A simple empirical model for calculating gain and excess noise in GaAs/Al$_{\xi}$Ga$_{1-\xi}$As APDs (0.3 $\leq \xi \leq$ 0.6)

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Abstract: In this paper, we present a simple empirical model for calculating gain and excess noise in heterojunction GaAs/Al$_{\xi}$Ga$_{1-\xi}$As APDs (0.3 $\leq \xi \leq$ 0.6), without going through the relatively complicated and time consuming Monte Carlo simulation, commonly used for such devices. In this model, we present a set of empirical formula which can predict a distribution function for ionization path length for a given electric field throughout multiplication region. To determine the optimized values of the adjustable parameters used in our empirical model, we train a back-propagation neural network. The results are in excellent agreement with those obtained by the random path length (RPL) technique.

Keywords: avalanche photodiode, Monte Carlo simulation

Classification: Photonics devices, circuits, and systems

References


1 Introduction

Heterostructure avalanche photodiode (HAPD) is known as a common detector to long-haul optical communication and Geiger mode applications due to high internal gain [1]. Random nature of impact-ionization processes causes uncertainty in determining the positions at which such events occur, within the multiplication region. Such uncertainties generate excess noise in APDs, which in turn increases as the number of impact-ionization events increases. In recent years, some research groups have tried to develop techniques to reduce the excess noise factor [2, 3, 4]. In doing so, one must shrink the width of the multiplication region. In modeling a thin multiplication region, when its width becomes comparable to the size of the dead space, the history on which each ionizing carrier depends, wave-like behavior of carriers at the heterointerface and the random nature of impact-ionization complicate device simulation and noise optimization [5].

Monte Carlo (MC) simulation, in general, is an efficient method to calculate multiplication and excess noise factors in APDs. A common technique is to solve the coupled integral equations [6], which was later on improved by considering a ‘softness’ factor and the history dependence of the ionizing carrier [7]. The random path length (RPL) is another popular technique in implementing Monte Carlo simulation for calculating multiplication and excess noise factors for APDs and HAPDs [8, 9]. In the latter technique, impact-ionization coefficients are calculated by Monte Carlo simulation, first. Then, these coefficients are used in RPL model by which two distribution functions are defined for the impact-ionization path length; one for the wide band gap and another one for the narrow band gap material. Furthermore, to fit the numerical results obtained from RPL to experimental data, one has to select a suitable size for the dead space. Although these techniques give the gain and excess noise very rapidly and accurately, they are relatively compli-
icated. In particular, adjustment of the dead space size is a critical issue and demands for a highly skilled user.

Knowing all these, our aim in this paper is to introduce a new simple, less time consuming, and yet relatively accurate alternative empirical model to estimate the gain and excess noise in GaAs/AlGaAs HAPDs. In this model, we have used a set of empirical formula to approximate a distribution function for the impact-ionization path length. We have also incorporated the effects of mole fraction, heterointerface position, as well as the electric field within the multiplication region in this distribution function. Using the newly obtained distribution function in conjunction with the rejection technique one can predict the position of each impact-ionization event. To adjust the fit parameters in our empirical model, we have trained a backpropagation neural network (BP-NN) [10].

Rest of this paper is organized as follows. In Section 2, we present the procedure by which the empirical model is obtained. We also briefly discuss about the BP-NN which is trained for optimizing the model fit parameters. In Section 3, we demonstrate the results for multiplication, and excess noise factors for both electrons and holes in GaAs/AlGaAs HAPDS, obtained by the empirical model. We also compare these results with those obtained by RPL model. Finally, we close this paper by conclusion, in Section 4.

2 Empirical model

By observation of the distribution functions of the impact-ionization path length for electrons and holes, obtained by Monte Carlo simulation for APDs and HAPDs, we have realized that general behavior of such distributions can be approximated by superposition of a linear and an exponential function. As an example, in Fig. 1, we illustrate the normalized distribution function for the ionization path length of electrons calculated by common Monte Carlo for

![Fig. 1. Normalized ionization events, as a function of ionization path length for electrons in a GaAs-APD with electric field $E = 900\,\text{kV/cm}$. The solid curve is the function obtained Monte Carlo simulation, whereas the dotted curve is the one predicted by the empirical model of (1).](image-url)
a GaAs APD with an electric field of $E = 900 \text{ kV/cm}$ (solid curve). Figure 1 also shows a well fitted distribution function (dotted curve) defined by

$$h(x) \cong \begin{cases} 
(x - a) \\
(b - a) \\
\exp[-\beta(x - b)] \\
0 
\end{cases} \quad \text{for } a \leq x \leq b \quad \text{for } b \leq x \leq c \quad \text{elsewhere}$$

(1)

where $x$ is the variable representing the ionization path length, $a$ is the characteristic length at which the linear function starts, $b$ is the characteristic length at which the function is unity, and $c$ is the characteristic length at which the exponential part of the distribution function falls down to 1% of its peak value, and the field dependent parameter $\beta$ is $\sim 3.3 \times 10^{-5} E \text{ (V/cm)} + 20.3$ for electrons and $\sim 3.3 \times 10^{-5} E \text{ (V/cm)} + 5.3$ for holes. Note that $E$ is the electric field (in terms of V/cm) within the multiplication region. The above mentioned characteristic lengths, for GaAs/Al$_x$Ga$_{1-x}$As HAPDs, are all functions of $E$, $\xi$ (0.3 $\leq \xi \leq 0.6$), and heterointerface relative position within the multiplication region of width $W_m$, $y$, are found to be

$$a = \left(\frac{\alpha_1 E + \alpha_2 \xi y}{100 E^{1.04}}\right)$$

$$b = \frac{(c - a)}{\alpha_3} + a$$

$$c = \left(\frac{\alpha_4 E + \alpha_5 \xi^{1.47} y}{\alpha_6 E^\gamma - E}\right)$$

(2)

where

$$y \equiv \frac{W_{WG}}{W_m} = \frac{W_{WG}}{W_{WG} + W_{NG}}$$

(3)

in which $W_{WG}$ and $W_{NG}$ are the widths of the wide and narrow gap portions of the multiplication region, respectively. Parameters $\alpha_i$ ($i = 1, 2, \ldots$, and 6) and $\gamma$, in (2) are adjustable fit parameters, whose values for electrons and holes in GaAs/Al$_x$Ga$_{1-x}$As HAPDs can be optimized by a trained BP-NN [10].

We use, $E$, $\xi$, and position as the inputs for the trained BP-NN, to obtain the optimized values for the model fit parameters $\alpha_i$ and $\gamma$, as the network outputs. Then, we feed these outputs into (2) to calculate the optimized values for the characteristic lengths $a$, $b$ and $c$. When comparing these predicted results with those obtained from MC simulation, the generated error signal, $(Er)$, for each parameter, is defined by [10]

$$Er = 0.5(R_p - R_{MC})^2$$

(4)

where $R_p$ is the predicted parameter obtained from (2) and $R_{MC}$ is the corresponding result obtained from MC simulation. The error gradient in (4) is also defined by

$$\frac{\partial Er}{\partial W_{i,j,k}} = \left(\frac{\partial Er}{\partial R_p}\right) \left(\frac{\partial R_p}{\partial out_{i,k}}\right) \left(\frac{\partial out_{i,k}}{\partial W_{i,j,k}}\right)$$

(5)
where \( W_{i,j,k} \) is the weight between the \( j \)th neuron in layer \( k-1 \) and the \( i \)th neuron in layer \( k \), \( \text{out}_{i,k} \) is the output of the \( i \)th neuron in the \( k \)th layer. The first partial derivative in (5) is \( R_R-R_{MC} \) and the third is the same as that of standard BP. The second partial derivative is computed individually for each of the unknown parameters to be estimated.

The best architecture for our neural network model is obtained with two hidden layers that contain 4 and 6 neurons, respectively. The activation function has the sigmoidal shape. Number of epochs is 40000 and learning rate is 0.15. The inputs \( E, y \) and \( \zeta \) are varied by intervals of \([0.33–1.2\,\text{MV/cm}]\), \([0.25–0.75]\) and \([0.3–0.6]\), respectively. The minimum error for learning set is obtained 0.028. The fit parameters obtained from this NN model is shown in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \alpha_3 )</th>
<th>( \alpha_4 )</th>
<th>( \alpha_5 )</th>
<th>( \alpha_6 )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td>2.044</td>
<td>1.011 \times 10^6</td>
<td>5.904</td>
<td>0.217</td>
<td>9.051 \times 10^6</td>
<td>0.038</td>
<td>1.289</td>
</tr>
<tr>
<td>Hole</td>
<td>2.068</td>
<td>1.236 \times 10^6</td>
<td>6.785</td>
<td>0.238</td>
<td>1.016 \times 10^7</td>
<td>0.032</td>
<td>1.293</td>
</tr>
</tbody>
</table>

Table I. The empirical model fit parameters obtained from a trained BP-NN.

Using these fit parameters, we can use the empirical model to predict the gain and excess noise factors for HAPDs. To perform such prediction, we inject an electron into the multiplication region, from wide band gap material. Using (1) and by rejection technique, impact-ionization path length, by which the electron travels before it gets involved in an ionization event, is selected. After the first impact-ionization event, an electron-hole pair is generated. Now, we should select the new ionization path length for each of these three carriers (two electrons and a hole), before getting involved in ionization events. Such a multiplication process is repeated until the entire generated carriers move out of the multiplication region. As a result, for any individual injection, indicated by a positive integer number, \( i \), there is a corresponding gain, \( M_i \), by which the total number of generated carriers moving out of the multiplication region can be counted. For a large number of injections, the gain average, \( M \), is called the multiplication factor. In our simulation, \( i = 1, 2, \ldots, 10000 \). The excess noise factor, \( F \), is defined by the ratio of the average of the square of \( M_i \) to the square of the gain average \([2]\):

\[
F = \frac{\langle M_i^2 \rangle}{M^2}
\]

3 Results

Figure 2 shows the results of multiplication factors, \( M \), obtained by the empirical model and compares them with those obtained by RPL model for GaAs/Al\( _{\xi} \)Ga\( {1-\xi} \)As HAPDs with multiplication width of \( W_m = 100 \)nm, Al mole fractions of \( \xi = 0.3, 0.45, \) and 0.6 and the heterointerface relative positions of \( y = 0.25 \) and 0.5.
Fig. 2. Multiplication Factors, $M$, as a function of electric field, $E$, for GaAs/Al$_{\xi}$Ga$_{1-\xi}$As HAPDs with $W_m = 100$ nm, $\xi = 0.3$, 0.45, and 0.6 and (a) $y = 0.25$ and (b) $y = 0.5$.

Fig. 3. Excess noise factor, $F$, as a function of multiplication factor, $M$, for GaAs/Al$_{\xi}$Ga$_{1-\xi}$As HAPDs with $W_m = 100$ nm, $\xi = 0.3$, 0.45, and 0.6 and (a) $y = 0.25$ (a) and (b) $y = 0.50$.

The corresponding values of the excess noise factors, $F$, calculated by the empirical model are illustrated in Fig. 3. In this figure, the results predicted by our proposed model are also compared with those obtained by RPL. Comparisons illustrated by both figures demonstrate the excellent capability of our model in predicting both gain and excess noise factors for GaAs/Al$_{\xi}$Ga$_{1-\xi}$As HAPDs.

4 Conclusion

We have presented a simple empirical model to predict distribution functions for ionization path length in GaAs/Al$_{\xi}$Ga$_{1-\xi}$As APDs, by which one can calculate the device gain and excess noise factor. Our empirical model includes a set of equations with fit parameters having information about heterointerface position and size of the electric field within the multiplication region, and Al mole fraction. This model not only predicts the result which
fit those obtained by RPL model in an excellent manner, but also it reduces the simulation time (on a 2.8 GHz P4), at least, by 17 and 100 times for RPL and common Monte Carlo methods, respectively.

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