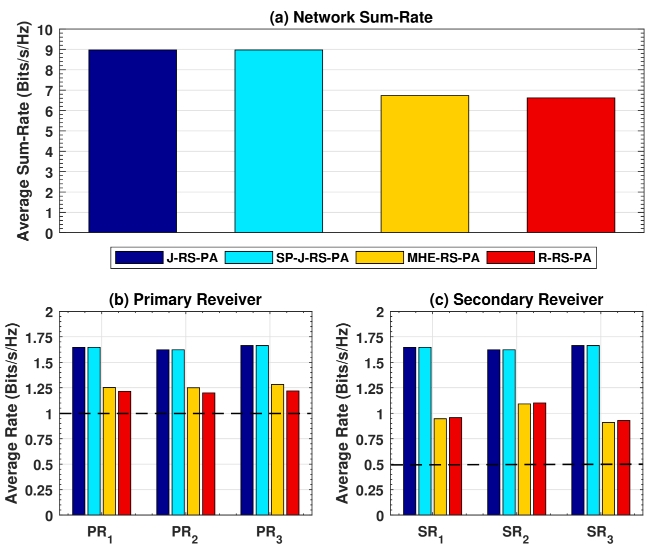

Fig. 3. Average (a) Network Sum-Rate, (b) Rate Per Primary Receiver, and (c) Rate Per Secondary Receiver (Bits/s/Hz)

In Fig. 3a, the average network sum-rate is illustrated, where it can be seen that the proposed **SP-J-RS-PA** scheme yields identical network sum-rate to the **J-RS-PA** scheme, and is superior to the other two schemes. Figs. 3b and 3c demonstrate the average rate per primary and secondary receiver, respectively. It is clear that all primary and secondary receivers satisfy the minimum rate requirements of $\mathbb{R}^P_{\min} = 1$ and $\mathbb{R}^S_{\min} = 0.5$ bits/s/Hz, respectively, which is due to the optimized power allocation.

Figs. 4a and 4b show the percentage of outage events of the primary and secondary receivers, respectively. It is evident that the proposed **SP-J-RS-PA** scheme as well as the **J-RS-PA** scheme incur about 2% of outage events, but are significantly less than the **MHE-RS-PA** and **R-RS-PA** schemes.

In Fig. 5, the average transmit energy of the primary and secondary transmitters is illustrated. Particularly, one can see that the primary transmitter is allocated higher transmit energy than the secondary transmitter under all schemes. In addition, the **SP-J-RS-PA** and **J-RS-PA** schemes are allocated higher transmit energy than the other schemes, which—along with optimal relay selection—maximizes the network sum-rate.

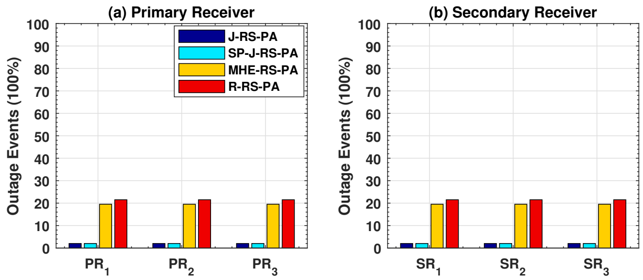

Fig. 4. Percentage of Outage Events Per (a) Primary Receiver, and (b) Secondary Receiver

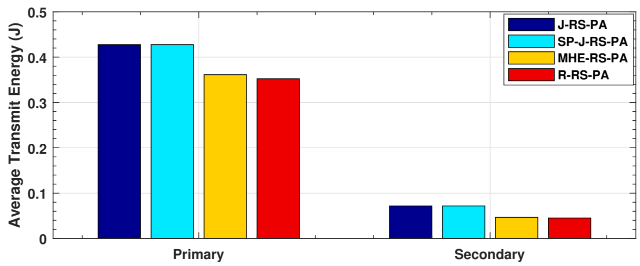

Fig. 5. Average Transmit Energy (J) of Primary and Secondary Transmitters

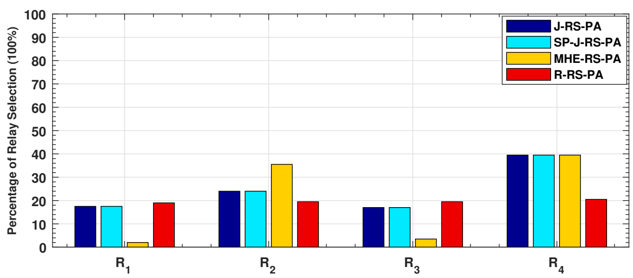

Fig. 6. Percentage of Relay Selection (100%)

Fig. 6 shows that the **SP-J-RS-PA** and **J-RS-PA** schemes yield identical percentages of relay selection over all relays, where R_4 is selected the most due to its location being relatively closer to all primary and secondary receivers than the other relays, as well as it relatively higher harvested energy. On the other hand, the **MHE-RS-PA** scheme selects relays R_2 and R_4 the most due to their relatively higher harvested energy than the other two relays, as per **Remark 8**. Lastly, the **R-RS-PA** scheme yields rather uniform percentage of relay selection, as would be expected.

VI. Conclusions

This paper has studied the problem of joint relay selection and power allocation for network sum-rate maximization in energy-harvesting NOMA multicast cognitive radio networks. Particularly, the formulated J-RS-PA problem has been shown to be non-convex and computationally-intensive. In turn, a low-complexity solution procedure has been proposed to optimally solve the power allocation over each relay, and then select the relay that maximizes the network sum-rate. Simulation results have been presented to validate the proposed solution procedure, and illustrate that it yields the optimal network sum-rate in comparison to the J-RS-PA scheme; however, with lower computational complexity, while satisfying the QoS requirements for all primary and secondary receivers.

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