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Low Sampling-rate Approach for ECG Signals with Compressed Sensing Theory

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Abstract

Wireless Body Area Networks (WBANs) consist of tiny Biomedical Wireless Sensors (BWSs) and a Gateway (GW) to connect to the external databases in the hospital and medical centres. The GW could connect the BWSs, to a range of wireless telecommunication networks. These wireless telecommunication networks could be either a mobile phone network, a standard telephone network, a dedicated medical centre or using public Wireless Local Area Networks (WLANs) also known a Wi-Fi system. The electrocardiogram (ECG) signals are widely used in health care systems because they are non-invasive mechanisms to establish medical diagnosis of heart diseases. The current ECG systems suffer from important limitations: limited patient’s mobility, limited energy, limited on wireless applications. The main drawback of current ECG systems is the location-specific nature of the systems due to the use of fixed/wired applications. That is why; there is a critical need to improve the current ECG systems to cover security handling and to achieve extended patient’s mobility. With this in mind, Compressed Sensing (CS) procedure and the collaboration of Block Sparse Bayesian Learning (BSBL) framework is used to provide a robust low sampling-rate approach for normal and abnormal ECG signals. Advanced WBANs based on our approach will be able to deliver healthcare not only to patients in hospital and medical centres; but also in their homes and workplaces thus offering cost saving, and improving the quality of life. Our simulation results based on two proposed algorithms illustrate 15\% incensement of Signal to Noise Ratio (SNR) and a good level of quality for the degree of incoherence between the random measurement and sparsity matrices.

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1. Introduction

WBANs as a special purpose of Wireless Sensor Networks (WSNs) are expected to be breakthrough Information Communication technology (ICT) in healthcare areas such as hospital and home care, Mobile Health (MH), Electronic Health (EH), telemedicine, and physical rehabilitation. WBANs are expected to be breakthrough Information Communication technology (ICT) in healthcare areas such as hospital and home care, mobile health, electronic health, and physical rehabilitation. Long-term records of ECG signals in WBANs have become commonly used to collect information from the heart for diagnostic and therapeutic purposes. That is why the quantity of data grows significantly and compression is required for reducing the storage, transmission times, and power consumption. The ECG signals generally illustrate the redundancy between adjacent heartbeats due to its semi-periodic structure. It is evident that this redundancy provides a high fraction of common support between consecutive heartbeats that is a good candidate for compression. The compressed sensing is a revolutionary idea for the acquisition and recovery of sparse signals that enables sampling-rate significantly below the classical Nyquist-rate. The normal and abnormal ECG signals based on CS theory in WBANs provide new wireless healthcare systems low data rate, very small transmitting power requirement, and longer battery life for diagnostic and therapeutic purposes. The CS theory says a small number of random linear measurements of bio-sparse signals contain enough information to collect, process, transmit, and recover the original signal [1]. The signal representing sparsity in any orthogonal basis can be well reconstructed using $\ell_1$ norm minimization, while satisfying the Restricted Isometry Property (RIP) condition for random measurement matrix $\Phi$ which offers by compressed sensing theory and orthogonal $\Psi$ in any domain [2]. On the other hand, the BSBL framework is proposed for signals with block structure such as abnormal ECG signals. By employing this framework, the abnormal ECG signal can be partitioned into a concatenation on non-overlapping blocks and a few of blocks are non-zero. This framework has a pruning mechanism, which pruned out blocks. That is why even if a signal has no clear block structure, the BSBL framework is still effective. In fact, the block partition can be evaluated as regularization to measure covariance matrix of the signal. This paper presents a contribution of CS approach with BSBL framework to establish a robust sampling procedure to reduce the load of sampling-rate for normal and abnormal ECG signals. The WBANs with CS approach and the collaboration from BSBL framework can offer two important advantages compared to current health monitoring systems. The first advantage is the mobility of patients due to use of ambulatory health monitoring systems. Second advantage is to control and investigate ECG signals from outside of hospital and medical centers in order to increase an ability of prevention and early diagnosis. By this convenient means, elderly people can keep track of their health conditions on their Smart phones or any portable device without the frequent visit to their doctor’s offices [3]. The normal and abnormal ECG signals based on our new sampling procedure provide low data rate, very small transmitting power requirement, and longer battery life and also serve the goal of reducing healthcare costs because of monitoring several patients simultaneously. The contribution of this paper lies in the use of our new algorithm to minimize PRD. The simulation results indicate that good level of quality of SNR can be achieved when PRD decreases by 25% and SNR increases by 15%. The structure of this paper is organized as follows: Section 2 gives an overview about CS theory in general and specifically for WBANs. Section 3 proposes our new algorithm based on collaboration of CS theory and BSBL framework. The reminder of the paper is categorized in the following way: the simulation results, including results on SNR and PRD are presented in Section 4. Section 5 offers main contribution of this work and relation to prior work. The conclusion is drawn in Section 6.

2. Overview of Compressed Sensing Theory
Our goal in the digital-CS theory as a new sampling scheme is to reduce the load of sampling-rate by
decreasing the number of samples after the Analog to Digital Convertor (ADC) required to completely
describe a signal by exploiting its compressibility [4]. An important aspect of CS theory is that our
measurements are not point samples but more general linear functional of the signals [5]. Any
compressible or sparse signal in \( \mathbb{R}^N \) can be expressed like [6], [7]:
\[
D = \sum_{i=1}^{N} C_i \Psi_i.
\]
Therefore, the compressed signal \( \mathcal{C} \) is found as:
\[
[\mathcal{C}]_{M \times 1} = [\Phi]_{M \times N} [D]_{N \times 1}.
\]
Thus, the compressed signal is found:
\[
[\mathcal{C}]_{M \times 1} = [\Phi]_{M \times N} [\Psi]_{N \times M} [C]_{N \times 1} = [\Theta]_{M \times N} [C]_{N \times 1}.
\]
Thus, CS scenario has two important steps. First step in CS offers a stable measurement matrix \( \Phi_{M \times N} \)
to ensure that the salient information in any compressible signal is not damaged by the dimensionality
reduction from \( D \in \mathbb{R}^N \) down to \( \mathcal{C} \in \mathbb{R}^M \). In the second step, the CS theory offers a reconstruction algorithm
under certain condition and enough accuracy to recover original signal \( D \) from the compressed signal.
Therefore, we can exactly reconstruct the original signal \( D \) with high probability via 1 norm. The
condition which guarantees the correctness of this recovery is given like:
\[
M \geq cK \log(N / K),
\]
which c is constant and \( M \) is number of random linear measurements [7]. To guarantee the robust
recovery with high probability and enough accuracy, the random dictionary matrix \( \Phi \) must have RIP
property [8]. This property provides theoretical guarantees to recover \( K \)-sparse using a system of \( M \) linear
equations with \( K \) unknowns. That is why the random dictionary matrix \( \Phi \) must verify the following
conditions:
\[
(1 - c) \| \Phi_S y \|_2^2 \leq (1 + c) \| y \|_2^2,
\]
where \( \Phi_S \) is a sub-matrix of matrix \( \Phi_{M \times N} \) with \( S \leq M \), \( y \) is a given vector, and \( c \) is constant [8].
Figure 1 illustrates CS theory in WBANs.

As it can be seen the biomedical signals are compressed by wireless sensors. The collected compressed
biomedical data are then transmitted wirelessly to Access Points (APs) at hospital, ambulance, and
helicopter. The APs are recovered compressed biomedical data for diagnostic and therapeutic purposes. The received vector in GW can be written as:

\[ \mathbf{C}_{M \times 1} = [\mathbf{\Phi}]_{M \times N} [\mathbf{D}]_{N \times 1}. \]  

(6)

Consequently, the received vector in GW is a condensed representation of the sparse events like:

\[
\begin{pmatrix}
C_1 \\
\vdots \\
C_M
\end{pmatrix} = 
\begin{pmatrix}
\Phi_{11} & \cdots & \Phi_{1N} \\
\vdots & \ddots & \vdots \\
\Phi_{M1} & \cdots & \Phi_{MN}
\end{pmatrix}
\begin{pmatrix}
D_1 \\
\vdots \\
D_N
\end{pmatrix}.
\]

(7)

Our simulation results show that by employing the CS, the WBANs can achieve a higher transmission, a lower time delay and higher probability of success of data transmission. Therefore, a combination of CS theory to WBANs is an optimal solution for achieving robust WBAN with low sampling rate and power consumption.

3. Proposed Approach

We demonstrate the incorporation of a Block Sparse Bayesian Learning (BSBL) framework with CS approach to compress any ECG signal for normal and abnormal scenarios, including sparse and non-sparse signals to achieve better performance. The BSBL framework is partitioned the ECG signal into a concatenation of non-overlapping blocks, and a few of blocks are non-zero [9]. More specifically, the number of non-zero blocks is the same as the number of random linear measurements in CS approach. This approach can prune out blocks in abnormal ECG signals [10]. Therefore, if abnormal ECG signals have no clear block structure, the BSBL framework and the collaboration from CS theory is still effective to compress and recover abnormal ECG signals. The CS based on BSBL framework can compress normal and abnormal ECG signals with high probability and enough accuracy. In order to generate an approximate real-time transmission for collected the ECG signals the length of each block should be short [11]. At the same time, we want to incorporate heartbeats in one block to recover the ECG signal with fewer samples. Table 1 illustrates the entire algorithm based on CS theory and BSBL framework. The main objective of this algorithm is to recover the original ECG signal with high probability and enough accuracy for patient monitoring purposes.

Table 1: Proposed Reconstruction Approach

<table>
<thead>
<tr>
<th>Algorithm: Reconstruction Approach for normal and abnormal ECG signals based on CS theory and BSBL framework</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Require</strong>: Matrix ([\Theta] = [\Phi][\Psi]), Number of random measurements ((M)), block distance (d_0), and number of non-zero blocks (i_{\theta \in M}).</td>
</tr>
<tr>
<td>1: Initialize (i = 0).</td>
</tr>
<tr>
<td>2: ([\Theta]_{\theta} ) is constructed by selecting the columns of (\Theta) in (d_0).</td>
</tr>
<tr>
<td>3: Find (C_0) by solving (C_0 = \arg \min_C | C - \Theta_0 C |_2^2)</td>
</tr>
</tbody>
</table>
4: Calculate the following features:
\[ R_0 = C - \Theta_0 C_0 , CR_0 = R_0 / C , \]
\[ PRD_0 = \frac{|| R_0 ||_2}{|| C ||_2} \]

5: While halting criterion \( d_0 \)

6: \( i = i + 1 \)

7: Solve \( C_i = \arg \min \_C \frac{|| C - \Theta_i C ||^2_2}{\iota} \)

8: Calculate the following features:
\[ R_i = C - \Theta_i C_i , CR_i = R_i / C , PRD_i = \frac{|| R_i ||_2}{|| C ||_2} \]

10: end while

11: provide \([C]\)

12: Reconstruct \([D] = [C][\Psi]\)

4. Simulation Results

In this Section, features of ECG signals such as Compression Ratio (CR), SNR, and PRD are simulated. The following assumptions were made for simulation:

- Three sensing matrix possibilities are examined for random sensing matrix \( \Phi \): (1) Bernoulli Toeplitz matrix, (2) Gaussian Circulant matrix, and (3) Binary Toeplitz matrix [12].
- The SPARCO toolbox is used for testing sparse reconstruction algorithm.
- Experiments are carried out over a 10-minutes ECG signal from MIT-BIH database.
- One hundred repetition’s are averaged for our simulation results. To validate the simulation results ECG signals from records 100,107,115 and 117 of MIT-BIH are investigated.
- The mean of ECG blocks is rounded in the sliding window to the nearest multiple of \( 2^L \), where \( L \) is the BSBL level.
- To simulate SNR for ECG signals the following equation is used [13].

\[ SNR = -20 \log_{10} (0.01 PRD) \]  

The implementation of sensing matrix \( \Phi^{M \times N} \) is simulated for Gaussian distribution, sparse binary sensing, and Uniform distribution [14].

- The sparse sensing matrix with nonzero entries equal to \( \pm 1/ \sqrt{2} \) is used for sparse binary matrix [15].
- The permissible parameters were adopted of IEEE802.15.3, IEEE802.15.5, and IEEE802.16e protocols which support low power communication in WBANs [16].

- The random sensing matrix \( \Phi^{M \times N} \) is applied to all the records of the MIT-BIH ECG database [17].
- The SPGL1 (Spectral Projected Gradient for L1 minimization) toolbox is used to determine Large-scale one-norm regularized least squares in the following equation:

\[ \min \| c \|_1 \text{ subject to } C = \Phi D. \]  

To validate the simulation results, the BPBQ (Basis Pursuit DeQuantizer) toolbox is used for recovery of sparse signals from quantized random measurements to solve:
\[
\begin{align*}
\arg \min_{\mathbf{c} \in \mathbb{R}^N} & \| \mathbf{c} \|_1 \\
\text{subject to} & \| \mathbf{C} - \mathbf{\Phi D} \|_p \text{ for } p \geq 2.
\end{align*}
\]

The simulation results were obtained for an input signal of \(N=512\) samples and a 12-bits resolution for the input signal and the measurement signal \(\mathbf{C}\). The random binary matrix is applied to all the records of the MIT-BIH ECG database to optimize the number of non-zero entries in order to simulate Signal-to-Noise Ratio (SNR). To compare \(\text{PRD}\), random binary sensing matrix is applied to all the records of the MIT-BIH ECG database, and the output \(\text{PRD}\) is measured. The mutual coherence \(\mu(\mathbf{\Phi}, \mathbf{\Psi})\) as an important parameter between random matrixes \(\mathbf{\Phi}\) and sparsity basis \(\mathbf{\Psi}\) is decreased by increasing the number of non-zero entries in matrix \(\mathbf{\Phi}\). Figure 2 illustrates the mutual coherence versus the number of non-zero entries in matrix \(\mathbf{\Phi}\).

Fig. 2: The mutual coherence \(\mu(\mathbf{\Phi}, \mathbf{\Psi})\)

The random binary matrix is applied to all the records of the MIT-BIH ECG database to optimize the number of non-zero entries in order to simulate Signal-to-Noise Ratio (SNR) [18]. Figure 3 reports the resulting average output SNR in terms of the number of non-zero elements in the random binary matrix \(\mathbf{\Phi}\).

Fig. 3: SNR in terms of number of non-zero entries

As depicted in the Figure 3 the satisfying quality for SNR can be achieved by minimizing CR, which will result in an increase in the number of non-zero elements. As it can be seen, the output SNR saturated after the number of non-zero entries \(M=15\), which is the reference value for the rest of simulation results. As it can be seen, the Binary Toeplitz matrix provides the best SNR and compression performance with the highest energy efficiency with the same number of randomly placed. Figure 4 shows simulation results on power consumption for random binary matrix with CS theory.
In the simulation of the proposed algorithm in the Table 2, only received ECG signals at GW with $SNR \geq 50dB$ are categorized as successful trials.

![Graph](image)

Fig. 4: Power consumption versus SR

It can be seen; the power consumption can be reduced by 65% by employing CS theory. Table 2 compares the simulation results on sampling rate and power consumption for random binary matrix with CS theory.

Table 2: Comparing SR and PC

<table>
<thead>
<tr>
<th>$N$ in ECG</th>
<th>CR</th>
<th>SR</th>
<th>PC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1024</td>
<td>15.24</td>
<td>26%*(NR)</td>
<td>33%*(PC in non-CS)</td>
</tr>
<tr>
<td>2048</td>
<td>25.78</td>
<td>29%*(NR)</td>
<td>37%*(PC in non-CS)</td>
</tr>
<tr>
<td>3074</td>
<td>34.54</td>
<td>33%*(NR)</td>
<td>42%*(PC in non-CS)</td>
</tr>
</tbody>
</table>

Table 2 indicates that satisfying quality on sampling rate and power consumption can be achieved when CR does not exceed of 30.

5. Relation to Prior Works

The emerging application of CS theory in medical areas has been potentially powerful to provide wireless healthcare systems. However, the success of CS theory heavily relies on the sparsity of ECG signal. Therefore, the CS approach is ineffective for non-sparse signals like abnormal ECG signals. That is why the CS theory and the collaboration from BSBL framework can provide robust algorithm procedure for normal and abnormal ECG signals due to the lake of research on this field. The main contribution of this paper lies in the use of CS approach and collaboration of BSBL framework to establish new low sampling-rare algorithm for normal and abnormal ECG signals. The work by A.M.R.Dixon [8] considers only CS theory for normal Ecg signals and the work by S.M.Jadhay [12] emphasizes on normal-ECG signal without CS theory. While the present study is provided a new sampling approach for normal and abnormal-ECG signal based on low sampling approach.

6. Conclusion
The ECG signal is widely used in WBANs because it is a noninvasive way to provide medical diagnosis of heart diseases. This paper has presented two new algorithms with a contribution of CS approach, BSBL framework to establish low sampling-rate approach for ECG signals. This paper has proposed a modified sampling approach for normal and abnormal ECG signals based on CS theory and the collaboration from BSBL framework. As expected, the proposed algorithm exhibits better performance on SNR, CR, and PRD. Our simulation results based on two proposed algorithms illustrate 15% incensement of Signal to Noise Ratio (SNR) and a good level of quality for the degree of incoherence between the random measurement and sparsity matrices.

7. Future Work

We have examined the benefit of CS theory and BSBL framework for one record of normal and non-normal ECG signals. Our future work involves developing the CS theory and BSBL framework for other records of ECG signals including abnormal records.

References